CRACK TIP PLASTICITY UNDER MIXED MODE LOADING

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INTRODUCTION

Literature is abundant with applications of Dugdale strip yield zone [1] which models the plastic yield zone ahead of a stationary crack (see for example [2,3]) as well as the extending plastic yield zone ahead of a moving crack (see for example [4,5,6]). Studies on the Dugdale strip yield zone associated with mixed-mode crack extension, however, are few due to the lack of an analytical solution for a Z-shaped crack with prescribed variable crack pressure distribution. The limited experimental results in Reference [7] suggest that with suitable modifications, the Dugdale strip yield zone of a straight crack could model the elastic-plastic field surrounding the strip yield zone associated with mixed-mode crack-tip loading. In this paper, we report on a modified Dugdale strip yield model which was used to model the mixed-mode crack-tip loading associated with a slanted crack in a tension plate.

MIXED-MODE DUGDALE STRIP YIELD MODEL

The modified Dugdale strip yield model with two prescribed tangential forces, Q_u and Q_l, which model the residual stresses generated by the wake of an extending ductile crack is shown in Fig. 1. This simplified modeling of the residual stress field reproduced the trailing isochromatics left behind in the wake of rapidly extending ductile cracks in polycarbonate fracture specimens [6]. The mixed-mode crack tip loading is modeled by the variable normal surface tractions, σy, along the strip yield zone and two constant tangential forces, Q_u and Q_l, prescribed at the physical crack tip. Only the variable normal surface tractions are assumed to govern the length of a Dugdale strip yield zone, r_y. Details of the crack tip stress fields generated by the variable normal surface tractions as well as by the two tangential surface forces are given in Reference [8].

The final Dugdale strip yield model for mixed-mode crack tip loading is constructed by superposing the nonsingular mode I and mode II crack tip stress fields as well as that of the two tangential surface forces.

PHOTOELASTIC VERIFICATION

The abovementioned modified Dugdale strip yield model for mixed-mode crack tip loading was verified through an experimental program consisting of static photoelastic analysis of thin, 1.6 mm thick, and 50.8 mm wide polycarbonate tension specimens with slanted central notches. Standard tensile tests of annealed polycarbonate plates, from which the specimens were machined, showed that this material has a well-defined static yield point of 68.9 MPa, flows under stress and exhibits tensile instability with accompanying Lueder's bands. The starter crack consisted of a 14-mm length sawed crack with fatigued crack tips and was slanted 0, 30, 45 and 60 degrees to the specimen width direction. The central notch tensile specimens were loaded under increasing load and the changing photoelastic patterns surrounding the crack were photographed continuously. The residual isochromatics after complete unloading of the specimens were also recorded. Fig. 2 shows the isochromatics surrounding a straight crack, i.e., 0-degree central notched (CN) specimen. The apparent trifurcation of the two Dugdale strip yield zones is due to the two shadows casted by the surface dimplings due to the out-of-plane flow of the strip yield zone. Fig. 3 shows the isochromatics surrounding a 30-degree slanted central notched (SCN) specimen. The strip yield zones in this specimen as well as all other specimens were oriented in the specimen width direction. This inevitable orientation of the strip yield zone, regardless of crack slant angle, under mixed-mode crack tip loading differs with the fatigue cracks which emanated from a slanted crack in aluminum specimens and which gradually extended into the width direction [9]. The unambiguous straight strip yield zones, which were oriented in the width direction of the polycarbonate specimens, provided the physical insight of using a straight crack with a modified Dugdale strip yield zone which modeled the Z-shaped crack-strip yield zone.

Lengths of the Dugdale strip yield zones, r_y, ahead of the crack tips of the slanted cracks were measured directly from the photoelastic records. Fig. 4 shows the variations in normalized Dugdale strip yield zone length, r_y, with increasing applied load for 0, 30, 45 and 60-degree slanted central notched (SCN) specimens. Also shown for comparison is the theoretical length of the Dugdale strip yield zone for mode I crack tip loading in an infinite plate. The reasonable agreements between the theoretical strip yield zone length of the horizontal crack and the measured strip yield zone lengths of the horizontal as well as the slanted cracks are additional justifications for our use of the Dugdale strip yield zone associated with a straight crack. The crack tip stress field, which is represented by a polynomial function of the crack tip coordinates, of the modified Dugdale strip yield model was then fitted to the recorded isochromatics in the elastic region surrounding the strip yield zone. An overdeterministic least square fitting routine [10] was used to determine the coefficients of the polynomial stress function which was then used to generate the theoretical isochromatics of the modified Dugdale strip yield model. Isochromatics inside the plastic yield zone surrounding the crack tip were generated by converting the calculated elastic isochromatics to plastic isochromatics using an estimated strain-optic law [11]. Agreements between the experimental and theoretically generated isochromatics justified the existence of a modified Dugdale strip yield model as developed. Figs. 5 and 6 are computer generated graphical displays of the recorded and fitted isochromatics. Agreements between the recorded and theoretically generated isochromatics in the vicinity and ahead of the strip yield zone are good. Similar agreements between the recorded and theoretically generated isochromatics were observed in other 0 and 30-degree CN and SCN specimens, respectively, as well as in 45 and 60-degree SCN specimens. As expected, substantial discrepancies between the recorded and theoretically generated isochromatics are noted in the region behind the strip yield zone where the actual Z-shaped crack-strip yield zone is replaced by a theoretical straight crack of Fig. 1.

STABLE CRACK GROWTH CRITERION

The modified Dugdale strip yield model for mixed-mode crack tip loading was then used to model the elastic-plastic states of a stably growing crack emanating from the two ends of an initially slanted central notch in a polycarbonate tension specimen under increasing load. Physical parameters, which are computed through the modified Dugdale strip yield model, were then scanned to identify those which govern stable crack growth and the onset of ductile fracture. Two such parameters are the critical crack opening displacement (CTOD) and the critical crack tip opening angle (CTOA). Fig. 7 shows the CTOA variations during stable crack growth of a straight crack and a Z-shaped crack of 0- and 30-degree initial slant angles, respectively. Identical results were obtained for Z-shaped cracks with initial slant angles of 45- and 60-degrees. The relative constancy of the CTOA for mode I as well as mixed-mode crack tip loadings suggests that CTOA is a viable stable crack growth criterion for thin polycarbonate fracture specimens.

CONCLUSIONS

A modified Dugdale strip yield model which models the elastic-plastic state surrounding the strip yield zone has been presented. The CTOA extracted by using this modified Dugdale strip yield model remained constant during stable crack growth of straight and initially slanted cracks in thin polycarbonate tension specimens.

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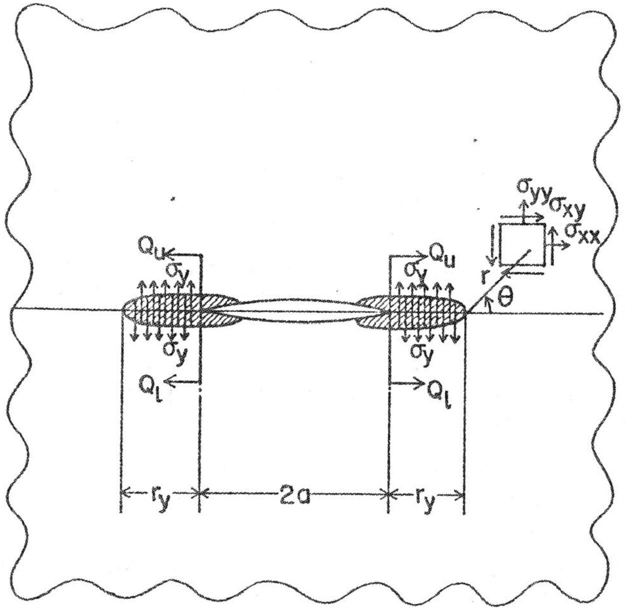

Fig. 1 Modified Dugdale Strip Yield Model

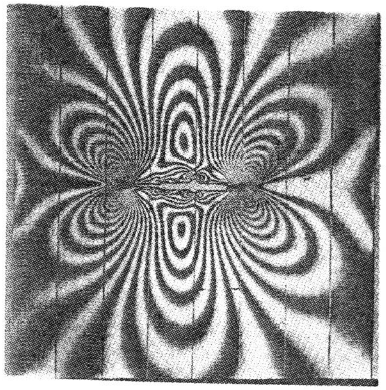

Fig. 2 Isochromatics of 0^o CN Specimen

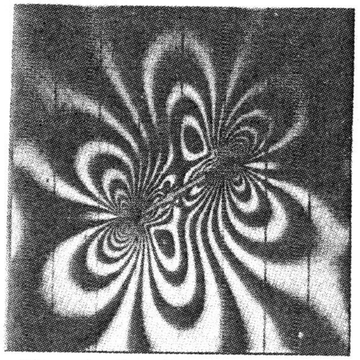

Fig. 3 Isochromatics of 30^o Crack Specimen

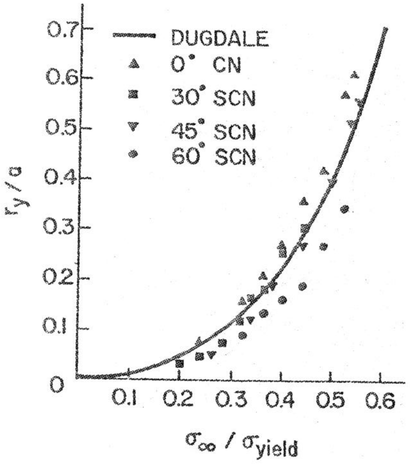

Fig. 4 Normalized Strip Yield Zone Length vs Applied Load

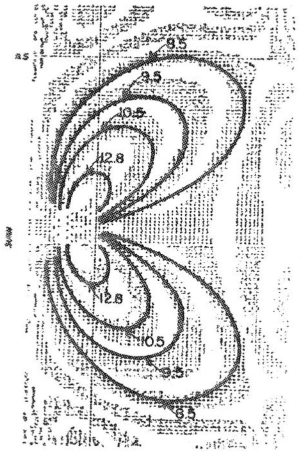

Fig. 5 Computer Generated and Actual (Solid Curves) isochromatics 0^o CN Specimen

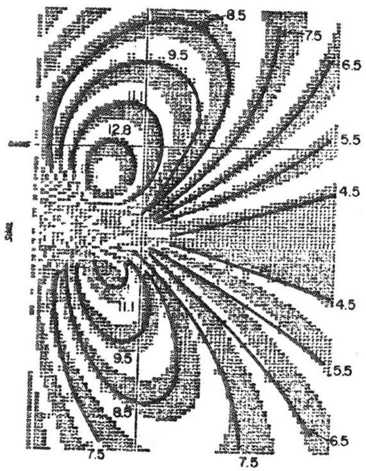

Fig. 6 Computer Generated and Actual (Solid Curves) Isochromatics 30° SCN Specimen

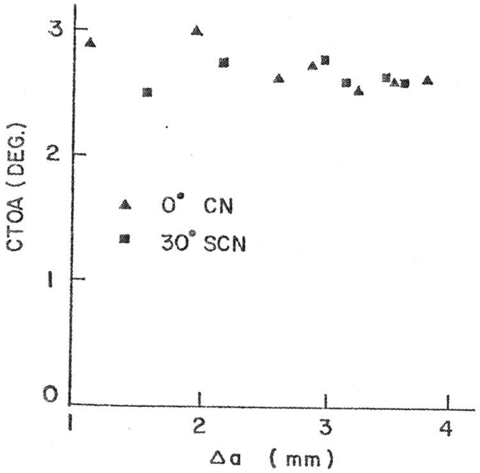

Fig. 7 CTOA During Stable Crack Growth of 0° CN and 30° SCN