

THE SHAPE VARIATION OF A SURFACE CRACK SUBJECTED  
TO REPEATED GENERAL ECCENTRIC TENSION

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ABSTRACT

The shape variation law of a surface crack subjected to general eccentric tension (GET) is studied in the present paper by experiments. An empirical formula describing this law is suggested. The comparison of this empirical formula with the test data from other references is made and agreement is excellent. The suggested empirical formula has also been used to predict the residual fatigue life of specimens. And some problems related to the estimation of fatigue life are discussed.

INTRODUCTION

Surface cracks are often encountered in welded structures and components. The study of the shape variation law of a surface crack is very important for the estimation of the residual fatigue life of such structures and components.

Although many testing results have been published, this problem is far from being resolved, because of the complexities and difficulties in testings and measurements. Many results published in recent years were related to axial tension (AT) and pure bending (PB). Actual stress field in components is more complex. In the present paper a study of the shape variation law of a surface crack under general eccentric tension (GET) fatigue loading has been made. The AT and PB loadings are considered as two special cases of GET with  $H = 0$  and  $\infty$ .

The functional relationship of  $a$ ,  $c$ ,  $t$ , and  $H$  is designated as shape function, where  $a$  = depth,  $c$  = half length of the surface crack,  $t$  = plate

thickness and  $H = e/t$ ,  $e =$  eccentricity. The shape function represents the variation of aspect ratio ( $\zeta = a/c$ ) of a surface crack during fatigue crack growth.

#### THE SHAPE FUNCTION OF A SURFACE CRACK

The total number of specimens with surface crack is 60; each group of 20 specimens being tested under AT, PB and GET loadings. The detail of testing procedures is given in Ref. [1].

Two assumptions have been made in the present paper:

- a) the material of the specimen is isotropic, and
- b) the initial sizes of the surface crack ( $a_0, c_0$ ) are small enough in comparison with the plate thickness  $t$ .

From the experimental data taken at constant amplitude fatigue tests, it was found that on  $\eta - \xi$  plane ( $\eta = c/t$ ,  $\xi = a/t$ ) through the data points, corresponding to the beach marks on the fracture of specimen, a best-fit curve could be drawn, and the equation of this curve is called the shape function of a surface crack (Fig. 1). If the experimental data were drawn on  $\zeta - \xi$  plane and the best-fit curve degenerated into a straight line (Fig. 2), evidently, it is appropriate to write the shape function in linear equation by using linear regression of the experimental data

$$\zeta = A + B\xi \quad (1)$$

Under out-of-plane PB loading, from the test data by linear regression, the coefficients A and B are obtained

$$A = 1.067 \quad \text{and} \quad B = -1.05$$

While under AT loading, the coefficients A and B are

$$A = 1.032 \quad \text{and} \quad B = -0.188$$

For GET loading, the coefficient A could be considered as a constant and equal to the mean of the A's for PB and AT, i.e.  $A = 1.05$ , but the coefficient B should be a function of H. Thus, the shape function for GET loading has the form

$$\zeta = 1.05 + B(H)\xi \quad (2)$$

From testing data of a series of GET specimens with different H, using eq. (2), the corresponding values of B(H) are determined, and the B(H) versus H plot has been drawn (Fig. 3). This plot suggested that B(H) could be expressed as

$$B(H) = -(1.046H^m + 0.188D)/(H^m + D) \quad (3)$$

such that for the special cases of PB ( $m = \infty$ ) and AT ( $m = 0$ ), the coefficient B reduced to -1.05 and -0.188 respectively.

In order to determine the exponent  $m$  and coefficient D in eq. (3) experimentally, this equation had better be rewritten in a linear form

$$y = \alpha + mx \quad (4)$$

here  $\alpha = \log(1/D)$ ,  $x = \log H$  and  $y = \log [(B - 0.188)/(1.05 - B)]$ .

Using the above expression to treat the testing data of GET specimens (Fig. 3) by linear regression, the following results are obtained

$$\alpha = 0.92, \quad D = 0.12, \quad \text{and} \quad m = 1.07$$

Thus, the coefficient B(H) in GET loading is expressed as

$$B(H) = -(1.05 H^{1.07} + 0.0226)/(H^{1.07} + 0.12) \quad (5)$$

and it leads, ultimately, to an expression for the shape function of a surface crack under GET loading (Fig. 2)

$$a/c + [(1.05H^{1.07} + 0.0226)/(H^{1.07} + 0.12)]a/t = 1.05 \quad (6)$$

or, for convenience of application (Figs. 1,4), it can also be expressed in another form

$$a/t = 1.05 / [(1.05H^{1.07} + 0.0226)/(H^{1.07} + 0.12) + 1/(c/t)] \quad (7)$$

A comparison of shape function proposed in the present paper with the test data from other references is made, and shown in Figs. 4 and 5 with good agreement.

## DISCUSSION

1) From Fig. 1, it is evident that all curves with different  $H$  converge to a straight line  $\xi = \eta$  when  $\zeta \leq 0.1$ . It means that the shape of a small surface crack ( $a \ll t$ ,  $c \ll t$ ) can always be treated as a semicircle ( $a/c=1$ ) during fatigue crack growth. This conclusion is very important for fatigue life prediction of a small surface crack.

2) In Fig. 1 the curves of GET loading with  $H \geq 2$ , coincide with the curve of  $H = \infty$ . Therefore, in engineering practice, for components under GET loading with  $H \geq 2$ , they could be treated as under PB loading and the shape function of a surface crack might be expressed by eq. (2), with  $B = -1.05$ .

3) From the testing results of welded specimens it has been found that the aspect ratio ( $a/c$ ) of a surface crack in the region of weld increased more rapidly than that in the base metal. The following empirical relationship is proposed

$$(a/c)_{B.M.} = (0.85 \sim 0.95)(a/c)_W \quad (8)$$

### SHAPE FUNCTION APPLIED TO PREDICTION OF FATIGUE LIFE

The fatigue crack growth rate of a surface crack along plate-width is expressed as

$$dc/dN = \alpha(\Delta K)^m \quad (9)$$

from which, the fatigue life may be predicted by

$$N = \int_{c_0}^{c_f} \frac{dc}{\alpha(\Delta K)^m} \quad (10)$$

For GET loading condition, eq. (10) could be written as

$$N = \int_{c_0/t}^{c_f/t} \left[ \frac{(a/t)^{1/2}}{\phi} \right] \left[ \frac{(\alpha_3 + 6H\alpha_4)}{(6H+1)} \right]^{-m} d(c/t) \quad (11)$$

The integral in the right hand side of eq. (11) should be computed by numerical integration. As the surface crack is considered to be a semicircle (i.e.  $a/c = 1$ ) when  $a/t \leq 0.1$ , this integral may be computed in two parts, i.e.

$$N = N_0 \pm N_1 \quad (12)$$

To calculate the first part,  $N_0$ , the aspect ratio is  $a/c = 1$  thus,  $\phi = \frac{1}{2}\pi$ ,  $\alpha_3 = 1.22$  and  $\alpha_4 = 0.766$ , then

$$N_0 = \left( \frac{1}{4\pi} \right)^{\frac{m}{2}} \left[ \frac{(6H+1)}{(1.22+4.6H)} \right]^m \times (0.316-c/t)^{1-\frac{m}{2}} \times 4/(2-m) \quad (13)$$

As the first part of the fatigue life  $N_0$  already obtained by eq. (13), the numerical integration of  $N$  is simplified.

Using the shape function proposed in the present paper we have predicted the residual fatigue life of specimen 8 AS-1 in Ref.[3] under AT loading with  $a_0 = 6.09$  mm and  $a_f = 14.44$  mm. The predicted residual life is  $11.4 \times 10^3$  cycles while the measured life is  $10.3 \times 10^3$ . The error is only 10.5%, which is believed mainly due to the scatter of the coefficient  $\alpha$  and exponent  $m$ . By the same process, we have treated the specimen M-1 under PB loading in Ref.[1] with  $a_0 = 1.106$  mm and  $a_f = 4.806$  mm. The predicted residual life is  $16.6 \times 10^3$  cycles with error of 2.6% only, compared with the measured life of  $17.0 \times 10^3$  cycles. Other test results in Ref.[3] have also been verified, and it is found that the shape function proposed in the present paper is sufficiently accurate for predicting the residual life of specimens with surface cracks.

## CONCLUSIONS

- 1) Semiellipse is a good mathematical model for a growing surface crack.
- 2) The shape function of a surface crack in a plate under GET loading is recommended as

$$a/c + \left[ \frac{(1.05H^{1.07} + 0.023)}{(H^{1.07} + 0.12)} \right] a/t = 1.05$$

- 3) A semicircle is the stationary shape of a small surface crack ( $a/t \leq 0.1$ ) during fatigue crack growth.
- 4) Before coalescence of two coplaner adjacent surface cracks, eqs.(6) and (7) could be used as shape functions under GET, AT and PB fatigue loading.

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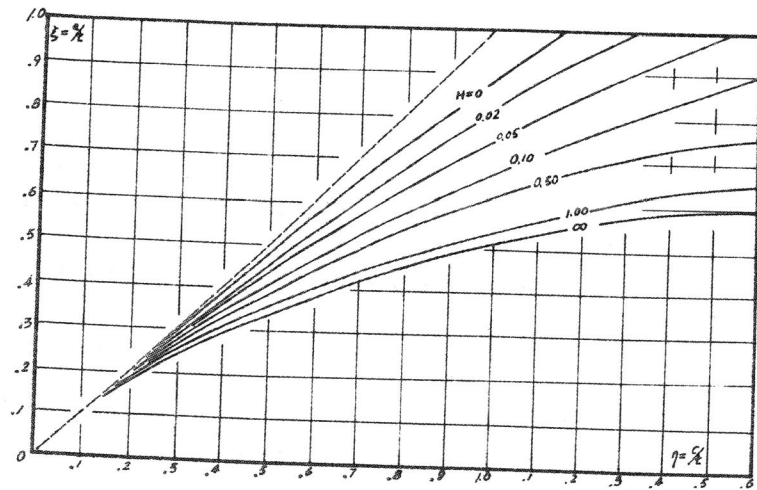


Fig. 1

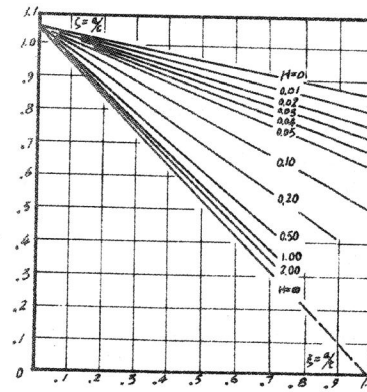


Fig. 2

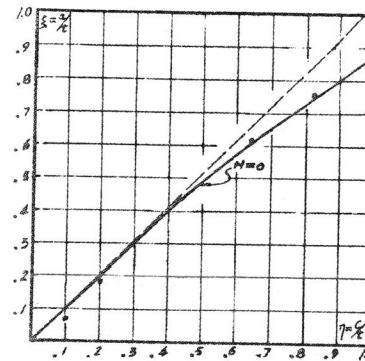


Fig. 4

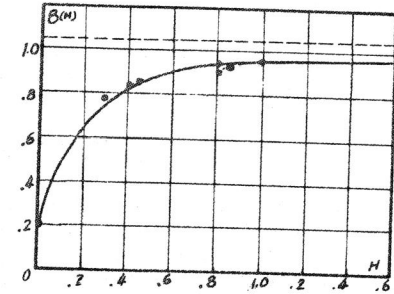


Fig. 3

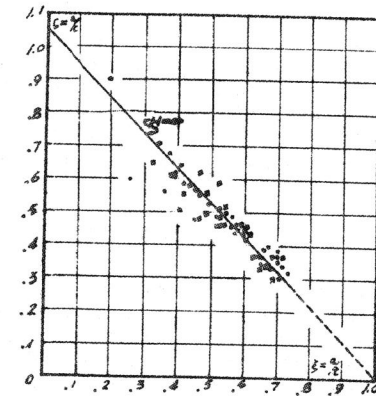


Fig. 5