

## RESIDUAL STRENGTH ANALYSIS OF RIVETED STIFFENED PANELS

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### INTRODUCTION

Fracture analysis of riveted stiffened panels plays an important part in the damage tolerance design of aircraft structures. In this paper, a method for analyzing residual strength of riveted stiffened panels is proposed. It is based on analytic method considering the plasticity of rivets and stringers. By using  $K_{Ic}$  criterion, the carrying capacity or damage tolerance can be found.

Also, the computations (including finite element method<sup>[6]</sup>) and experiments on residual strength are done for a number of cracked panels with stringers of different types. The results show that experimental data are in excellent agreement with the computed results.

In the computations, the Kron's Method of Tearing is extended to include plasticity iteration with the result that computing efficiency is improved.

### FINDING RIVET FORCES BY USING PLASTICITY ITERATION

First the equations of compatibility between cover sheet and stringer are established for relative displacement of one point with respect to another, each group of two points being centers of two rivets. In establishing the equations, it is assumed that buckling damage near the crack tip is negligible and that rivet forces are considered as concentrated forces applied at the centres of the rivet holes. The equations can be written as follows:

$$\left\{ \begin{aligned} & \sum_{j=1}^{\ln} A_{ij} Q_j + \frac{Y_i}{A_s E_{si}} \sum_{j=1}^{n+i-1} Q_j + f_{vi} Q_i - B_i S^* + \\ & \frac{S^* E_{si}^{(1)}}{E E_{si}} Y_i = 0 \quad (i=1, n+1, 2n+1, \dots) \end{aligned} \right. \quad (1)$$

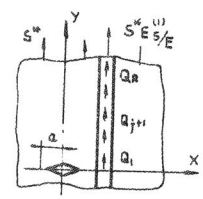


Fig.1 between-stringers crack

$$\left\{ \begin{aligned} & \sum_{j=1}^{\ln} (A_{ij} - A_{i-1,j}) Q_j + \frac{Y_i - Y_{i-1}}{A_s E_{si}} \sum_{j=i}^m Q_j + \\ & + f_{vi} Q_i - f_{vi-1} Q_{i-1} \\ & - (B_i - B_{i-1}) S^* + \frac{S^* E_{si}^{(1)}}{E E_{si}} \times \\ & \times (Y_i - Y_{i-1}) = 0 \end{aligned} \right. \quad (2)$$

where  $m=n$  for  $2 \leq i \leq n$ ;  $m=n+2$  for  $n+2 \leq i \leq 2n, \dots$

where  $l$  is the number of stringers,  $n$  is the number of rivets for every stringer.

The above equations are applicable to intact stringers. When the  $i$ th stringer fails Eq.(3) will be used instead of the corresponding one in Eqs.(1)

$$S^* \frac{A_s E_{si}^{(1)}}{E} + \sum_{j=(i-1)n+1}^{in} Q_j = 0 \quad (3)$$

For details about the parameters in Eqs.(1) and (2) see Ref.[1]. It is important to notice differences between under-stringer crack and between-stringer crack.

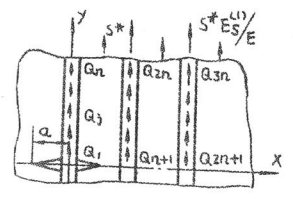


Fig.2 under-stringer crack

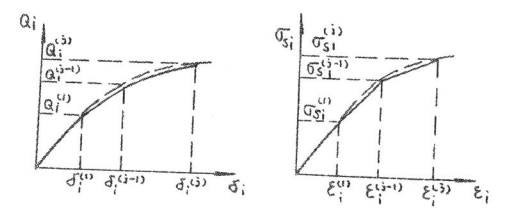


Fig.3 compliance curves for rivet and stringer

The rivet forces are obtained from Eqs.(1) and (2) by using increment method through plasticity iteration. The compliance of each ( $i$ th) rivet and stringer section can be written as follows:

$$f_{vi}^{(j)} = \frac{\delta_i^{(j)} - \delta_i^{(j-1)}}{Q_i^{(j)} - Q_i^{(j-1)}} \quad (4)$$

$$E_{si}^{(j)} = \frac{\sigma_i^{(j)} - \sigma_i^{(j-1)}}{\epsilon_i^{(j)} - \epsilon_i^{(j-1)}} \quad (5)$$

FAILURE CRITERION AND RESIDUAL STRENGTH CALCULATION

1. Calculation of Fracture Characteristic Parameters

The fracture characteristic parameters can be obtained as follows:

The reduced coefficient of stress intensity factor  $C$  is

$$C = K/S^* \sqrt{\pi a} \quad (6)$$

where  $K$  is the stress intensity factor<sup>[1]</sup> and is given by

$$K = S^* \sqrt{\pi a} - \sum_{j=1}^{\ln} \bar{K}_j Q_j \quad (7)$$

When unstable crack propagation occurs in sheet, gross stress  $S_c$  is given by

$$S_c = K_c / C \sqrt{\pi a_c} / \sec \pi a_c / B \quad (8)$$

When  $i$ th stringer fails gross stress  $S_{sci}$ , is given by

$$S_{sci} = \sigma_{bsi} \psi_i / L_i \quad (9)$$

where the  $i$ th stringer overloading coefficient  $L_i$  is given by

$$L_i = 1 + \frac{E}{S^* A_s E_{si}^{(1)}} \sum_{j=(i-1)n+1}^{in} Q_j \quad (10)$$

2. Calculation of the R-Curve of Stiffened Panel

The relationship curve between the applied stress  $S_r$  and the crack length  $a_r$  during the crack propagation process is called the R-curve of

stiffened panel. It can be calculated from the  $K_{R^0} \sqrt{a_0}$  curve of sheet material and initial damage by using iteration method.

After obtaining the  $S_{R^0} \sqrt{a_0}$  curve of sheet [1], the corresponding stress of stiffened panel  $(S_R)_i$  is solved from Eqs.(1), (2) and Eq.(6):

$$(S_R)_i^{(j)} = (S_R)_i^0 / (C_R)_i^{(j)} \quad (11)$$

where  $j$  denotes the  $j$ th iteration.

$$\text{If } |(S_R)_i^{(j)} - (S_R)_i^{(j-1)}| \leq \epsilon$$

( $\epsilon$  is given a small value) (12)

then  $(S_R)_i$  can be found, namely,  $(S_R)_i = (S_R)_i^{(j)}$ . Curve-fitting all the point  $(S_R)_i, (a_R)_i$  gives the R-curve of stiffened panel.

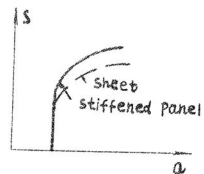


Fig.4 The R-curve of stiffened panel

### 3. The $K_R$ Criterion and Failure Criterion

The failure of stiffened panel is closely related to sheet, stringers and rivets. So we propose the following failure criterion, It is based on the relationship between failure curves of rivets, stringers and R-curve of stiffened panel. We call it  $K_R$  fracture criterion (Fig.5). Keeping in mind engineering considerations, we can select the failure criterion from among the following cases:

- Unstable Crack propagation of sheet
- Failure of one stringer
- Failure of two stringers
- Failure of two rivets

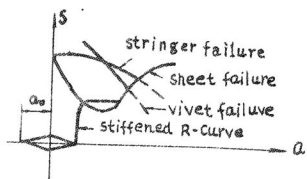


Fig. 5  $K_R$  criterion of Stiffened Panels

### 4. The Computation of Residual Strength of Riveted Stiffened Panels

The fail-safe load  $S_c$  and the critical damage  $a_c$  can be found when the initial damage  $a_0$  is given or  $a_0$  and  $a_c$  can be found when the fail-safe load is given, by using  $K_R$  criterion. The simplified flow chart diagram for residual strength calculation of stiffened panel is presented in Fig.7. The detailed calculation procedure is shown in Ref.[1]

### NUMERICAL EXAMPLES AND ANALYSIS OF RESULT

We have computed and found experimentally the residual strength of the lower

panels of some aircraft wings. A number of cracked panels are tested with sizes 300mm×560mm, 640mm×1300mm, and 720mm×1300mm. Rectangular, L-, and Z-stiffeners are used. Both photographic and compliance methods are employed. The experimental setup is shown in Fig.6[5]. Part of the calculated and experimental results is listed in the following tables and Fig.8.

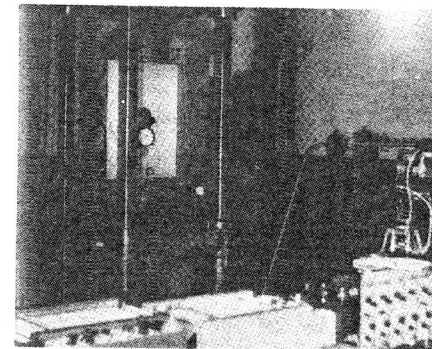


Fig.6 Experimental setup

Fig. 8 shows that when considering elasticity only one stringer is fails. It is not in agreement with experiment. So the plasticity of rivets and stringers must be considered. And all the results (see tables ) show that experimental data of of panels are in excellent agreement with the plasticity computations. The proposed method of residual strength analysis is satisfactory for purposes of engineering application.

Moreover, the method requires comparatively small computer storage and gives comparatively good accuracy so the proposed method is acceptable.

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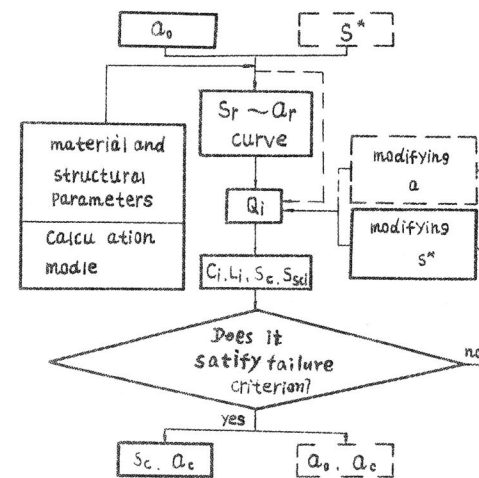


Fig.7 the flow chart diagram for residual strength calculation of stiffened panel

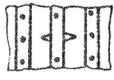
Table (1) The results of fail-safe load  $S_c$  and critical damage  $a_c$  ( $2a_0=60\text{mm}$ )

computation procedure adjusted to allow for stringer failure				one stringer has failed prior to computation	
$S_c(\text{kg}/\text{mm}^2)$		$2a_c(\text{mm})$		$S_c(\text{kg}/\text{mm}^2)$	$2a_c(\text{mm})$
analysis	test	analysis	test	analytic method	
28.77	28.78	91.78	91.33	24.83	105.94
failure of one stringer				failure of two stringers	



Table (2) The results of tolerable damages ( $S^*=24.5\text{kg}/\text{mm}^2$ )

computation procedure adjusted to allow for stringer failure		
	analytic method	finite element method
$2a_0(\text{mm})$	140	135
$2a_c(\text{mm})$	250	243
failure of one stringer		



Note: for the results of fail-safe load  $S_c$  and critical damage  $a_c$  see Fig.7

Table (3) The results of fail-safe load  $\sigma_c$  and critical damage  $a_c$  ( $2a_0=60\text{mm}$ )

computation procedure adjusted to allow for stringer failure			
$S_c(\text{kg}/\text{mm}^2)$		$a_c(\text{mm})$	
analytic method	test value	analytic method	test value
18.37	18.92	75.2	76.5
unstable crack propagation of sheet			

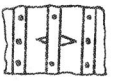


Table (4) The results of fail-load  $S_c$  and critical damage  $a_c$  ( $2a_0=192\text{mm}$ )

computation procedure adjusted to allow for stringer failure			
$S_c(\text{kg}/\text{mm}^2)$		$a_c(\text{mm})$	
finite element method	test	finite element method	test
16.9	17.8	153	153.3
unstable crack propagation of sheet			

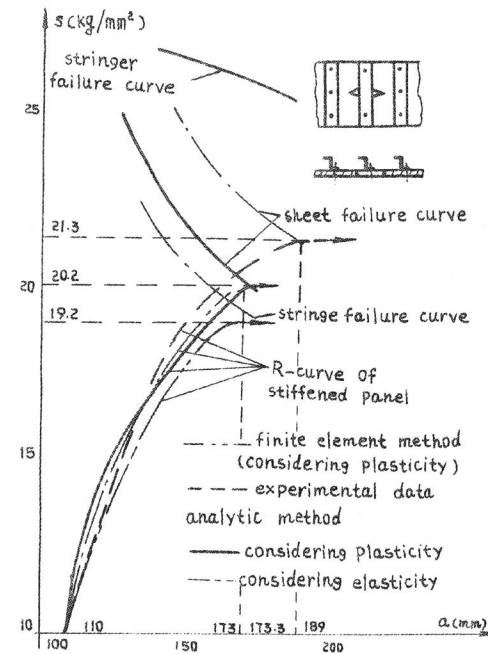


Fig. 8 Residual strength curve for the lower panel of a transport airplane

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