APPLICATION OF THE EQUIVALENT DISPLACEMENT OF CRACK TIP TO PRESSURE VESSEL WITH SLANT CRACK

Li Zezhen (李泽震) Chen Shuyi (陈树义)
General Machinery Institute, China
Zhou Zegong (周则恭)
Taiyuan Heavy Machinery Institute, China

ABSTRACT

The equivalent displacement of crack tip δ_E is used as the descriptive parameter for mixed mode in a pressure vessel with slant crack. The resulting formula under the linear elastic condition is in between S criterion and $\sigma_{\theta\max}$ criterion. Other formulas that we got for the vessel crack initiation pressure, burst pressure, bulge effect and flow stress under elastic plastic condition are in accord with the experimental result of some pressure vessel with slant notch. By means of these methods, we have assessed a ϕ1800mm and a ϕ1400mm pressure vessel with hundreds slant cracks and have made it on service successfully since 1976.

I. THE LINEAR ELASTIC CASE

The equivalent displacement of crack tip δ_E is regarded as the vector sum of the opening displacemnet of crack δ_0 (mode I) and slide displacement of crack δ_s (mode II), that is:

$$\delta_E = \sqrt{\delta_0^2+\delta_s^2} \tag{1}$$

when σ/σ_y and $\tau/\tau_y<0.55$, we have got[1]

$$\left[\left(\frac{\sigma}{\sigma_y}\right)^2 K_I^4 + \left(\frac{\tau}{\tau_y}\right)^2 K_{II}^4\right]^{\frac{1}{4}} = K_{1c} \tag{2}$$

As shown in the Fig. 1, the cylindrical vessel with a slant crack is acted by internal pressure

$$K_I = \sigma_\theta\sqrt{\pi a} = \frac{PR}{2t}\sqrt{\pi a}\,(1+\sin^2\theta) \qquad (3)$$

$$K_{II} = \tau_\theta\sqrt{\pi a} = \frac{PR}{2t}\sqrt{\pi a}\,\sin\theta\cos\theta \qquad (4)$$

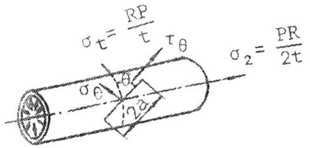

Fig. 1

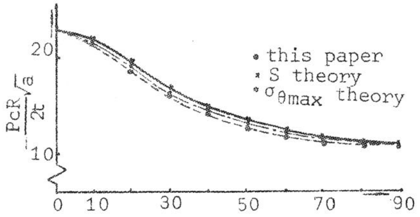

Fig. 2

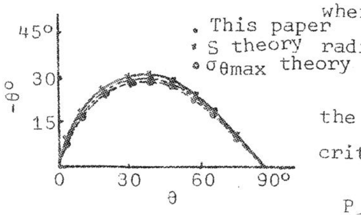

Fig. 3

where t is the thickness of wall, and R is the radius of cylinder.

Substituting formula (3–4) into (2), we get the calculating formula for the non-dimensional critical pressure:

$$\frac{P_cR}{2t}\sqrt{a} = [(1+\sin^2\theta)^2 + 4\sin^2\theta\cos^2\theta(1+\sin^2\theta)^2 + 3\sin^4\theta\cos^4\theta]^{-\frac{1}{4}}\,\frac{K_{1c}}{\sqrt{\pi}} \qquad (5)$$

The value determined by formula (5), S criterion and max. tangential stress ($\sigma_{\theta max}$) theory are shown in Fig. 2. One can find that the value obtained by formula (5) is between S criterion and $\sigma_{\theta max}$ criterion. The angle of initiation crack:

$$\theta_0 = tg^{-1}\frac{\sqrt{3}\tau_\theta}{\sigma_\theta} = tg^{-1}\frac{\sqrt{3}\sin\theta\cos\theta}{1+\sin^2\theta} \qquad (6)$$

It is in accord with S criterion and $\sigma_{\theta max}$ criterion, (Fig. 3.)

II. THE ELASTIC PLASTIC CASE

For elastic plastic case, it is obvious that critical pressure can not be calculated by formula (5). But,

$$\delta_0 = \frac{\delta\sigma_{py}a}{\pi E}\,\ln\sec\frac{\pi\sigma}{2\sigma_{py}} \qquad \delta_S = \frac{q\tau_{py}a}{\pi E}\ln\sec\frac{\pi\tau}{2\tau_{py}} \qquad (7)$$

where σ_{py}, τ_{py} are tensile and shear yield stress in plastic zone.

After substituting it into formula (1)

$$\delta_E = \frac{8a}{\pi E}\sqrt{(\sigma_{py}\ln\sec\frac{\pi\sigma}{2\sigma_{py}})^2 + (\tau_{py}\ln\sec\frac{\pi\tau}{2\tau_{py}})^2} = \delta_c \qquad (8)$$

The equivalent stress σ_p^* satisfies Von Mises yield condition

$$\sigma_p^* = \frac{PR}{2t}\,[(1+\sin^2\theta)^2 + 3\sin^2\theta\cos^2\theta]^{\frac{1}{2}} \qquad (9)$$

In view of consistence of sizes of plastic zone produced by tensile stress and shere stress, one can obtain $\frac{\sigma_p^*}{\sigma} = \frac{\sigma_y}{\sigma_{py}}$, $\frac{\sigma_p^*}{\tau} = \frac{\sigma_y}{\tau_{py}}$ then, $\frac{\sigma}{\sigma_{py}} = \frac{\tau}{\sigma_{py}} = \frac{\sigma_p^*}{\sigma_y}$

Formula (8) can be simplified as follow (considering the coefficient of bulge effect M_θ)

$$\delta_E = \frac{8\sigma_y a}{\pi E}\,\ln\sec\frac{\pi}{2}\,\frac{M_\theta\sigma_p^*}{\sigma_y}, \quad \sigma_y' = \sqrt{\sigma_{py}^2 + \sigma_{py}^2} \qquad (10)$$

So the pressure for crack initiation is

$$P_c = \frac{2t\cdot 2}{R\pi}\,\frac{\sigma_y'}{M_\theta\sqrt{(1+\sin^2\theta)^2+3\sin^2\theta\cos^2\theta}}\,\cos^{-1}\exp(-\frac{\pi E\delta_c}{8\sigma_y a}) \qquad (11)$$

For a straight crack we have

$$P_c = \frac{4t}{R\pi}\,\frac{\sigma_y}{M_\theta\sqrt{(1+\sin^2\theta)^2}}\,\cos^{-1}\exp(-\frac{\pi E\delta_c}{8\sigma_y a}) \qquad (12)$$

or

$$\delta_c = \frac{8\sigma_y a}{\pi E}\,\ln\sec\frac{\pi}{2}\,\frac{M_\theta(P_cR/t)}{\sigma_y} \qquad (13)$$

Using the following formulas of linear interpolation and curvilinear interpolation, we obtain the coefficient of bulge effect M_θ of slant crack

$$M_\theta = \{1+[0.32+(1.61-0.32)\frac{2\theta}{\pi}]\cdot\frac{a^2}{Rt}\}^{\frac{1}{2}} \qquad (14)$$

$$M_\theta = [1+0.32(1+4\sin\theta)\frac{a^2}{Rt}]^{\frac{1}{2}} \qquad (15)$$

III. BURST PRESSURE

The calculating formula of vessel burst stress for plastic case is

$$\sigma_F = \frac{\sigma_0}{M_\theta}, \quad \sigma_0 = \sigma_y + \frac{1}{2}(\sigma_u - \sigma_y)\frac{\sigma_u}{\sigma_y} \tag{16}$$

where σ_0 is flow stress. The calculating formula of flow stress above is the emprical formula we have got[2]. It is in accord with the data of burst test of more than 180 vessels made in China GMRI and abroad.

The burst pressure of vessel with slant crack is

$$P_F = \frac{2t\frac{\sigma_0}{M_\theta}}{R[(1+\sin^2\theta)^2+(\sin\theta\cos\theta)^2]^{\frac{1}{2}}} \tag{17}$$

IV. THE EXPERIMENTAL STUDY OF THE PRESSURE VESSEL WITH SLANT CRACK

Do these calculating formulas accord with practice? For this, we made a group of model vessels from 18MnMoNb steel. The condition of their heat treatment is classified into two classes, such as normallize temper, and QT-temper. The chemical composition, mechanical properties and fracture toughness of base material and weld seam are shown in Table 1 and Table 2.

Table 1 Chemical Composition

Material	Composition %						
	C	Si	Mn	M_0	P	S	Nb
Weld Seam	0.14	0.28	1.40	0.50	0.017	0.014	0.026
Base Material	0.21		1.50	0.51	0.01		0.041

Table 2 Mechanical Properties and Fracture Toughness

Condition of Heat Treatment	σ_y kg/mm²	σ_u kg/mm²	δ_c (mm) test results	min value	mean value
Q-T Base Material	61.33	73.33	0.095, 0.092	0.092	0.0935
Q-T Electroslag Weld Seam	65.33	75.33	0.075, 0.087, 0.087 0.076, 0.072, 0.088	0.072	0.0808
Q-T Manual Well Seam	65.50	77.50	0.143, 0.155	0.143	0.149
Normalized Base Material	45.83	61.00	0.088, 0.11	0.088	0.099
Normalized Electroslag Weld Seam	43.67	62.83	0.077, 0.074	0.074	0.075

Table 3

Cylinder No.	θ	a	R	t	δ_c	Vessel CD(δ_c)	Crack initiat pressure			burst stress			I mode Displacement		II mode Displacement		Equivalent Displacement	
	(°)	mm	mm	mm	mm	formula (13) by AE	for-mula (12) P_c	formula (11) P_c	P_c deter-mined by AE	determined by AE	formula (15) (17)	formula (14) (17)	Calculated Value	Measured Value	Calculated Value	Measured Value	Calculated Value	Measured Value
C401	0	40.6	41.4	3.6	0.076	0.0850	180	180	190	25.59	27.00	27.00	0.085	0.079	0	0	0.085	0.079
C402	15	31.0	41.3	3.5	0.076	0.0716	183	179.1	180	25.57	24.94	27.00						
C403	28.3	21.8	41.5	3.43	0.076	0.0580	207	191	185	28.90	28.55	31.14	0.070	0.071	0.024	0.014	0.074	0.072
C404	45	14.5	41.33	3.54	0.076	0.0672	205	232	235	35.70	34.69	37.40	0.092	0.060	0.030	0.040	0.097	0.072
C708	90	9.5	41.44	3.53	0.076	0.0787	227	227	228	44.37	40.85	41.14	0.077	0.074	0	0	0.077	0.074
CM3	90	10.0	41.34	3.62	0.099	0.0653	249	249	220	48.53	38.85	38.85	0.062	0.074	0	0	0.062	0.074
T508	15	33.0	41.38	3.65	0.081	0.0599	194	189	175	31.54	26.43	28.80	0.070	0.079	0.016	0.016	0.072	0.081
T509	29.5	33.0	41.41	3.55	0.081	0.0785	170	159	165	31.25	25.04	27.88	0.096	0.095	0.034	0.025	0.102	0.098
T510	45	22.6	41.32	3.48	0.081	0.0787	195	185	190	35.88	30.78	33.86	0.093	0.063	0.031	0.023	0.098	0.067
T512	60	17.0	41.42	3.60	0.081	0.0910	225	219	225	39.67	36.65	39.30	0.092	0.145	0.023	0.023	0.095	0.146
T101	90	13.75	41.45	3.50	0.081	0.0770	214	214	210	46.50	40.40	40.40	0.077	0.089	0	0	0.077	0.089
TM3	90	13.5	41.34	3.52	0.094	0.0493	223	223	180	46.74	39.20	39.20	0.054	0.058	0	0	0.054	0.058

Artificial notchs are made on the model vessel which are at an angle of 0°, 15°, 30°, 45°, 60°, 90° respectively. We cut through the notchs with electric spark first, and then cut 5–6mm narrow notchs at the both end of the model vessel with a saw blade of 0.1mm thickness. We increase pressure slowly and measure the initiation crack pressure of the model vessel by accustic emission techniques.

The comparision of the experimental results with its design value is shown in Table 3.

(1) The formula of non-dimensional critical internal pressure and angle of crack initiation under the condition of linear elasticity is between S criterion and $\sigma_{\theta max}$ criterion.

(2) Under the elastic-plastic condition the design value is basically in accord with the actual measured value. It shows that the formulas mentioned in this paper can be used to estimate the pressure of crack initiation and the bursting pressure of the pressure vessel, and the crack tip equivalent displacement of the vessel.

(3) Mode I and mode II displacements all have an effect on crack initiation. According to the formulae we obtain δ_0 and δ_s in accord with with the results measured by the feeler pin type cip-gage as shown in Table 3.

REFERENCES

[1] Li Zezhen, Chen Shuyi and Xu Peizu, The CD theory of general crack tip displacement and it's application in pressure vessel, J. of Chemical and Petroleum Machinery, No. 2(1982).

[2] Li Zezhen, He Churen and Chen Shuyi. The calculating formula of burst pressure for pressure vessel with crack, J. of Chemical and General Machinery, No. 2(1982).