PROBABILISTIC FRACTURE MECHANICS OF FATIGUE CRACK PROPAGATION AND ITS APPLICATION TO DAMAGE TOLERANCE DESIGN

Masahiro Ichikawa, Mituo Hamaguchi and Takeo Nakamura
Dept. of Mech.Eng., Univ. of Electro-Communications, Japan

In order to establish the probabilistic fracture mechanics approach to damage tolerance design, the statistical characteristics of m and C in the fatigue crack propagation law $da/dN=C(\Delta K)^m$ was investigated experimentally on 2024-T3 Al alloy. It was shown that (i) m and logC follow normal distributions, and (ii) strong negative correlation exists between m and logC as expressed by $C=C_0K_0^{-m}$ where C_0 is a log-normally distributed random variable and K_0 is a constant. The distribution of the fatigue life as calculated based on the above characteristics (i) and (ii) was in good agreement with the experimental distribution.

INTRODUCTION

It is useful to combine fracture mechanics and reliability engineering for refining fracture mechanics approach to design. This combined approach is often called probabilistic fracture mechanics(PFM). In order to establish the PFM-based design procedure, it is necessary to characterize the statistical nature of m and C in the fatigue crack propagation law of the following form

$$\frac{da}{dN} = C(\Delta K)^m \qquad (1)$$

Several methods were proposed in literature concerning how to treat m and C in the calculation of the probability of failure. However, experimental basis was insufficient. In the previous paper,[1] we conducted a preliminary study by analysing Tanaka et al's data[2] on 0.04 % carbon steel. The study showed that m and logC follow normal distributions and that strong negative correlation exists between m and logC. The similar trend was observed by Sakai et al[3] for a pure Al. However, the former data was obtained for the case of bending, and the latter data was obtained using a rather small number(=12) of specimens. Thus, in the present study, we conduct fatigue crack growth tests under axial loading using a moderate number(=30) of specimens.

EXPERIMENTAL PROCEDURE

The material used is 2024-T3 Al alloy. Its chemical composition is given in w/o as follows: Cu, 4.35; Si, 0.13; Fe, 0.30; Mn, 0.66; Mg, 1.42; Cr, 0.01; Zn, 0.03; Ti, 0.02; Zn+Ti, 0.05. The mechanical properties are as follows: 0.2 % proof stress 327 Mpa; tensile strength 478 Mpa; elongation 19.9 %. From the sheet of 1 mm thickness, the center notched specimens as shown in Fig.1 were prepared by machining. Crack growth tests were conducted under axial loading using a closed-loop hydraulic testing machine. The testing condition was as follows: stress range $\Delta\sigma$=59.4 Mpa; stress ratio R=0.20; repeating frequency f=18.4 Hz; test temperature T=30° C. The crack growth was monitored using a travelling microscope. 30 specimens were tested under one and the same condition.

EXPERIMENTAL RESULTS AND DISCUSSION

The crack growth rate da/dN was evaluated by the secant method. The stress intensity range ΔK was calculated as $\Delta K=\Delta\sigma\sqrt{\pi a}\sqrt{\sec(\pi a/W)}$ where W is the specimen width. Plotting da/dN against ΔK on log-log coordinates, and applying the least square method to the data points in the range of $10.4 \leqq \Delta K \leqq 22.3$ $\mathrm{Mpa\cdot m^{1/2}}$ (which corresponds approximately to the range of $10^{-5} \leq da/dN \leq 10^{-3}$ mm/cycle), m and logC were evaluated for each of the 30 specimens. The results were plotted on normal probability paper in Fig.2 and Fig.3. The unit of C is $(\mathrm{mm/cycle})(\mathrm{Mpa\cdot m^{1/2}})^{-m}$. It is seen that both m and logC follow normal distributions. This trend is similar to that found for the 0.04 % carbon steel in the previous paper.[1] The mean value and the standard deviation of m are 2.939 and 0.247, respectively, and those of logC are -7.156 and 0.310, respectively. It is noted that the coefficient of variations of m and logC are about 10 % and 5 %, respectively, for both of the 2024-T3 Al alloy and the 0.04 % carbon steel.

It follows from Eq.(1) that variability of m brings different dimensions of C among the specimens. At first sight, this might seem unnatural. However, this is actually natural because of the following reason. From dimensional consideration, it is reasonable to consider that Eq.(1) has essentially the following form

$$\frac{da}{dN} = C_0\left(\frac{\Delta K}{K_0}\right)^m \tag{2}$$

where C_0 and K_0 are quantities which have the same dimensions as da/dN and ΔK, respectively. The theoretical equation by Yokobori et al[4] based on a physical model has the form of Eq.(2). From Eqs.(1) and (2) one has

$$C = C_0 K_0^{-m} \tag{3}$$

Eq.(3) implies that the dimension of C can be different among the specimens dependent on the value of m.

logC was plotted against m in Fig.4. It is seen that a straight line fits the data points fairly well. This result implies that C is related to m by Eq.(3) and that the values of K_0 and C_0 are approximately constant among the specimens. Then, it follows from Eq.(2) that the log da/dN - log ΔK line for each specimen passes near the common point (pivot point) of $\Delta K = K_0$ and da/dN=C_0. One might suspect that the correlation in Fig.4 comes simply from the nature of the regression line on the log da/dN - log ΔK coordinates. If this were the case, the pivot point would be located close to the centroid of the data points on the log da/dN - log ΔK coordinates. Applying the least square method to Fig.4, K_0 and C_0 are obtained as K_0=17.6 Mpa·$m^{1/2}$ and $C_0=3.20\times10^{-4}$mm/cycle. On the other hand, th centroid is found to be at ΔK=13.8 Mpa·$m^{1/2}$ and da/dN=1.54×10^{-4} mm/cycle. Hence, the two points are not close to each other. It should be also noted that Fig.4 represents the inter-specimen correlation of one and the same material, not the inter-material correlation.

The deviation of the data points from a straight line in Fig.4 may be taken into consideration by treating C_0 as a random variable. $\log C_0$ was evaluated for each specimen from the relation $\log C_0 = \log C + m \log K_0$ and

plotted on normal probability paper in Fig.5. In a first approximation, logC may be regarded to follow a normal distribution. The mean value and the standard deviation of $\log C_0$ are -3.495 and 0.042, respectively.

CALCULATION OF DISTRIBUTION OF FATIGUE LIFE AND PROBABILITY OF FAILURE

Several methods were employed in literature to take account of variablity of m and C in the calculatiion of the fatigue life and the probability of failure:

Method (a): Eq.(2) is used in which m and $\log C_0$ are assumed to follow normal distributions, and K_0 to be a constant.

Method (b): this is the same as (a) except that C_0 is assumed to be a constant.

Method (c): Eq.(1) is used in which logC is assumed to follow a normal distribution, but m is assumed to be a constant.

The method (a) was used by Okamura et al[5] in the reliability analysis of piping. The methods (b) and (c) were examined by Sakai et al.[3] Since (c) is inconsistent with the experimental fact that m has variabilty, (a) and (b) will be compared here. From Eq.(2), the number of repeated cycles N for crack growth from a=a_1 to a =a_2 is calculated as

$$N = \frac{1}{C_0}\left(\frac{K_0}{\Delta\sigma\sqrt{\pi}}\right)^m \cdot \int_{a_1}^{a_2}\left(a\cdot\sec\frac{\pi a}{W}\right)^{-\frac{m}{2}} da \tag{4}$$

Taking a_1=9 mm and a_2=23 mm which corresponds to the range of ΔK for which the present data analysis was made, the distribution of N was calculated by Monte Carlo simulation. As the input data, the distributin parameters of m and $\log C_0$ are used. The result is shown in Fig.6 together with the experimental distribution. It is seen that the method (a) is in better agreement with the experimental distribution.

The distribution of the fatigue life of a component containing crack-like defects can be calculated based on the above method (a). Then, the pobability of failure P_f for a given number of cycles is evaluated. Using P_f as a measure of safety, one can decide the allowable stress level, the acceptable size of a defect, the appropriate period of inspection, etc. along with the concept of damage tolerance design.

CONCLUSIONS

In order to establish the probabilistic fracture mechanics approach to fatigue design, the statistical characteristics of m and C in the crack propagation law $da/dN=C(\Delta K)^m$ was investigated experimentally on 2024-T3 Al alloy under axial loading using 30 specimens. The following conclusions were obtained.

(1) m and logC follow normal distributions.

(2) Strong negative correlation exists between m and logC. Their relation can be expressed as $C=C_0K_0^{-m}$ where C_0 is a log-normally distributed random variable and K_0 is a constant.

(3) The distribution of the crack propagation life as calculated based on the above characteristics (1) and (2) was in good agreement with the experimental distribution.

(4) By evaluating the probability of failure using the above method of calculating the distribution of the fatigue life , it is possible to decide the design parameters in the damage tolerance design.

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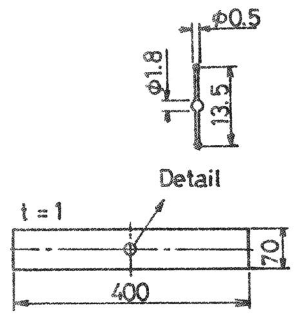

Fig.1 Shape and dimension of the specimen.

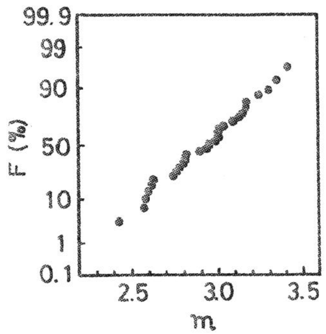

Fig.2 Distribution of m (normal probability paper)

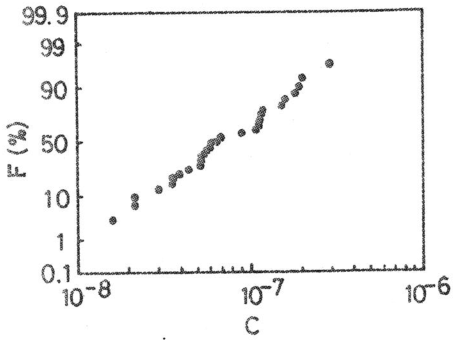

Fig.3 Distribution of logC (normal probability paper)

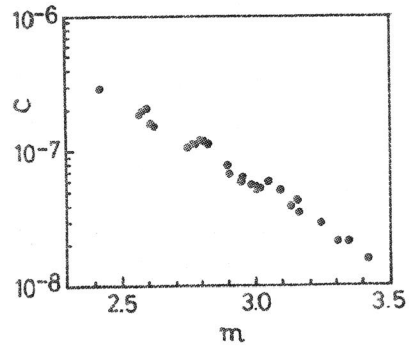

Fig.4 Correlation between m and logC.

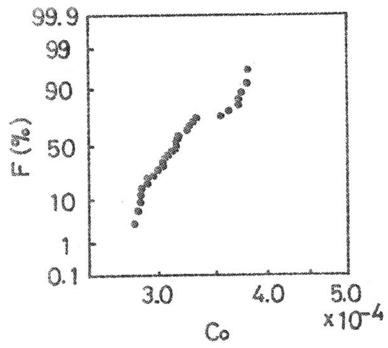

Fig.5 Distribution of $\log C_0$ (normal probability paper)

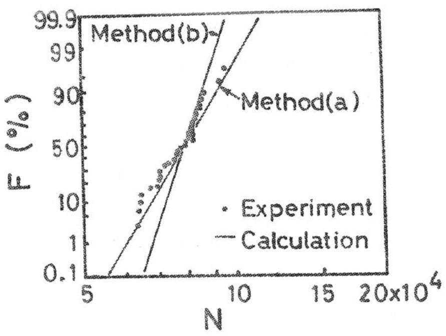

Fig.6 Comparison of the calculated distribution of fatigue life and the experimental one.