STRESS-INTENSITY FACTORS FOR RADIAL CRACKS AROUND CIRCULAR HOLES-METHOD OF EVALUATION AND COMPUTING GRAPHS

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INTRODUCTION

Among the accidents of failure and damage in ships and offshore structures, the most cases are due to the occurrence of growing cracks originating from the cut-outs or discontinuities such as the rivet holes in some structural components. Therefore, the computing graph of SIF for radial cracks around the circular holes has its practical use for the ship designers in designing their related structural components on the fail-ure-safe basis. In the present paper, we present a rather effective method evaluating approximately the SIF for the radial cracks around the holes, and provide some computing graphs accordingly. The method is, in a sense, an extension of the Bowie's method [1]–[3] for the case of single radial crack emanating from a circular hole to the general case of the radial cracks around the multiple holes by the method of superposition. Without loss of generality, the paper treated for simplicity, the problems of structural member with twin-holes carrying respectively the same radial crack under the uniform tensile load, so as to illustrate the general method of solution. As the result, the computing graphs of SIF for the cracks with various arrangements are plotted in Fig. 1 to Fig. 3 respectively. The calculating curve in Fig. 1 for the case that the distance between the hole-centers M=8 is found to be in good agreement with that obtained by the finite element method using plane isoparametric elements. Thus, we may conclude that the effect of the residual stresses around the otherwise free hole boundaries in present method on the final value of the SIF for the cracks can be neglected, so long as the geometry

condition at the crack tip is fulfilled, and the computing graphs shown in Fig. 1–Fig. 3 are reasonably accurate for the purpose of engineering ap-plications.

METHOD OF SOLUTION

For the general case of the radial cracks around the multiple holes, we use the method of superposition and assume the expression for the sum of the plane normal stresses in complex stress functions as follows:

$$\sigma_x + \sigma_y = 4\mathrm{Re}[\sum_i \pm \phi_i'(\zeta_i)/\omega_i'(\zeta_i)] \tag{1}$$

If the first term in above expression corresponds to the case of single hole or cut-out, then the rest of the terms may be regarded as the total effect on that hole due to the presence of other holes. In fact, each of these terms is chosen to satisfy the stress boundary conditions of the respective hole. Moreover, the stress function $\phi(\zeta) = \sum_i \pm \phi_i(\zeta_i)$ must also satisfy the stress conditions at infinity. The choice of the sign in front of each individual stress function depends solely on the position of its crack. For example, in the simplest case of an infinite plate with twin-holes carrying co-linear cracks, the complex stress function is obtained by superposing the term $[\phi_2'(\zeta_2)/\omega_2'(\zeta_2)]$ for the second hole to the expression $[\phi_1'(\zeta_1)/\omega_1'(\zeta_1)]$ for the first hole. The additional term to the first hole will cause some stresses on its hole boundary, the twin-hole problem then can be treated as a single-hole problem but with some "initial stress" around the hole boundary. Consequently, the same solving procedure as that previously used by Bowile [1], [2] for the single-hole problem may be fol-lowed, except that in this case the complex stress function and its de-rivative are written in the forms as:

$$\left.\begin{aligned} \phi(\zeta) &= \phi_1(\zeta_1) \pm \phi_2(\zeta_2) \\ \text{and} \quad \phi'(\zeta) &= \phi_1'(\zeta_1) \pm \phi_2'(\zeta_2) \end{aligned}\right\} \tag{2}$$

The negative signs are taken in eq. (2) for the case of twinholes with inside cracks as shown in Fig. 1, since the crack directions are opposite to each other. Here, ζ_2 can be solved by inverting the function $Z=\omega_2(\zeta_1)$ expressed as

$$\zeta_2 = -\frac{B}{2} + \frac{1}{2}\sqrt{B^2-4} \tag{3}$$

$$B = \frac{4\cos\alpha+[(1+\cos\alpha)(Z-MR)/R+(1-\cos\alpha)]^2}{2(1+\cos\alpha)(Z-MR)/R} \tag{4}$$

Above expression maps the point $\zeta_2=1$ onto the crack tip in Z-plane, i.e. Z=R+L. Referring to the Bowie's work, we may assume

$$\phi(\zeta) = C_1T[\frac{\zeta}{4} + \sum_{n=1}^{\infty} \alpha_n\zeta^{1-n} + \sum_{n=1}^{\infty} \alpha'_n\zeta_2^{1-n}]$$

and

$$\phi'(\zeta) = C_1T[\frac{1}{4} + \sum_{n=1}^{\infty} (1-n)\alpha_n\zeta^{-n} + \sum_{n=1}^{\infty} (1-n)\alpha'_n\zeta_2^{-n}] \tag{5}$$

where T represents the uniform tensile stress at infinity. For $M \to \infty (B \to -\infty)$, that is $\zeta_2 \to \infty$, eq.(5) degenerates to the case of single-hole with a radial crack. Finally, the complex stress intensity factor will be:

$$K = K_I - K_{II} = 2\sqrt{\pi}\lim_{\zeta\to 1}\phi'(\zeta)/[\omega''(\zeta)]^{1/2} \tag{6}$$

For the case of twin-holes with inside co-linear cracks, two mapping functions which map individually the unit circle and its exterior in ζ-plane onto the respective cut-out of length (2R+L) and its exterior in the Z-plane are

$$Z=\omega_1(\zeta) = \frac{R}{1+\cos\alpha}[\zeta+\frac{1}{\zeta}+(1-\cos\alpha)+(1+\frac{1}{\zeta})(\zeta^2-2\zeta\cos\alpha+1)^{1/2}] \tag{7}$$

and

$$Z=\omega_2(\zeta) = \frac{R}{1+\cos\alpha}[\zeta+\frac{1}{\zeta}+(1-\cos\alpha)+(1+\frac{1}{\zeta})(\zeta^2-2\zeta\cos\alpha+1)^{1/2}]+MR \tag{8}$$

Being analytic outside the unit circle, the mapping function $Z=\omega(\zeta)$ can be expressed in the series form as:

$$Z = \omega(\zeta) = C_1[\zeta+\sum_{n=1}^{\infty} A_n\zeta^{1-n}] \tag{9}$$

where C_1, A_1, A_2,.........., A_n are all real constants. Expanding eq. (7) also in a series form, we have

$$Z=\frac{R}{1+\cos\alpha}\Big[2\zeta+2(1-\cos\alpha)+\frac{1}{4}(5-4\cos\alpha-\cos2\alpha)\zeta^{-1}+\sum_{n=2}^{\infty}\Big\{\sum_{i=1}^{n-1}[R_i(R_{n-i}+R_{n+1-i})$$
$$+S_i(S_{n-i}+S_{n+1-i})+R_1R_n+S_1S_n-2(R_n+R_{n+1})]\Big\}\zeta^{-n}\Big] \tag{10}$$

Then all constants in eq. (9) can be determined by comparing the coefficients with that of eq. (10). According to Bowie's truncation method, we also truncate the infinite series in eq. (9) to finite terms, i.e.

$$\omega_T(\zeta) = C_1[\zeta+\sum_{n=1}^{k} A_n\zeta^{1-n}+A_{k+1}\zeta^{-k}+A_{k+2}\zeta^{-(k+1)}] \tag{11}$$

where the truncation index k should be chosen so as to satisfy the following equations:

$$\omega'_T(1) \doteq 0 \quad \text{and} \quad \omega''_T(1) \doteq Q \tag{12}$$

The constant Q is the value obtained directly from the second derivative of eq. (7) and evaluated at $\zeta=1$, i.e.

$$Q = \frac{R}{1+\cos\alpha}[2+\sqrt{\frac{1-\cos\alpha}{2}}+\sqrt{\frac{2}{1-\cos\alpha}}] \tag{13}$$

Substituting eq. (11) into eq. (12), we get after some manipulations

$$A_{k+1}=\frac{1}{k}[\sum_{n=1}^{k}(n-1)n\,A_n+(k+2)-(k+2)\sum_{n=1}^{k}(n-1)A_n-\frac{Q}{C_1}]$$

$$A_{k+2}=\frac{1}{k+1}[(k+1)\sum_{n=1}^{k}(n-1)A_n-(k+1)-\sum_{n=1}^{k}(n-1)n\,A_n+\frac{Q}{C_1}] \tag{14}$$

The effective truncation number of terms k is the number which makes the sum $(|A_{k+1}|+|A_{k+2}|)$ a minimum. After truncation, the number of terms chosen for the complex stress functions depends on that of $\omega_T(\zeta)$, i.e

$$\phi(\zeta) = C_1T[\frac{\zeta}{4}+\sum_{n=1}^{k+2}\alpha_n\zeta^{1-n}+\sum_{n=1}^{k+2}\alpha'_n\zeta_2^{1-n}]$$

Similarly,

$$\psi(\zeta) = C_1T[\frac{\zeta}{2}+\sum_{n=1}^{k+2}\beta_n\zeta^{1-n}+\sum_{n=1}^{k+2}\beta'_n\zeta_2^{1-n}] \tag{15}$$

where the constant C_1 has been determined as mentioned earlier; the coefficients α_n and $\alpha'_n(n=1,2,\ldots,\ldots,k+2)$ remain as yet to be determined.

Denoting $N\equiv k+2$, $A_{N-1}\equiv A_{k+1}$, $A_N\equiv A_{k+2}$ and substituting each part of expression both in eq. (11) and eq. (15) into the corresponding individual cut-out boundary conditions, we finally obtain two similar systems of equations for α_n and α_n' respectively. They are

$$\alpha_p + \sum_{n=1}^{N-p} \alpha_{n+p} A_n (1-n) + \sum_{n=1}^{N-p} A_{n+p} \alpha_n (1-n) + A_p/4 = \begin{cases} 0, & p\neq 2 \\ -\frac{1}{2}, & p=2 \end{cases} \qquad (16)$$

in which $p = 1,2,\ldots\ldots, N$. The two sets of linear algebraic equations can be solved simultaneously for the coefficients α_n and α_n'.

Thus, the stress function $\phi(\zeta)$ is completely determined. Substituting the values of $\phi'(1)$ and $\omega''(1)$ into eq. (6), we then obtain the stress intensity factor K_I.

NUMERICAL RESULTS

The numerical results for three cases of the crack arrangement, i.e. for inside cracks,,outside cracks and all cracks emanating from the same side of two circular holes under uniform tensile load T are plotted as follows:

1) The case of inside co-linear cracks emanating from two circular holes

In this case, due to symmetry, $K_{IA}^{(1)} = K_{IB}^{(1)} \equiv K_I^{(1)}$. With the nondimensional parameter L/R as abscissa and $K_I^{(1)}/T\sqrt{\pi(L+R)}$ as ordinate, the calculating results are shown in Fig. 1. The dotted line in Fig. 1 represents the calculating result by F.E.M. for M=8.

2) The case of co-linear cracks emanating from the same side of two circular holes

Obviously, in this case, $K_{IA}^{(2)} \geq K_{IB}^{(2)}$ The calculating results are shown in Fig. 2, in which the dotted lines represent the dimensionless value of $K_{IB}^{(2)}$, whereas solid lines for $K_{IA}^{(2)}$.

3) The case of outside co-linear cracks emanating from two circular holes.

Again, due to symmetry, $K_{IA}^{(3)} = K_{IB}^{(3)} \equiv K_I^{(3)}$. The calculating results are shown in Fig. 3.

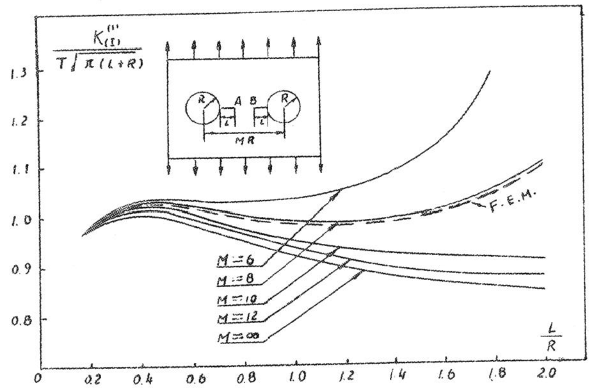

Fig. 1

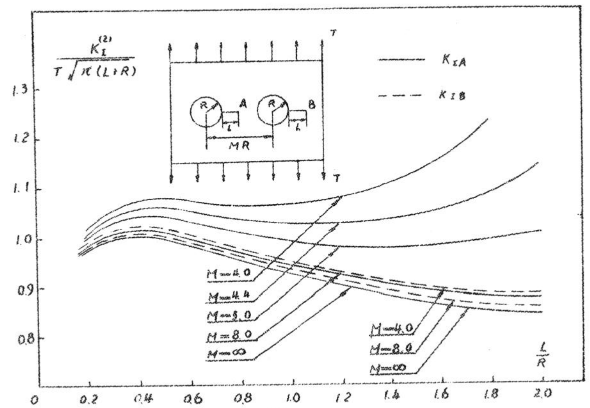

Fig. 2

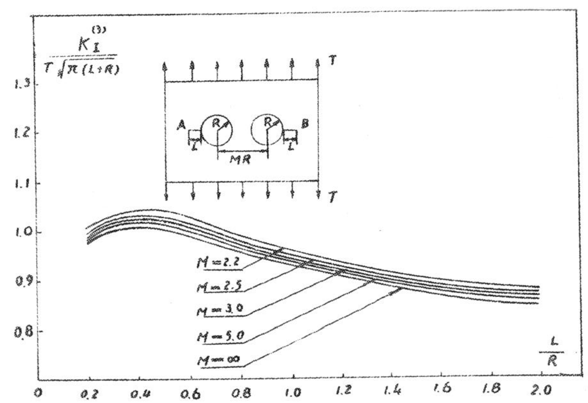

Fig. 3

DISCUSSION

1. As shown in Fig. 1, 2 and 3, for the special case $M=\infty$, i.e. the case of single cut-out, the error of these curves is within ±4% as compared with Bowie's solution. When the result of $M=8$ in Fig. 1 is compared with that calculated by F.E.M., the discrepancy between them for the range $0.2 \leqq L/R \leqq 2.0$ is less than ±1%. Thus, the validity of this approximate method is justified.

2. The effect of the adjacent cut-out on $K_I^{(1)}$ is the largest in the first case of inside co-linear cracks. As shown in Fig. 1. With the increase of the value M, the interaction between the cut-outs decreases rapidly. For instance, the error caused by neglecting the effect of adjacent cut-out for $L/R \leqq 1$ is around ±11% for $M=6$, it reduces to ±3% for $M=10$. Therefore, the effect of adjacent cut-out must be considered, otherwise larger error are likely introduced.

3. In the second case (Fig. 2), since $K_{IA}^{(2)} > K_{IB}^{(2)}$, i.e. the effect of adjacent cut-out on $K_{IA}^{(2)}$ is larger, the error caused by neglecting the effect of adjacent cut-out for $M=8$ with $L/_R=2$ is less than ±2% for $K_{IA}^{(2)}$, and ±1% for $K_{IB}^{(2)}$.

4. In the case of Fig. 3, $K_I^{(3)}$ is very close to that of the single hole case, hence the effect of adjacent cut-out may be negligible.

REFERENCES

[1] Bowie, O.L., Methods of Analysis and Solutions of Crack Problems, Ed. Sih, G.C., Noordhoff, pp. 1–55 (1973).

[2] Bowie, O.L., J. Mathe. and Physics, Vol. 35, p. 60 (1956).

[3] Neal, D.M., Int. J. Frac. Mech., Vol. 6, p. 393 (1970).