ON INCLINED CRACK UNDER COMPRESSIVE LOADING

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ABSTRACT

The effect of compressive loading on an inclined crack is examined in this investigation. Four fracture criteria, namely the maximum hoop stress, the strain energy density, the potential energy release rate, and the energy-momentum tensor, are reviewed. A modified model is proposed to include the frictional effect of the sliding mode under compressive loading. The predictions of the initial direction of crack growth are compared to experimental results over a wide range of inclined crack angles.

INTRODUCTION

The inclined crack problem under mixed mode loading has been a controversial topic in recent literature in fracture mechanics. A popular representation of a two-dimensional mixed mode loading case is a straight crack oriented at an angle β to the uniaxial tension (Figs. 1 & 2). When $\beta = 90^0$, the classical Griffiths crack is resumed and once the critical fracture load is reached, the crack will start to propagate in the direction of its own plane. For values of β other than 90^0, similarity is lost in that propagation starts with crack initiation angle θ_0 different from zero. The problem involves the prediction of initial crack growth angle θ_0 and the magnitude of the applied load σ_{cr} at which growth occurs.

In an early attempt to solve this problem Erdogan and Sih [1], Ewing and William [2, 3] made use of the maximum hoop stress at the crack tip as a criterion. Later Sih [4, 5, 6] proposed a new concept using the strain energy density criterion. Extension of the original Griffith's fracture criterion to the inclined crack problem was pursued by several reseachers [8, 9, 10, 11] using potential energy release rate G. Another alternative approach was proposed by Tirosh [12] using Eshelby's energy-Momentum tensor [13].

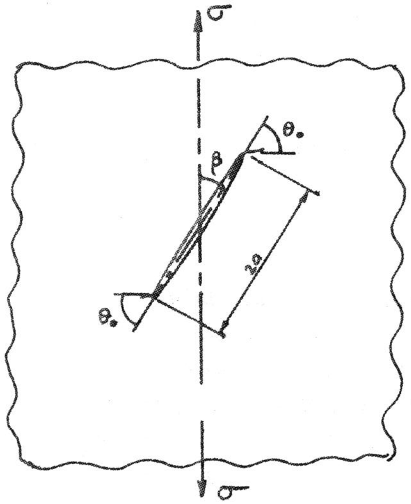

Fig.1 Inclined crack under uniaxial tension

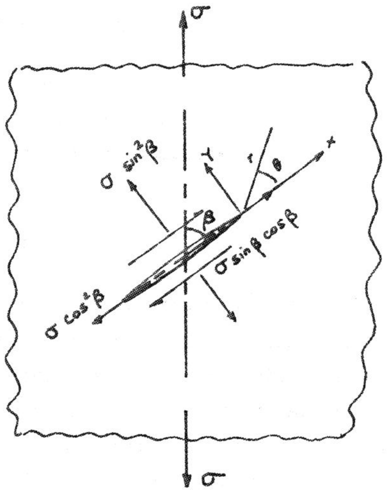

Fig.2 Stress field

In all the above mentioned criteria, the evaluation of the stress field at the crack tip is necessary. The solution obtained from Williams' analysis [14, 15] in the form of series expansion is given as:

$$\sigma_r = \frac{K_I}{\sqrt{2\pi r}}\left[\frac{5}{4}\cos\frac{\theta}{2} - \frac{1}{4}\cos\frac{3\theta}{2}\right] + \frac{K_{II}}{\sqrt{2\pi r}}\left[-\frac{5}{4}\sin\frac{\theta}{2} - \frac{3}{4}\sin\frac{3\theta}{2}\right]$$

$$+ \sigma_t \cos^2\theta + \ldots\ldots$$

$$\sigma_\theta = \frac{K_I}{\sqrt{2\pi r}}\left[\frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{3\theta}{2}\right] + \frac{K_{II}}{\sqrt{2\pi r}}\left[-\frac{3}{4}\sin\frac{\theta}{2} - \frac{3}{4}\sin\frac{3\theta}{2}\right]$$

$$+ \sigma_t \sin^2\theta + \ldots\ldots$$

$$\tau_{r\theta} = \frac{K_I}{\sqrt{2\pi r}}\left[\frac{1}{4}\sin\frac{\theta}{2} + \frac{1}{4}\sin\frac{3\theta}{2}\right] + \frac{K_{II}}{\sqrt{2\pi r}}\left[\frac{1}{4}\cos\frac{\theta}{2} + \frac{3}{4}\cos\frac{3\theta}{2}\right]$$

$$+ \sigma_t \sin\theta\cos\theta + \ldots\ldots \qquad (1)$$

where $\sigma_t = (\cos^2\beta - \sin^2\beta)$ (2)

The stress intensity factors of the inclined cracks are given by

$$K_I = \sigma\sqrt{\pi a}\,\sin^2\beta$$

$$K_{II} = \sigma\sqrt{\pi a}\,\sin^2\beta\,\cos\beta \qquad (3)$$

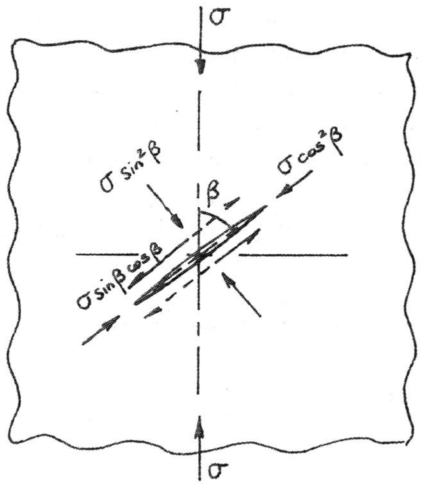

Fig 3 Crack under uniaxial compression

The first two terms of equation (1) give the singularity as $r \to 0$. σ_t appears in none of these terms and is thus independent of the parameter r. As fracture is a localised phenomenon at the crack tip, most of the predictions reported made use of the first two terms and the second order effect (σ_t) has been ignored. Cotterell [16] has discussed the inclusion of this term and demonstrated that it has a significant effect on crack path prediction under uniaxial tension loading.

When a line crack is loaded in compression it is evident that the crack faces tend to close on themselves, and the frictional effect along the crack face must be taken into consideration. Cotterell [18] predicted the fracture in compression by a stress criterion applied to a slender ellipse, and hence the above close-up problem did not appear in his analysis. However, his predictions did not conform especially well to his experimental results. McClintock and Walsh [19] modified the original Griffith criterion to include friction along the crack face but these effects disagreed with the data presented by Hoek and Bieniawski [20]. Swedlow [15] put forth a model of uniaxial compressive loading, trying to modify equations (1) and (3) to take account for the closure and frictional effect. The values of K_I and K_{II} in equations (3) will have a change in sign because of the compressive applied stress ($-\sigma$) Swedlow disallowed negative values of K_I, since they correspond to crack closing on itself, equivalent to the crack tending to propagate in the backward direction, with material merging near the crack tip zone.

Consider the stress distribution under compression in figure 3, the shearing stress which gives rise to mode II stress intensity factor K_{II}

is partially cancelled by the frictional resistance which is equal to $\mu\sigma \sin^2\beta$. Hence K_{II} should be $-\sigma\sqrt{\pi a}\,(\sin\beta\cos\beta - \mu\sin^2\beta)$, provided that $\cos^2\beta > \mu\sin^2\beta$ or $\cot^2\beta \geq \mu$. When $\cos^2\beta < \mu\sin^2\beta$, the sliding motion will be completely impeded and hence $K_{II} = 0$. The model for uniaxial compression is therefore modified as:

$$\sigma_r = \frac{K_{II}}{\sqrt{2\pi r}}\left[-\frac{5}{4}\sin\frac{\theta}{2} + \frac{3}{4}\sin\frac{3\theta}{2}\right] - \sigma\sin^2\theta\sin^2\beta + \sigma_t\cos^2\theta$$

$$\sigma_\theta = \frac{K_{II}}{\sqrt{2\pi r}}\left[-\frac{3}{4}\sin\frac{\theta}{2} - \frac{3}{4}\sin\frac{3\theta}{2}\right] - \sigma\cos^2\theta\sin^2\beta + \sigma_t\sin^2\theta \qquad (4)$$

$$\tau_{r\theta} = \frac{K_{II}}{\sqrt{2\pi r}}\left[\frac{1}{4}\cos\frac{\theta}{2} + \frac{3}{4}\cos\frac{3\theta}{2}\right] - \sigma\sin\theta\cos\theta\sin^2\beta - \sigma_t\sin\theta\cos\theta$$

where $K_{II} = -\sigma\sqrt{\pi a}\,(\sin\beta\cos\beta - \mu\sin^2\beta)$ (5)

$$\sigma_t = -(\cos^2\beta - \sin^2\beta) \qquad (6)$$

EXPERIMENTAL RESULTS

The verification of the modified model for inclined cracks under compression was carried out using 1/8" thick PMMA sheets with cracks of different inclinations tested by a Instron Universal Testing Machine. Predictions of the fracture angle θ_0 for various inclination angles were calculated using i) maximum hoop stress criterion, ii) minimum strain-energy-density criterion, and iii) energy-momentum-tensor criterion. Both theoretical and experimental results of the inclined crack compression problem are shown in figures 4 and 5. The theoretical curves are obtained using value $\frac{r}{a} = 0.01$. There were two initial fracture angles (emerging from both ends of the crack) in each test specimen. In most cases, these two angles differ from each other by a few degrees. A close examination of the crack growth process during the test revealed that the crack propagation started first at one end of the original inclined crack. The second one emerged after the first had advanced a small distance of a few mm. The first fracture angle is believed to be the one that conforms more to the theoretical prediction because the second one emerges from a branched crack situation for which the stress intensity factors will surely differ from that of the line

crack.

In figures 4 and 5, both Swedlow's and the modified form were presented with different values of coefficients of friction (μ). The discrepancies between Swedlow's and the modified form in the three criteria are indeed small, but the most striking feature is the substantial deviation among the theoretical curves, especially between the most popular $\sigma\theta_{max}$ & S_{min} criteria (These large theoretical discrepancies have not been encountered in inclined crack problems with tensile normal stress). The theory which conforms best with experimental data is the maximum hoop stress criterion. Cotterell's data with elliptical cavities in uniaxial compression test were also presented in figures 4 and 5 for comparison. The severe scattering of Cotterell's results is probably due to geometrical deviation of the elliptical cavity from a line crack.

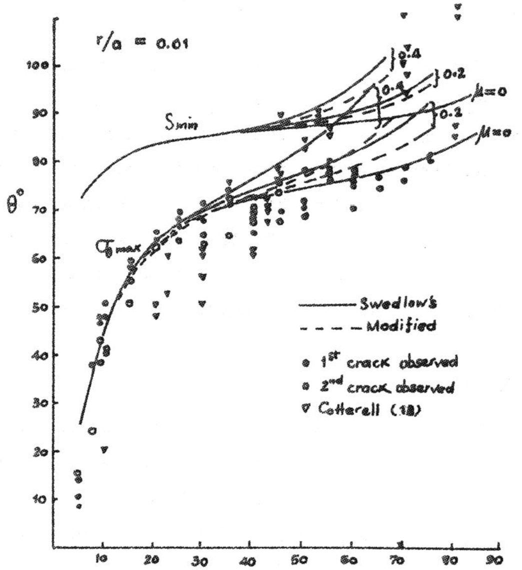

Fig 4 Crack initiation angle

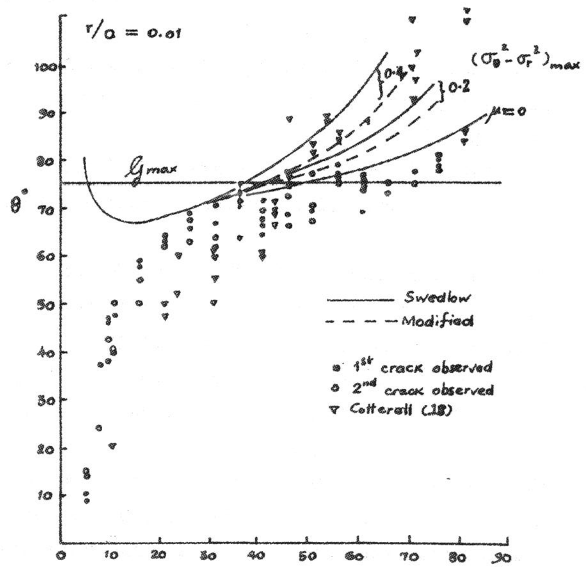

Fig 5 Crack initiation angle

CONCLUSION

The present work has proposed a modified form for the prediction of the crack initiation angle of an inclined crack under compressive loading. Sliding friction between the crack surfaces was found to have some influence on the fracture angle. For this reason the experimental results were found to have a bigger scatter than those obtained under tensile loading only. Of the four fracture criteria examined, the maximum hoop stress seems to give a consistent prediction over a wide range of inclined crack angles.

REFERENCES

[1] F. Erdogan & G.C. Sih, "On the Crack Extension in Plates under Plane Loading and Transverse Shear", Journal of Basic Engineering, Vol. 85D, pp. 519-525, (1963).

[2] J.G. William & P.D. Ewing, "Fracture under Complex Stress - The Angled Crack Problem", International Journal of Fracture Mechanics, Vol. 8, No. 4, pp. 441-446, (1972).

[3] P.D. Ewing, J.L. Swedlow & J.G. Williams, "Further Results on the Angled Crack Problem", International Journal of Fracture, Vol. 12 No. 1, pp. 85-93, (1976).

[4] G.C. Sih, "Strain-Energy-Density Factor Applied to Mixed Mode Crack Problems", International Journal of Fracture, Vol. 10, No. 3, pp. 305-321, (1974).

[5] G.C. Sih, "Some Basic Problems in Fracture Mechanics and New Concepts", Engineering Fracture Mechanics, Vol. 5, pp. 365-377, (1973).

[6] G.C. Sih & B. Macdonald, "Fracture Mechanics Applied to Engineering Problems - Strain Energy Density Fracture Criterion", Engineering Fracture Mechanics, Vol. 6, pp. 361-386, (1974).

[7] G.C. Sih, "Discussion on 'Fracture under Compress stress - the Angled Crack Problem'", International Journal of Fracture Mechanics, Vol. 8, pp. 441-446, (1972).

[8] K. Palaniswamy & W.G. Knauss, "On the Problem of Crack Extension in Brittle Solids under General Loading", Report SM 74-8, Graduate Aeronautical Laboratories, California Institute of Technology (1974).

[9] M.A. Hussain, S.L. Pu & J. Underwood, "Strain Energy Release Rate for a Crack under Combined Mode I & Mode II", Fracture Analysis, ASTM STP 560, American Society for Testing & Materials, pp. 2-28, (1974).

[10] R.J. Nuismer, "An Energy Release Rate Criterion for Mixed Mode under Fracture", International Journal of Fracture, Vol. 11, No. 2, pp. 245-250, (1975).

[11] B.A. Bilky & G.E. Cardew, International Journal of Fracture, Vol. 11, pp. 708-712, (1975).

[12] Jehuda Tirosh, "Incipient Fracture Angle, Fracture Loci & Critical Stress for Mixed Mode Loading", Engineering Fracture Mechanics, Vol. 9, pp. 607-616, (1977).

[13] J.D. Eshelby, "Energy Relations & the Energy-Momentum Tensor in Continuum Mechanics", Inelastic Behaviour of Solids, pp. 77-115, McGraw-Hill, N.Y. (1970).

[14] M.L. Williams, "On the Stress Distribution at the Base of a Stationary Crack", Journal of Applied Mechanics, Vol. 24, pp. 109-114, (1957).

[15] J.L. Swedlow, "Criterion for Growth of the Angled Crack", Crack & Fracture, ASTM STP 601, ASTM, pp. 506-521, (1976).

[16] B. Cotterell, "Notes on the Paths & Stability of Cracks", International Journal of Fracture, Vol. 2, pp. 526-533, (1966).

[17] I. Finnie & A. Saith, "A Nate on the Angled Crack Problem & the Directional Stability of Crack", International Journal of Fracture, Vol. 9, pp. 484-486, (1973).

[18] B. Cotterell, "Brittle Fracture in Compression", International Journal of Fracture, Vol. 8, No. 2, pp. 195-208, (1972).

[19] F.A. McClintock & J.B. Walsh, "Friction on Griffith Cracks in Rocks under Pressure", Proc. 4th U.S. Congress Applied Mechanics, 1962, Berkeley, ASME, N.Y., pp. 1015-1021, (1963).

[20] E. Hoek & Z.T. Bieniawski, "Brittle Fracture Propagation in Rock under Compression", International Journal of Fracture, Vol. 1, pp. 137-155, (1965).