A COMPARISON OF THREE DESIGN METHODS FOR ESTIMATING VALUES OF J AND COD APPLIED TO A CRACK WITH YIELDED LIGAMENT

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ABSTRACT

Some elastic-plastic fracture design procedures are briefly reviewed. The COD, R-6 and EnJ methods are applied in a comparative study of a buried crack with a ligament that yields.

INTRODUCTION

Failure by fracture of the remaining ligament of an already notched component is a well-known, albeit not very common occurrence. The micro-structural nature of the fracture is well known for most metals. Factors that for steel, permit separation on the micro-scale by brittle cleavage at low temperature or high strain rate, and ductile tearing by micro-void coalescence and shearing at higher temperatures are widely discussed in the literature. Design to avoid such fractures primarily by material selection has long been practised, and normal quality control procedures during fabrication are usually sufficient to avoid unintentional defects. Nevertheless, some cracks do occur in both fabrication and service so that the continued fitness of a component to serve its purpose must be assessed. For materials of higher toughness which nevertheless occasionally fail with less than the ductility expected from the smooth bar tensile behaviour, elastic-plastic fracture mechanics (epfm) is now being used. A general background is given in several recent books, such as (1), (2), (3). Some epfm design methods are summarised in Part I of this paper. Comparative studies of a particular problem are made by three methods in Part II.

1. EXISTING DESIGN METHODS

The so-called lefm shape factor, Y, permits the estimation of the elastic stress intensity factor, K, for many crack configurations where $Y = f(a/W)$ for two dimensions (a is crack length, W is width of the body)

or where $Y = g(a/B\ ,\ a/\ell)$ for three dimensions, (B is thickness and ℓ is crack size in the perpendicular direction). Thus, $K = Y\sigma\sqrt{a}$, where σ is the stress in the uncracked body and data for Y are available in collections such as (4), (5), (6). Test methods to determine the fracture toughness under plane strain conditions, K_{Ic}, have been standardised in several countries.

The methods of epfm are less well established than those of lefm. Test methods for determining toughness using full thickness test pieces, have been standardised for the crack opening displacement method (COD) by B.S.I. (7) and for rather small pieces using the J-contour integral method, by A.S.T.M. (8). Methods of estimating the applied severity of COD in terms of stresses in a structure, together with discussion on how to idealise actual complex crack shapes into stylised configurations, treatment of crack growth by fatigue and the final fracture, assessed in terms of a critical value of COD are given (9). A method historically derived from the two separate arguments of lefm and plastic collapse (10) but now presented as one particular epfm method known as R-6, is detailed (11). It calls on toughness expressed as K_{Ic} or in terms of J_{Ic} preferably derived from a full thickness test. A J-estimation procedure called EnJ, intended for use in a wide range of engineering situations has been proposed (12), (13). It also calls for toughness values from a full thickness J test, for which no standard as yet exists in the U.S.A. or the U.K., although tentative suggestions for such a test are made (11) and (14). Another recent proposal is the EPRI estimation scheme (15) and related fracture diagram (16). It is directed more specifically to certain configurations mainly of interest to the nuclear pressure vessel and pipework industry. It requested values of the hardening exponent in the estimation of toughness from computed data and calls for K_{Ic} or J_{Ic} from the standard A.S.T.M. test methods for toughness values.

Comparison of the main features of epfm design methods

In the broadest sense, it was shown (12) that all the foregoing methods were basically related. If COD (δ) were written $J = m\sigma_y\delta$ where σ_y is yield stress and m has value $1 \lesssim m \lesssim 3$, (often m = 2) then the ordinate scales of non-dimensional COD, $\Phi = \delta/2\pi e_y\bar{a}$ (9) and K_r for R6(11) and J or $\sqrt{G/J}$ (15), (16), can clearly be related to J/G_y used (12), (13), by

$$J/G_y = JE/Y^2\sigma_y^{\,2}a = (J/G)(\sigma/\sigma_y)^2 = (1/K_r)^2(\sigma/\sigma_y)^2$$

$$= \frac{m\sigma_y\delta E}{Y^2\sigma_y^{\,2}\bar{a}} = \frac{m\delta}{Y^2 e_y \bar{a}} = 2m\delta$$

where G_y is the value of G at $\sigma=\sigma_y$ and $e_y=\sigma_y/E$. $\bar{a}$ is an equivalent crack (9) notionally related to the real crack by $Y^2a = \pi\bar{a}$. Clearly, all the ordinates can be seen as normalised values of J, subject to the uncertainty in the value of m. The abscissa scales are nominally strain, e/e_y, in COD (9) and EnJ (12, 13) and collapse load ratio $S_r = Q/Q_c$ in R-6 (11) and the EPRI diagram (15). These can be related if a load deflection diagram is specified. The simplest concept is if a power law relation is used, $Q = Aq^n$, where Q is load, q is displacement and $0 \leqslant 1 \leqslant n$, whence the work done $w = Qq/(n+1)$. For that case $J = \eta w/Bb$ where B is thickness, b is ligament width and $\eta = f(a/W,n)$ defined by $\eta = (b/w)(\partial w/\partial a|_q)$. The simplest design diagram is then a linear one, ordinate J/η, abscissa w/Bb. Since work done, w, is not usually a convenient design parameter, it can be re-expressed in terms of Q to give a schematic version of R-6 or EPRI diagram, or in terms of q to give a schematic version of COD and EnJ. If the relation between load, Q, and displacement, q, is taken in a tangent rather than power law form and the variables of gemoetry and degree of deformation assumed separable, then the J-load (or δ-load) relationship is of the well known ln sec $\pi\sigma/2\sigma_y$ form which was the precursor of the ln sec $\pi Q/2Q_c$ term now used in R-6, where Q_c is the "local collapse load" of the structure local to a crack.

These conceptual relations, whilst helpful in perceiving the interrelation of the methods are not adequate for comparative studies. In the abscissa R-6 and EPRI are normalised with respect to a nominal collapse load and in fact, both COD and EnJ accept σ/σ_y where σ is the uncracked body stress instead of e/e_y in the near lefm regime, so that all diagrams start with an abscissa related to load but thereafter COD and EnJ relate to strain. Collapse load is then introduced because J becomes very large as collapse is approached, so that it forms a natural limit and also because avoidance of it is often a design requirement in its own right. R-6 and EPRI use different definitions of collapse related to net section yield of a small region of a structure, and EPRI allow passage beyond that state by virtue of work

hardening. R-6 and EPRI diagrams therefore become "safe strength" design diagrams, rather than "fracture diagrams." COD and EnJ advocate the separate study of collapse, in part because evaluation for cracked structures may be difficult and J (or δ) is very sensitive to the many geometric features that affect the collapse load. These methods therefore imply that if collapse is important it deserves separate study, but otherwise J or δ can be treated as a function of strain. COD does not permit use beyond $e/e_y = 2$ without more detailed plastic analysis. EnJ permits entry to a strain controlled regime if the structural system is such that collapse will not occur close to net ligament yield for reason of displacement control, including that imposed where parallel elastic loading paths exist such as in many part through thickness crack cases. In that regime an effective strain is used whereby the nominal elastic value is augmented by a geometric multiplier defined below. Other features of difference are that treament of residual stress and regions of stress concentration call for allowances on the abscissa in the COD method, but on the ordinate for R-6 and EnJ. Both the latter also make some distinction between cases where residual and mechanical stress are elastic or, in their summation, exceed yield, although the treatments differ. Other major features of difference between COD, R-6 and EnJ on the one hand, and EPRI method on the other, are that the former use lefm shape factors with the effect of plasticity allowed for in the estimation procedures, whereas EPRI offers separate solutions for all cases considered but no estimate for other configurations not yet computed. EPRI also takes the hardening exponent, n, as a major variable, whereas the others do not. This probably arises from the inclusion of stainless steel pipework within the scope of EPRI, whereas the other methods are implicitly aimed at more conventional steel structures. For application to structural aluminium alloys with little hardening (9) uses an alternative estimate of COD that is more severe for a given value on the abscissa, and EnJ (12) notes it may not be applicable at the limits of either no hardening (or a long plateau of yield) or continued high hardening rate. For conventional steels of a moderate degree of hardening use of a modified yield stress appropriate to the mean strain in the ligament seems adequate. R-6 also uses a flow stress $\sigma_{fl}=(\sigma_y+\sigma_u)/2$ to simulate the effect of hardening on the collapse load for structural steels.

Recategorisation of deep cracks and an alternative concept

A feature of both R-6 and COD methods is that with yield of the net ligament ahead of a deep part through crack, the crack is recategorised as a fully through thickness crack of a size that is specified in each method but embraces an area appreciably larger than the crack itself. It is then argued that if such a recharacterised crack is acceptable, then the real crack obviously is, but if the recharacterised crack is not acceptable, nor is the real crack, despite the significant change in severity between the two cases. It is the unknown value of this change in severity that leads to a conservative recharacterisation method. EnJ does not directly use recharacterisation but employs the idea of a "cracked body structural strain" (cbse), itself based on the localisation of strain if the ligament yields. The concept was introduced (12) as a means of defining for practical purposes the strain at which the EnJ equations were to be entered. Clearly, with yielding in a statically indeterminate system the nominal condition is a compromise between a remote strain that would be elastic and a local ligament strain that with a deep notch could well exceed the yield strain. A localisation of strain also occurs, particularly for non-hardening material, when the ligament yields but the gross section does not. These geometric scale factors that control the localisation of strain may be a slip line field size associated with the ligament length or the distance of the crack from some determinate reference strain pattern. Where elastic material is in parallel with the crack, the general elastic state of stress and strain nearby, provides the reference. The features that dominate these effects are combined to give the "cracked body structural ratio" (cbsr). The cbse is taken as the (cbsr) (e/e_y) and for nominal elastic loading, $e/e_y = \sigma/\sigma_y$. In fact, only the plastic component of strain is focussed into the crack, there being a general elastic strain governed by the maximum load transmitted by the ligament, and the effective strain e_f/e_y takes this into account as well. Thus, after ligament yield, the concept is e_f/e_y = (nominal e/e_y + localised e_{pl}/e_y) = $f(\sigma/\sigma_y$, cbsr). Suggestions for the cbsr augmentation factor are listed Table 1.

THE EnJ ESTIMATION PROCEDURE

The EnJ procedure has only recently been proposed and is therefore re-stated here. The COD, R-6 and EPRI methods are now widely published (9),(11), (15), so the details are not repeated here. The basic EnJ equations are

For $e/e_y \leqslant 1.2$:- $J/G_y = (e/e_y)^2\ (1+0.5(e/e_y)^2)$ Eqn.1a

where e/e_y is taken as σ/σ_y

For $e/e_y \geqslant 1.2$:- $J/G_y = 2.5\ [(e/e_y)-0.2]$ Eqn.1b

where e/e_y is the effective strain (see Table 1)

If cbse $\geqslant$ 1 (see Table 1), $Y/\sqrt{\pi} > W/b$, or $W/b > 2$

the risk of collapse should be examined explicitly Eqn.1c

For residual stress, σ_r, or thermal stress, σ_{th}, J is estimated from Eqn. 1a, b, separately for mechanical stress, σ_m, and residual or thermal stress, to give J_m and J_r and/or J_{th}. The terms are combined to give the total J value by adding $J = (J_m^{\beta} + J_r^{\beta} + J_m^{\beta})^{1/\beta}$ where $0.5 \leqslant \beta \leqslant 1$

according to $\beta = 0.5 + (\sigma_m + \sigma_r + \sigma_{th})/\ 2\sigma_y$ (though not exceeding unity) i.e. $\beta = 1/2$ for elastic conditions and 1 for plastic. Use of the cbse concept may be necessary if restraint can induce reaction stresses of general yield level, but is not necessary for self-equilibriating compatibility stresses.

For stress concentrations, Eqn.1a is used with the appropriate lefm value of Y whilst cbse < 1. If cbse >1an estimate must be made of the uncracked body strain at the point of concentration to give the effective strain e_f/e_y.

When Eqn.1c calls for assessment of collapse, the user must decide whether the cracked body is liable to collapse at or soon after net ligament yield, or whether the load applied to the cracked region will be redistributed because of the configuration of the body and its loading system. If collapse will not occur, then Eqn.1b can be used to accommodate the localisation of strain that is the feature of displacement controlled loading. If collapse is possible, then closer study would have to be made of a safe limiting state but EnJ and COD methods do not include such an assessment within their own methodology. Nevertheless, Eqn.1c of EnJ is intended to draw attention to the circumstances where either collapse or the localisation of strain under displacement control seems likely.

TABLE 1

Suggested values of the cbse augmentation factor and the effective strain

	cbsr	cbse	e_f/e_y
a) Edge crack in tension	W/b	$(W/b)(\sigma/\sigma_y)$	$(\sigma/\sigma_y)\left[1+(W/b)\right] - 1$
b) Edge crack in bending	W/b	$(W/b)(\sigma/\sigma_y)$	$(\sigma/\sigma_y)\left[1+(W/b)\right] - (b/W)$
c) Edge crack tensile deformation control	$D/2b$	$(D/2b)(e/e_y)$	$(e/e_y)\left[1+(D/2b)\right] - (D/2W)$
d) Partthrough crack in tension	B/b	$(B/b)(\sigma/\sigma_y)$	$(\sigma/\sigma_y)\left[1+(B/b)\right] - 1$
e) Part through crack in bending	B/b	$(B/b)(\sigma/\sigma_y)$	$(\sigma/\sigma_y)\left[1+(B/b)\right] - (b/B)$
f) Buried crack in tension	B/b_1	$(B/b_1)(\sigma/\sigma_y)$	
i) If also	$\left[B/(b_1+b_2)\right](\sigma/\sigma_y) < 1$;		$(\sigma/\sigma_y)\left[1+(a/b_1)\right]$
ii) If also	$\left[B/(b_1+b_2)\right](\sigma/\sigma_y) \geq 1$;		$(\sigma/\sigma_y)\left[1+\frac{B}{b_1}\right] - \left(1+\frac{b_2}{b_1}\,\frac{b_2}{B}\right)$

σ/σ_y is the uncracked body elastic state

e/e_y is an applied deformation.

W = width; B = thickness; D = gauge length

b = ligament (b_1 is the smaller of two ligaments, b_2, the larger for a buried crack).

Attention is also drawn in an approximate way in EnJ to the effect of biaxial loading, the main effect being to alter the fully plastic condition at which collapse may occur.

II. COMPARISON OF ESTIMATES OF APPLIED SEVERITY FOR A BURIED CRACK IN A YIELDED LIGAMENT, BY COD, E-6 AND EnJ METHODS

Some comparative studies of surface flaws in circumferential welds in pipe lines were made by McHenry et al (17) using the COD method (9), a modified COD concept, an instability method and an empirical method. It is intended to apply the R-6 and EnJ methods to the problem of (17) elsewhere. Here, a similar problem of laying a large diameter pipe line is used. It is supposed that

pipes are welded together circumferentially and that during laying, or by earth settlement, the pipe is deformed as a beam in bending. In reality, other laying and proof testing stresses would require consideration but for simplicity no other parts of the operation are considered. It is supposed that during welding cracks may occur of an extent measured during trials to be up to 10^o in circumferential length by 6.5mm in radial depth, but they are unlikely in the final weld run, so that the postulated crack is buried within the thickness, by one weld pass of at least 3mm. The weld is not stress relieved. The pipe wall thickness is 22mm, and the diameter large in relation to thickness, for example, of the order of 1m. The yield stress of the pipe material is 450 MN/m^2 and tensile strength 600 MN/m^2. The nominal elastic bending stress induced during laying is 382 MN/m^2. The applied stress system, though caused by bending, is essentially a uniform tension across the wall thickness because of the large ratio of diameter to thickness. The problem is therefore a crack buried part way through the wall thickness subjected to a mechanical tensile stress of $0.85\sigma_y$ and a residual stress assumed, in the absence of stress relief heat treatment, to be at yield level of the pipe wall. No exact solution for the lefm shape factor Y for a buried crack of very high ellipticity ratio is known, so a solution is used from the literature (6) for a two-dimensional eccentric crack, length (in the wall thickness direction) 2a = 6.5mm, buried with the near surface ligament b_1 = 3mm, subjected to uniform tension. For the crack end nearer the surface, Y = 2.07.

Solution by EnJ

Using Eqn.1c, $B/b < 2$ (where B is the wall thickness 22mm and b is the whole remaining ligament 15.5mm) so the problem is not sufficiently "deep notch" to require special attention. The cbse for a ligament with elastic material in parallel (i.e. the remainder of the pipe circumference) is (Table 1, case f) cbse (22/3) (0.85) = 6.2. This is greater unity so that tensile collapse must be considered. However, $Y/\sqrt{\pi}=1.17 < B/b$ so that it is inferred that local collapse by other than tension is not likely here (as may be self-evident if the ligament is considered to be deformed by uniform displacement rather than pin-loading). Although the body is a thin walled component where the fully plastic moment does not greatly exceed first yield, the nominal stress is not considered sufficient to induce overall collapse by bending of the pipe for a relatively small reduction in section modulus implied by the crack even with no account taken of work hardening, so that a more exact study of collapse is not made. Nevertheless, using Table 1, case f, sub-section ii) is relevant implying both ligaments yield, so the effective strain is rather large at $e_f/e_y = 3.70$.

Thus, for the mechanical stress system, from Eqn. 1b,

$$J/G_y = 2.5(3.70 - .2) = 8.75$$

so with $Y = 2.07$

$$(JE/\sigma_y^2 a)_{mech} = 36.5$$

For residual stress, in the absence of a known pattern of the stress, assume $Y = 2.07$ as for mechanical stress, then $e_f/e_y = \sigma_r/\sigma_y = 1$, so from Eqn.1a

$$J/G_y = 1(1+.5) = 1.5$$

$$\therefore \quad (JE/\sigma_y^2 a)_{res} = 6.4$$

Since $\sigma_{mech}+\sigma_{res} > \sigma_y$, J values are added with $\beta = 1$

$$\therefore \quad \text{"total"}\ JE/\sigma_y^2 a = 42.9$$

$$\therefore \quad JE = 42.9\ (450 \times 10^6)^2\ (6.5\times10^{-3}/2)$$

$$= 282\ MN^2 m^{-3}$$

$$\therefore \quad J = .138\ MNm^{-1}$$

$$\text{or} \quad K_J = \sqrt{EJ} = 168\ MNm^{-3/2}$$

These values of J K_J, are the minimum values of toughness required in the weld metal and heat affected zone (HAZ) if $J < J_{Ic}$ (or $K < K_{Ic}$).

If it is supposed $J = 2\sigma_y\delta$, a value of $\delta = .153$mm is implied. If it is supposed there is a factor of safety of 2 between a "critical" crack and the "acceptable" crack as estimated by PD6493 (9) then a value of $\delta = .306$mm is implied for the toughness.

Outline solution by COD method, PD6493

In the notation of PD6493 (9) thickness B = e = 22mm; crack length through the thickness (crack height) 2a = t = 6.5mm, and the near face ligament b_1 = p = 3mm. Thus, t/e = 2a/B = 6.5/22 = 0.295 and p/e = 1.36; p/t = 0.46. Since p/t < 0.5, then the crack must be recategorised as a surface breaking part through crack of height t_{eff} = t + p = 6.5 + 3 = 9.5mm ∴

$t_{eff}/e = 0.43$. The circumferential length ℓ may be large, i.e. 10πd/360, where d is the pipe diameter so that it is almost certain $t_{eff}/\ell < 0.1$. Using Fig.12 of PD6493, entering at $t_{eff}/e = 0.43$ and using $t_{eff}/\ell \lesssim 0.1$, then $\bar{a}/e > 0.74$, whence the equivalent crack $\bar{a} \geq 16.3$mm.

The applied strain $e/e_y = (\sigma/\sigma_y)_{mech} + (\sigma/\sigma_y)_{resid} = 0.85 + 1 = 1.85$.

The non-dimensional COD, $\bar{\Phi}$, is given by Fig.14 of PD6493 or by the relation

$$\bar{\Phi} = e/e_y - .25 = 1.6$$

$$\text{Thus,} \quad \delta = \bar{\Phi}\ (2\pi\sigma_y\bar{a}/E) = 0.36\text{mm}$$

If $\delta < \delta_c$, then this is the value required for the weld metal and HAZ toughness in which a factor of safety, notionally 2, is already included as inherent to the PD6493 procedure.

Outline solution by R-6

The notation is B = t = 22mm; flaw size 2a x ℓ; depth of flaw centre line below the near surface is $t_1 = (b_1+a) = 6.25$mm.

The rules for a first recharacterisation of the crack are the same as for recategorisation in PD6493, i.e. $(t_1-a)/2a < 0.5$, so the crack is treated as a surface breaking semi-elliptical crack of size $a' = t_1+a = 9.5$mm. The length of the recategorised crack is the greater of ℓ' = 2ℓ or ℓ' = 4a'. For the long crack involved here ℓ' = 2ℓ is the greater but a'/ℓ' is so small (< 0.1) that the crack would be treated as a line crack. The recategorised crack is treated as effective size c/w in order to discuss collapse, where, for the line crack, c/w = a'/t = 9.5/22 = 0.432. If the crack were of more restricted circumferential length so that a'/ℓ' > 0.1, then c/w < a'/t and is taken as πa'ℓ'/4t(ℓ'+t) for which specific values of length are required.

If the crack were considered to be "pin-loaded" so that the eccentricity caused a bending action, then following Appendix 2 of R-6, the value of the abscissa $S_r > 1$ (S_r = 2.2 for the long crack and 1.2 for a short crack) so a solution could not be obtained. The further possibility of recharacterising the defect as a through wall crack and then estimating S_r for the collapse of the whole pipe cross-section rather than of "local collapse" of the area adjacent to the defect still exists. The toughness required would be very large indeed and that argument has not been pursued.

It seems reasonable, however, to treat the cracked section as if loaded by tensile deformation with "ends parallel" so that no bending is induced in the wall. No precise rules are given in R-6 but it would seem appropriate to treat the buried defect as if, for the purposes of estimating local collapse, the enveloping region were of size $\ell' \times t$, where $\ell' = \ell + t$ (as in Fig. A2.3 of R-6) and

$$S_r = S/S_c = \sigma_m \ell' t / \bar{\sigma}\{\ell' t - (\pi/4)(2a\ell)\}$$

where σ_m is the direct stress 382 MN/m^2 and $\bar{\sigma}$ the flow stress, $\bar{\sigma} = (450 + 600)/2$ as used in R-6. Specific values of t/ℓ are required. If, for example, $t/\ell = 0.2$, then $\ell' = 1.2\ell$ and $S_r = \{382/575\}\{1.2/(1.2 - .232)\} = 0.822$, whence $K_r = 0.82$, Using the same value of Y as before, i.e. Y = 2.07

$$K_p = 2.07(382)\sqrt{(3.25 \times 10^{-3})}/K_{Ic} = 45/K_{Ic}$$

For K_s a plastic zone correction is required. For no correction, with residual stress $\sigma_r = 450$, Y = 2.07, then $K_c(a) = 53/K_{Ic}$. Using the plane stress factor $(1/2\pi)(K/\sigma_y)^2$ gives

$$K_s = 69/K_{Ic}$$

If $K_r = K_p + K_s$ (as in earlier editions of R-6), then

$$0.82 = (69+45)/K_{Ic}$$

whence $K_{Ic} = 139$ MNm$^{-3/2}$

If K is further corrected to allow for the interaction of plasticity between the mechanical and residual stresses (as in R-6, Rev.2) then ratio $K_s(a)/K_s(a+r_p) = 53/69 = .766$, whence the correction term, ρ, (R-6, R v.2, App.4) is $\rho = .257$. Thus,

$$K_r = .82 = .257 + (53+45)/K_{Ic}$$

$$K_{Ic} = 174 \text{ MNm}^{-3/2}$$

TABLE 2. Comparison of values of toughness calculated by 3 methods for a particular buried crack problem.

Method	EnJ	COD	R-6
Calculated toughness	J=.138	δ=.360	K = 174
Equivalent) K MNm$^{-3/2}$	168	188	.174
Critical) J MNm^{-1}	.138	.171	.147
Value ^{x}) δ_J mm	.153	.190	.163
δ (Acceptable)	.306	.360	.326

x Using $K^2 = EJ$; $\delta_J = J/2\sigma_y$; δ contains a factor of safety inherent in COD but taken as $2\delta_J$ here for EnJ and R-6.

Discussion

In comparisons of COD based and other methods, not including R-6 or EnJ (17), the PD6493 method was found conservative primarily because it included a factor of safety, whereas the others did not. In a comparison of PD6493, R-6 and EnJ (18), quite close agreement was found if the factor of safety inherent in PD6494 was allowed for, although other matters of engineering judgement not specified in the codes also had to be agreed, even if the "correct" choice were unknown. Further points that may cause significant differences emerged in the present study. These are primarily the treatment of residual stress and the recategorisation of buried defects if local yield of the surrounding ligament occurs.

Residual stress is allowed for in R-6 and EnJ on the ordinate, whereas allows for it on the abscissa. However, R-6 enhances the importance of residual stress by use of a correction factor, ρ. Although collapse is not itself affected by residual stress, the effect is as if the origin for mechanical stress were to be moved part way along the abscissa (19). The reasoning is clear in terms of plastic zone sizes but the engineering concept seems to imply load control in that the permissible value of K_r is reduced as if the body were nearer to collapse, so that K_{Ic} is increased. (Compare $K_{Ic} = 139$ & $K_{Ic} = 174$ in the R-6 solution above). In EnJ, the interaction of residual and mechanical stress, by adding values (β = 1 in the example) is less severe than adding K values. The engineering concept is that when ligament yield can be tolerated, the local deformation is displacement controlled and there seems no reason to add terms more severe than J.

The second less conservative effect of using EnJ is that there is no recategorisation of a crack at ligament yield as in R-6 and COD. It has been argued that using the cbse concept, the value of J is assessed adequately when the ligament is strained to net section yield with elastic material in parallel with it. The recharacterisation step is quite severe since the enveloping crack is appreciably larger than the true crack. If that were coupled with a "pin-loaded" assumption, whereby bending of the wall is introduced because of the crack eccentricity, it would lead to a yet further large increase in the apparent severity, although in this problem that was avoided, as discussed in the R-6 solution.

The emphasis here has been on comparing estimates of applied severity. It is implied that to avoid fracture the values so estimated will be equal to or less than the toughness. Only brief discussion is offered here on which definition of toughness is the more appropriate. All three methods advocate a full thickness test to guard against the risk of cleavage, if that is possible. The value of toughness measured in a COD test (7) may be a true initiation value δ_i, or a value up to maximum load, δ_m, in which an unknown amount of slow stable growth is accepted. It can be argued that use of the PD6493 estimation procedure, which appears to be conservative by its known "factor of safety" may be offset by use of a maximum load toughness value, which might be well beyond initiation. Conversely, it may be thought that with use of an assessment value, such as R-6 or EnJ, in which there is no intenttional factor of safety, the toughness value should be restricted to the initiation value, unless other arguments are presented to show that a value in excess of that can be used without risk of unstable tearing. One method is, of course, to conduct a separate instability analysis but that seems incompatible with the use of a simple design procedure. Arguments on the circumstances in which a post-initiation value of toughness can be used without risk of unstable tearing, yet without explicitly measuring an R-curve and conducting an instability analysis are not yet clearly formulated. By analogy with the well known effects of size on the slope of terms $\partial G/\partial a|_a$ or $\partial J/\partial a|_a$ it seems unlikely that a ligament would necessarily remain stable if its size were less than that of the test component, as is likely in the present example. On the other hand, the ligament will be near plane stress and thus possess a toughness greater than the test piece by an unknown amount. In short, the assessment of an appropriate value of toughness may be more uncertain than the assessment of the applied severity.

Conclusions

1) Comparative studies made by COD (PD6493), R-6 and EnJ methods of a buried crack under uniform tension appear to show a more than two-fold difference for the required toughness (in terms of COD or J).
2) This large difference is reduced to some 20% (PD6493 being the most conservative) if the factor of safety inherent in PD6493 is also introduced into R-6 and EnJ so that like sees to be compared with like.
3) Nevertheless, the introduction or not of a "factor of safety" in the estimation procedure must be judged in relation to the use of an initiation toughness value or a maximum load toughness value, either being permissible in PD6493.
4) The circumstances when a post-initiation value of toughness can be used safely in conjunction with a simple design method and without an explicit analysis of ductile instability is not clear.

REFERENCES

(1) Knott, J.F., Fundamentals of Fracture Mechanics. Butterworth, London, 1973.

(2) Barsom, J.M. & Rolfe, S.T. Fracture & Fatigue Control in Structures, Prentice-Hill, New Jersey, 1977.

(3) Latzko, D.G.H. Post-yield Fracture Mechanics. Applied Science, London, 1979.

(4) Tada, H. et al. The Stress Analysis of Cracks Handbook. Del Research Corp., Hellertown, Pa, 1974.

(5) Sih, G.C. Handbook of Stress Intensity Factors, LeHigh Univ., Pa., 1973.

(6) Rooke, D.P. & Cartwright, D.J. Compendium of Stress Intensity Factors, HMSO, London, 1979.

(7) Methods for Crack Opening Displacement (COD) Testing, BS5762, Br. Standards Inst., London, 1979.

(8) Standard Test for J_{IC}, a Measure of Fracture Toughness. E813-81, ASTM, Philadelphia, 1982.

(9) Guidance on some Methods for the Derivation of Acceptance Levels for Defects in Fusion Welded Joints, PD6493, Br.Stand.Inst., Lond. 1980

(10) Dowling, A.R. & Townley, C.H.A. Effects of Defects on Structural Failure: a two-criteria Approach. Int.Jour.Press.Vess.& Piping, 3, pp.77-107, 1975.

(11) Assessment of the Integrity of Structures Containing Defects. R/H/R6-Rev.2. Central Electricity Generating Board, London, 1980.

(12) Turner, C.E. Further Developments of a J-based Design Curve and its Relationships to other Procedures. 2nd Int.Conf.El-pl Fr.Mechs. To be published, ASTM STP803.

(13) Turner, C.E. The J-estimation Curve, R-curve and Tearing Resistance Concepts leading to a Proposal for a J-based Design Curve against Fracture. Fitness for purpose validation of Welded Constructions. Welding Institute, Cambridge, 1981, paper 17.

(14) Sumpter, J.D.G. & Turner, C.E. A Method for Laboratory Determination of J_i Cracks & Fractures. ASTM STP601, 1976, pp.3-18.

(15) An Engineering Approach for Elastic-plastic Fracture Analyses. EPRI NP1413, Prog.1237, Electric Power Res.Inst., Calif., 1981.

(16) Shih, C.F., et al. An Engineering Approach for Examining Crack Growth and Stability in Flawed Structures. Int.Jour.Pressure Vessel & Piping, 9, 1981, pp.119-196

(17) McHenry, H.I. et al. Fracture Mechanics Analysis of Pipeline Girth Welds, ASTM STP668, 1979, pp.632-642.

(18) Burdekin, F.M. et al. Comparison of COD, R-6 and J-contour Integral Methods of Defect Assessment Modified to give Critical Flaw Sizes. Ibid (13), Paper 41.

(19) Chell, G.G. A Procedure for Incorporating Thermal & Residual Stresses into the Concept of a Failure Assessment Diagram. ASTM STP668, 1979, pp.581-605.