A SIMPLE ESTIMATION MODEL FOR A CENTER CRACKED PANEL IN TENSION

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INTRODUCTION

Although mechanical problems can adequately be treated by the presently highly developped numerical methods, from the engineer's point of view there is a certain need for analytical solutions even if they are based on simplifying assumptions. They can serve for quick estimations of a structural situation and the costly numerical methods can be applied if the situation is too complicated for a simple analytical procedure or if very precise results are required. It will be shown in the present paper that for the center cracked tension panel important quantities can be estimated by very simple formalisms.

ESTIMATION MODEL

In the estimation model two load ranges are distinguished:

- Large scale yielding. The applied force, F, is less than or just equal to the net section yield load, F_Y, which is defined by equality of the nominal net section stress, σ_n, and the stress σ_Y, representing the onset of yield.
- Net section yield, where $F > F_Y$.

The quantities calculated by the estimation model will be compared with experimental results.

Three important displacements of a center cracked tension panel are

- load line displacement, s,
- crack mouth opening displacement at the specimen's center line, v,
- crack tip opening displacement δ.

The linear elastic solutions for these quantities can be applied up to the yield load, F_Y, provided that the plasticity corrected crack length, a_{eff}, is used. This is in accordance with the findings of other authors [1-3] who demonstrated that linear elastic fracture mechanics with plastic zone correction works very well up to the yield load. In the present work Irwin's plasticity correction is used, i.e. $a_{eff} = a + 0.5(K^2/\pi\sigma_{0.2}{}^2)$, and the yield load is defined by $\sigma_Y = A\sigma_{0.2}$. For the sake of simplicity, A = 1 was chosen although in some instances beyond A = 0.9 the error in load may exceed 5 per cent (see also Ref. [3]).

The solutions for v and s were taken from Tada et al. [4]; the crack tip opening displacement is not considered here, it will be the subject of a future paper.

A further quantity of interest is the J-integral which can be replaced by G_{eff} for $F \leq F_Y$.

For $F > F_Y$ the deformations are treated like a tensile test on an ordinary tensile spesimen. The work-hardening law assumed

$$\varepsilon = \varepsilon_Y \left(\frac{\sigma}{\sigma_Y}\right)^{1/n} \quad \text{for } \varepsilon > \varepsilon_Y \qquad (1)$$

expresses the strains by taking the value at the yield point, ε_Y, and multiplying it with the magnification factor $(\sigma/\sigma_Y)^{1/n}$. It is assumed that the displacements s, v, and δ can be extrapolated from the yield point the same way, i.e.

$$\begin{bmatrix} s \\ v \\ \delta \end{bmatrix} = \begin{bmatrix} s_Y \\ v_Y \\ \delta_Y \end{bmatrix} \left(\frac{F}{F_Y}\right)^{1/n} \qquad (2)$$

with

$$\frac{F}{F_Y} = \frac{\sigma_n}{\sigma_Y}$$

If the gage length for s is much larger than the specimen width and additional elastic contribution is to be expected.

For the J-integral, an expression is used which consists of a linear elastic part, G, and a plastic contribution:

$$J = G + \frac{\sigma_n + \sigma_e}{2} \cdot s_p \qquad (3)$$

with σ_n: actual net section stress of the point under consideration (accounting for crack growth where applicable), σ_e : σ_n for which 10 per cent of s is plastic (the calculations showed that mostly $\sigma_e = (0.8 - 1)\sigma_{0.2} \approx 0.9\sigma_{0.2}$), and s_p = plastic portion of the load line displacement, s. Thus, $(\sigma_n + \sigma_e)/2$ is an average flow stress acting on the actual net section and the plastic portion of J is based on the solution for a rigid ideally plastic center cracked tension panel [5]. Eq(3) is similar to the method of Bucci et al. [5].

COMPARISON WITH EXPERIMENTAL RESULTS

In order the verify this simple model a number of calculations have been done using input data from an extensive experimental R-curve programme [6]. The materials investigated were a high strength aluminium alloy, the same alloy in a very soft condition and an alloy steel. All the calculations demonstrated a close coincidence with the experimental results. Due to the limited space only two examples can be shown in Figs. 1 and 2, namely s and J for one specimen. It is worth to note that crack growth started at $\sigma_n/\sigma_{0.2} \approx 1$, i.e. most of the range shown here is covered by crack growth.

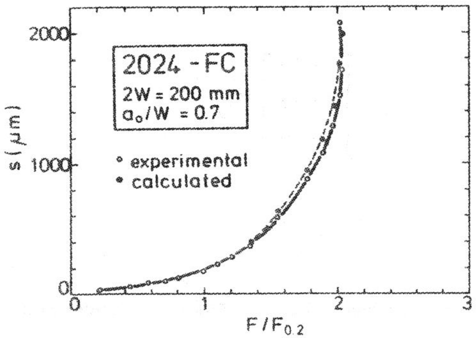

Fig. 1: Load-line displacement, s, for a center cracked panel of the low-strength aluminium alloy (2W = total width, B = thickness).

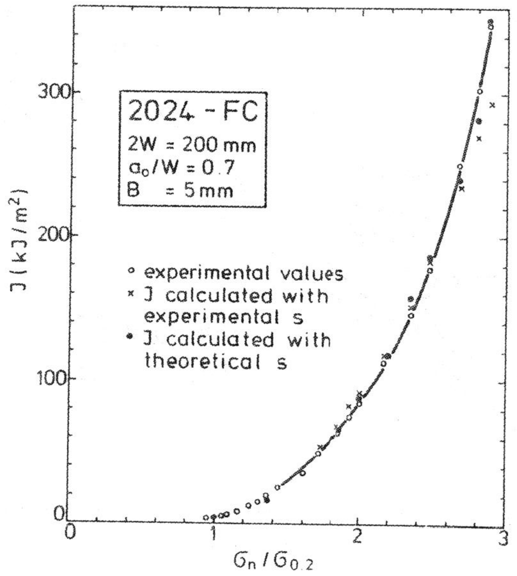

Fig. 2: J-integral of the specimen from Fig. 1

Eq(3) can be used for driving force calculations in order to predict instability. Two examples are shown in Fig. 3, one for the high-strength aluminium and one for the low-strength aluminium. A further feature is obvious from this diagram: although both materials have very similar R-curves and although both specimens have the same plan-view dimensions, their instability stress differs by almost a factor of two. This shows that in a fully yielded net section condition the R-curve alone is not sufficient to characterize a material's instability behavior. The reason is the strong influence of the material's flow properties on the driving force which can also be seen in Fig. 3.

A specific problem should be mentioned: technical materials usually don't work-harden according to a single power law; thus, for the work-hardening exponent an average value was determined the following way: the three materials investigated exhibited a double-linear work-hardening curve on a log-log plot and the arithmetic average of both slopes was taken.

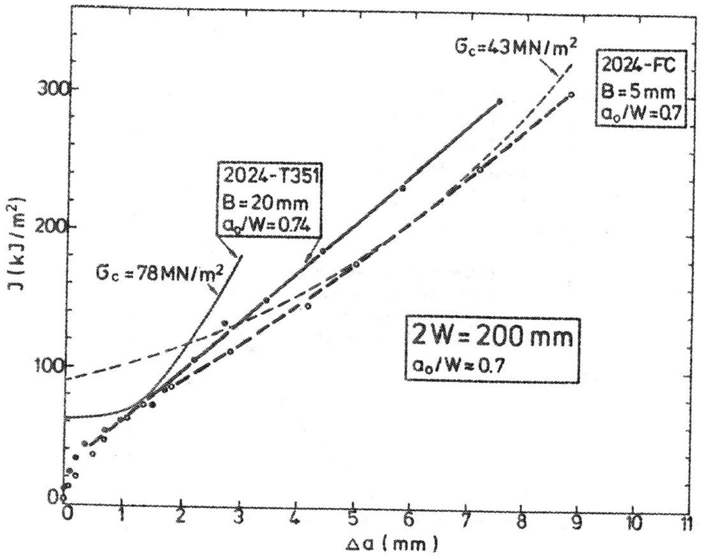

Fig. 3: R-curves and J crack driving force curves for the two aluminium alloys at instability. The true values for σ_c were 46 and 83 MN/m^2, respectively.

CONCLUSIONS

A simple model was developped which calculates the load line displacement, s, and the crack mouth displacement, v, by a two-step procedure: first the values of these quantities are calculated at the load of incipient ligament yielding using plasticity corrected linear elastic solutions. For higher loads a magnification factor given by the material's work-hardening law is used to calculate the actual s or v from their values at the yield point. Furthermore, the J-integral can be estimated from the plastic portion of s. Comparison with experimental values of three different materials demonstrated good coincidence.

REFERENCES

[1] McCabe, D.E., and Landes, J.D., An Evaluation of Elastic-Plastic Methods Applied to Crack Growth Resistance Measurements, Elastic-Plastic Fracture, ASTM STP 668, ASTM 1979, pp. 286-306.

[2] Shih, C.F., German, M.D. and Kumar, V., An Engineering Approach for Examining Crack Growth and Stability in Flawed Structures, General Electric Report No. 80CRD205, 1980.

[3] Schwalbe, K.-H., and Setz, W., R Curve and Fracture Toughness of Thin Sheet Materials, Journal of Testing and Evaluation, Vol. 9, No. 4, 1981, pp. 182-194.

[4] Tada, H., Paris, P.C., and Irwin, G.R., The Stress Analysis of Cracks Handbook, Del Research Corp., 1973.

[5] Bucci, R.J., Housel, W.S., Paris, P.C., Landes, J.D., and Rice, J.R., J Integral Estimation Procedures, ASTM STP 514, ASTM 1972, pp. 40-69.

[6] Hellmann, D., and Schwalbe, K.-H., Geometry and Size Effects on J_R and δ_R-Curves under Plane Stress Conditions, to be published in an ASTM STP.