J-INTEGRAL FOR SURFACE CRACKED BODY

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INTRODUCTION

In the problem of tearing instability of ductile materials, it is necessary to evaluate reliable values of J-integrals for real structural members with surface cracks such as line pipes and pressure vessels. The application of the three-dimensional finite element method is not practical at the moment since it is too time consuming to do the parametric study. The line spring model proposed by Rice[1,2] and developed by Parks[3] is one of the most respectable candidates to do the work. The authors[4] have developed a new finite element program based upon the line spring model, and they have shown that it is useful for the evaluation of J-integrals for surface cracked plates and shells. They have made some assumptions as first approximations which have to be reexamined for the model to be more reliable. In this paper some such improvements will be discussed.

FINITE ELEMENT METHOD BASED UPON THE LINE SPRING MODEL

Let us consider the problem of evaluating the values of J-integral along the leading edge of the surface crack for the plate subject to uniform tension and uniform bending, see Fig. 1. The problem is essentially three-dimensional, but it is simplified into the problem of stretching and bending of the plate with a line spring at the portion of the surface crack. The compliance of the line spring is chosen to be equivalent to the surface crack. In the line spring finite element method, flat shell elements are combined with the newly developed line spring elements. This method is more flexible since the configuration of the surface crack is not limited to the semi-ellipse only, the applied generalized force is not necessarily uniform, and general shell problem can be dealt with.

There have been some problems in the line spring model in the evaluation of J-integral. These are

(1) The plastic component of J-integral, J^p, is evaluated by the relation

$$J^p = m\sigma_Y \delta_t^{\,p} \tag{1}$$

where σ_Y: tensile yield stress, $\delta_t^{\,p}$: plastic component of the crack tip opening displacement, and m: numerical parameter depending upon the plastic constraint at the crack root. In the previous papers[3,4] the value of m was set constant for simplicity, i.e., m = 1.15, which is the value for the pure tension without any rotation. Therefore, this value should be more elaborated according to the real situation.

(2) The plasticity has been confined to the line spring only. But it will occur also in the portion other than the line spring when the applied load is large enough.

(3) The material has been assumed to be elastic-perfectly plastic. But work-hardening cannot be neglected in the ductile materials.

In this paper the first two problems will be discussed while the last has been discussed by Parks[5,6].

ABOUT THE PARAMETER m

The authors[7] have reported about the relation between J-integral and crack tip opening displacement for pin-loaded single-edge cracked (SEC) specimen. They considered the rigid-perfectly plastic material and deduced the same relation as eq. (1), where the parameter m was such a function of a/t as shown in Fig. 2(a). The result is available for the evaluation of m-value for the general SEC specimen used in the line spring model. If the bending moment M is defined about the mid-ligament point of the general SEC specimen, the relation between M and the axial force N is

$$M = \frac{t-b}{2}N = \frac{aN}{2} \tag{2}$$

from the diagram in Fig. 2(a). Therefore,

$$\frac{M_1}{N_1} = \frac{2a/t}{1-a/t} \quad \text{or} \quad \frac{a}{t} = \frac{M_1/N_1}{2+M_1/N_1} \tag{3}$$

where $N_1 = N/(2kb)$, $M_1 = M/(0.5kb^2)$ are normalized axial force and bending moment, respectively, and b and k are the ligamant length and yield stress in shear, respectively. The above equation shows that the parameter m is a function of M_1/N_1 if it is the function of a/t. Therefore, if we substitute eq. (3) into the relation shown in Fig. 2(a), we can get the relation between m and M_1/N_1 which is shown in Fig. 2(b). The figure shows that the value of m varies from 1.15 to 1.97 depending upon the ratio of M_1/N_1. This result agrees with that estimated by Parks[6].

THE LINE SPRING MODEL OF DUGDALE TYPE

As already mentioned, it has been assumed in the earlier analyses[3,4] that the plasticity is confined to the line spring only. Let us call it the model I analysis. When the applied load is large enough, this model is not realistic since dominant plasticity will also occur in the portion other than the line spring. This effect can be taken into account by the finite element method based upon the line spring model, that is, plasticity can be taken into account not only in the line spring element but also in the shell element by following the ordinary incremental procedure for elastic-plastic analysis of shell problem. This will be called the model II analysis. Fig. 3 shows an example of the results of the model II analysis. Three dimensional plastic zones which have developed in the shell elements are observed. One problem of the model II analysis is that it is time-consuming because a great number of incremental stages are necessary to obtain the reliable results. Therefore, the authors have developed a new model to overcome this shortage.

Generally the plastic zone around the crack develops three dimensionally as shown in Fig. 3. This plastic zone is modeled in such a way as shown in Fig. 4. It is assumed that the plasticity occurs only in the plane of crack, i.e., in x_1-x_3 plane, and that the plastic zone is regarded as a part of the crack in the same way as the Dugdale model in the two dimensional problem. Therefore, the deformation and the rotation are regarded as the crack opening displacement and rotation, respectively, at the same portion. This model could be called the line spring medel of Dugdale type or simply the model III. It is similar to that proposed by Erdogan, et al. [8,9,10] except for the point that it is solved by the finite element method in the authors' analyses.

The authors[4] have developed the line spring element (LSE) to express the compliance of the real surface crack in the finite element code of shell analysis. Now they need to develop a new element to express the compliance of the plasic zone. It could be called Dugdale type line spring element (DLSE). The compliance of the DLSE can be approximated by that of flat plate (plane strain) which is subject to the axial force N and the bending moment M at the same time, as the compliance of the LSE has been approximated by that of SEC specimen with the same crack depth, see Fig. 5.

Fig. 6 shows an example of the results analized by the three models mentioned above. There are little differences in J-values among these results when the applied stress $\sigma_\infty = N_\infty/t$ is less than the tensile yield

stress σ_Y, i.e., $\sigma_\infty/\sigma_Y < 1.0$. But remarkable difference is observed between the result of model I analysis and those of model II and III analyses in the range of $\sigma_\infty/\sigma_Y > 1.0$. This is because no plasticity is considered except for the line spring in the model I analysis. Therefore, the model II and III analyses are more realistic in the whole range of applied stress. The computing time up to the applied stress level of $\sigma_\infty/\sigma_Y = 1.0$ are 708, 1171, and 735 seconds in the model I, II, and III analyses, respectively, by the computer of HITAC M280H used in the University of Tokyo. This means that the model III analysis is about 50 % more economical than the model II analysis in the computing time.

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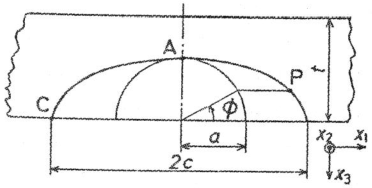

Fig. 1 Configuration of the surface crack

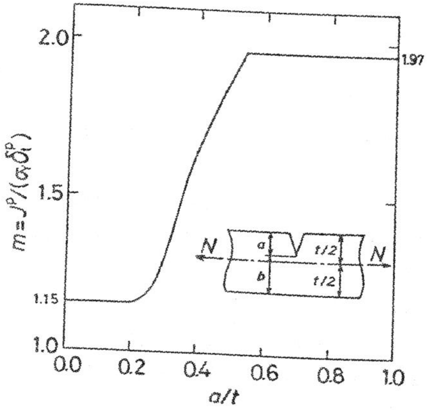

Fig. 2(a) Relation between m and a/t for the pin-loaded SEC specimen

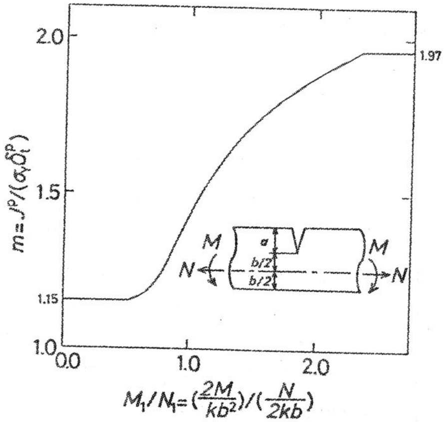

Fig. 2(b) Relation between m and M_1/N_1 for SEC specimen subject to combined axial force and bending

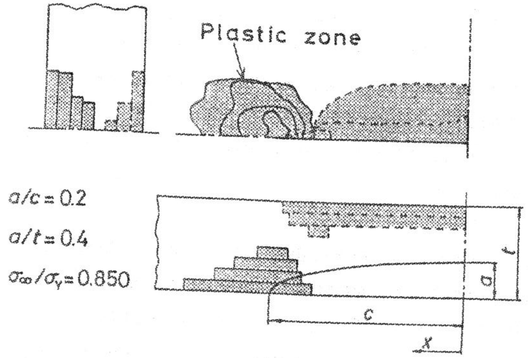

Fig. 3 Plastic zones occured in the flat shell elements, the model II analysis

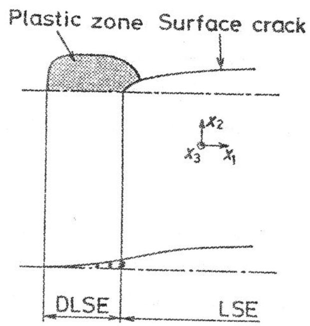

Fig. 4 Line spring model of Dugdale type

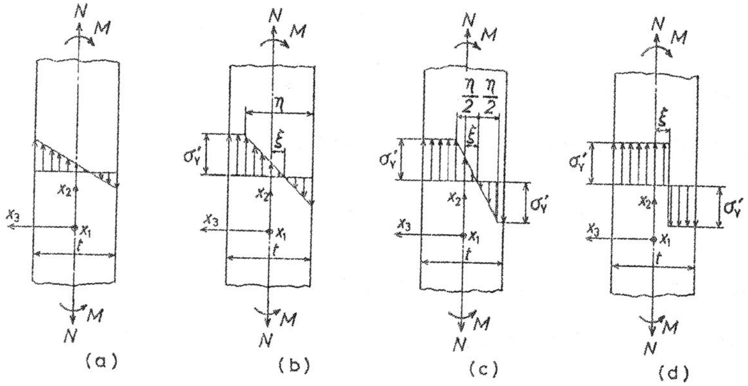

Fig. 5 Elastic-plastic stress state for plane strain flat plate subject to combined axial force and bending. $\sigma_Y' = \sigma_Y/\sqrt{1-\nu+\nu^2}$ where ν is the Poisson's ratio

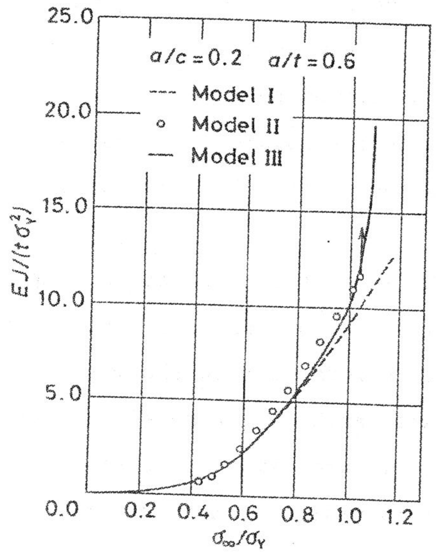

Fig. 6 J-integral at the center of the surface crack (point A in Fig. 1) versus applied stress, results of the analyses by three models