THE NONLINEAR LINE-SPRING MODEL FOR SURFACE CRACK ANALYSIS WITH PARTICULAR REFERENCE TO WORK HARDENING MATERIALS

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ABSTRACT

It is presented in this paper a nonlinear line-spring model for the elastic-plastic analysis of surface cracks with particular reference to the work hardening materials. For the cases of surface cracks commonly occurring in thin plates and cylindrical shells, the J-integrals are evaluated and the results are compared with those derived by D.M. Parks.

INTRODUCTION

Owing to its three-dimensional nature, there is no complete and accurate elastic-plastic solution available for surface cracks. Early in 1972, Rice and Levy[1] proposed a linear line-spring model for the elastic analysis of surface-cracks in plates. This approach has been proved to be reliable by a comprehensive three-dimensional finite-element calculation by Raju and Newman[2]. Recently, some researchers have renewed their interest in this model[3–6]. Parks tried to extend this model to the elastic-plastic analysis of a surface crack and asserted that this area is worth while to develop[3,4]. In references [5] and [6], the nonlinear consititutive relation of line-spring model was established adopting the D-M model of edge crack strip, and the elastic-plastic analysis of surface cracked plates was tentatively made. In the present work, we devote our attention to the nonlinear line-spring of work hardening material with Ramberg-Osgood constitutive relation, and take account of the plastic effects on crack tip by D-M model. The approximate generalized yield surface used by Parks[3] is revised so that the uncracked accurate yield surface may be regained as $a/t \to 0$. The rectified yield surface is then linearized approximately and transformed the line-spring constitutive relation of incremental form into the total deformation expression.

NON-LINEAR CONSTITUTIVE RELATIONS FOR THE LINE-SPRING OF WORK HARDENING MATERIALS

The semi-elliptical surface crack of length 2c and depth a as shown in Fig. 1(a) is replaced by a through crack of length 2c with distributed line-springs connecting the two crack surfaces, as shown in Fig. 1(b). The constitutive properties of the line-spring are simulated by an edge crack plane strain strip as shown in Fig. 1(c).

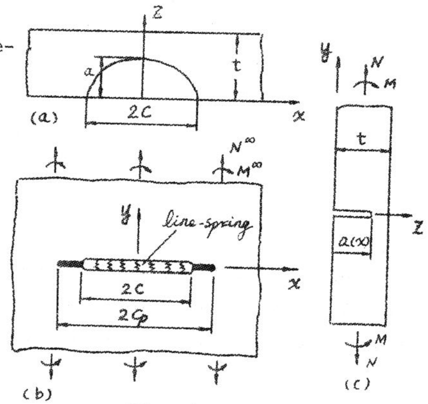

Fig. 1

First, consider this edge crack strip subjected to the combined action of membrane force N and bending moment M. It is assumed that material obeys Ramberg-Osgood stress-strain relation

$$\varepsilon/\varepsilon_s = (\sigma/\sigma_s) + \alpha(\sigma/\sigma_s)^n$$

where α and n are material constants, σ_s the yield stress, and $\varepsilon_s = \sigma_s/E$.

For the edge crack strip subjected to the membrane stress $\sigma_M = N/t$ only, the numerical fully plastic solutions were given by Shih et al. using finite element analysis[7]. The plastic stretch δ^P induced by the crack is expressed by

$$\overline{\delta}^P = \delta^P/t = \beta_1\zeta h_3(\zeta,n)\overline{\sigma}_M^n \qquad (1)$$

where $\zeta=a/t$, $\overline{\sigma}_M=\sigma_M/\sigma_s=N/t\sigma_s$, β_1 and $h_3(\zeta,n)$ are given in reference [7].

When the edge crack strip is subjected to the combined action of tensile stress σ_M and bending stress σ_B, its nonlinear constitutive relation is generally described by increment theory. Suppose that the plastic stretch δ^P and the plastic rotation θ^P are induced in the edge crack strip. Then the increment of plastic work done on the strip is equal to

$$dW^P = Nd\delta^P + Md\theta^P = t\sigma_M d\delta^P + t^2\sigma_B d\theta^P/6 = t^2\sigma_s\{\overline{\sigma}\}d\{\overline{q}^P\} \quad (2)$$

where, W^P = plastic work, $\{\overline{\sigma}\}^T = [\overline{\sigma}_M,\overline{\sigma}_B]$, and $\{\overline{q}^P\}^T = [\overline{\delta}^P,\overline{\theta}^P]$, with $\overline{\sigma}_M = \sigma_M/\sigma_s$, $\overline{\sigma}_B = \sigma_B/\sigma_s$, $\overline{\delta}^P = \delta^P/t$ and $\overline{\theta}^P = \theta^P/6$. Assume that the work hardening is isotropic so that the condition of hardening may be expressed by

$$\phi(\overline{\sigma}_M,\overline{\sigma}_B,\zeta) - F(W^P) = 0 \qquad (3a)$$

Then

$$d\phi = \{\frac{\partial\phi}{\partial\overline{\sigma}}\}[d\overline{\sigma}] = F'dW^P \qquad (3b)$$

Making use of the orthogonal condition between the two stress systems, the generalized plastic displacement increment $d\{\overline{q}^P\}$ may be written as

$$d\{\overline{q}^P\} = d\lambda\{\frac{d\phi}{d\overline{\sigma}}\} \qquad (4)$$

Substituting Eq. (4) into (2) and rearranging, gives

$$d\lambda = d\phi/F't^2\sigma_s[\frac{\partial\phi}{\partial\overline{\sigma}}]\{\overline{\sigma}\} \qquad (5)$$

thus

$$d\{\overline{q}^P\} = \{\frac{\partial\phi}{\partial\overline{\sigma}}\}d\phi/F't^2\sigma_s[\frac{\partial\phi}{\partial\overline{\sigma}}]\{\overline{\sigma}\}$$

in which F' may be determined from the uniaxial tension curve. Introducing the equivalent "univalent" stress $\overline{\sigma}_e$ in the strip, we have $\phi(\overline{\sigma}_e,\zeta)-F(W^P)=0$.

Consequently,

$$d\phi = \frac{\partial\phi}{\partial\overline{\sigma}_e}d\overline{\sigma}_e = F'dW^P = F't^2\sigma_s\overline{\sigma}_e d\overline{\delta}^P$$

Substituting Eq. (1) into the above equation and rearranging, gives

$$F' = \frac{\partial\phi}{\partial\overline{\sigma}_e}/t^2\sigma_s\beta_1\zeta h_3(\zeta,n)n\overline{\sigma}_e^n$$

Thus, Eq. (5) becomes

$$d\{\overline{q}^P\} = \beta_1\zeta h_3 n\overline{\sigma}_e^n\{\frac{\partial\overline{\sigma}_e}{\partial\overline{\sigma}}\}/[\frac{\partial\overline{\sigma}_e}{\partial\overline{\sigma}}]\{\overline{\sigma}\} \qquad (6)$$

in which $\overline{\sigma}_e$ may be determined by the generalized yield condition of the edge crack strip. Rice obtained an approximate yield condition according to the slip-line analysis of edge crack strip. We express the yield surface in the following parametric form

$$\{\overline{\sigma}\} = \{A(\alpha,\beta)\} \quad \text{and} \quad 2(\alpha-\beta) = tg\alpha - tg\beta \qquad (7)$$

To simplify numerical work. The yield surface is linearized as follows: $\bar{\sigma}_e = \bar{\sigma}_M + \frac{1}{3}k_1(\zeta)\ \bar{\sigma}_B$. The resulting yield surface, however, crosses the accurate uncracked yield surface as $\zeta\to 0$[3]. To overcome this difficulty, We can estimate the limiting yield condition from the limit analysis of a simple beam. Then, the linearized stress $\bar{\sigma}_e$ should approach to the following value as $\zeta\to 0$: $\bar{\sigma}_e = \bar{\sigma}_M + \frac{1}{3}k_2(\zeta)\ \bar{\sigma}_B$. Thus, the approximate equivalent stress may be expressed by

$$\bar{\sigma}_e = \bar{\sigma}_M + \frac{1}{3}k(\zeta)\ \bar{\sigma}_B \tag{8}$$

where, $k(\zeta) = k_1(\zeta)\zeta + K_1(\zeta)(1-\zeta^r)$, $r<1$. When the $\bar{\sigma}_e$ is expressed by equation (8), equation (6) can be integrated to obtain the total deformation expression

$$\{\bar{q}^p\} = \beta_1\zeta h_3(\zeta,n)\bar{\sigma}_e^{n-1}[D_t]\{\bar{\sigma}\} \tag{9}$$

where $$[D_t] = \begin{bmatrix} 1 & \frac{1}{3}k \\ \frac{1}{3}k & \frac{1}{9}k^2 \end{bmatrix}$$

THE GOVERNING EQUATIONS AND J-INTEGRAL FORMULA

Generally, the nonlinear constitutive relation can only be expressed in the following incremental form

$$d\{\bar{q}\} = d\{\bar{q}^e\} + d\{\bar{q}^p\} = [S_t^{ep}]\ d\{\bar{\sigma}\} \tag{10}$$

where $[S_t^{ep}] = 2\sigma_s(1-\nu^2)[C]/E + \beta_1\zeta h_3(\zeta,n)n\bar{\sigma}_e^n\ [D_t]/[\frac{\partial\bar{\sigma}_e}{\partial\bar{\sigma}}]\{\bar{\sigma}\}$, $[C] = \begin{bmatrix} \alpha_{MM} & \alpha_{MB} \\ \alpha_{BM} & \alpha_{BB} \end{bmatrix}$

$\alpha_{\lambda\mu} = \int_0^{\zeta_e} g_\lambda(\zeta)\ g_\mu(\zeta)d\zeta$, $g_M(\zeta)$ and $g_B(\zeta)$ may be taken from reference [8], $\zeta_e = a_e/t$, and a_e is the Irwin's equivalent crack size with plastic zone correction[7].

From the D-M model solutions for thin plate with a through crack, as shown in Fig. 1(b), the following equations are obtained

$$\{\bar{q}\} = 4\sigma_s c[B](I(\bar{x})\ \{\bar{\sigma}^\infty\} - \int_0^1 H(\bar{x},\bar{t})\{\bar{\sigma}(\bar{t})\}d\bar{t})/Et \tag{11}$$

where $I(\bar{x}) = (\bar{c}_p^2 - \bar{x}^2)^{\frac{1}{2}} - \pi G(\bar{x},\bar{c}_p)/2$, $G(\bar{x},\bar{c}_p) = \int_0^{\bar{c}_p} F(\bar{x},\bar{t})d\bar{t}/s$, $s = \pi/2 - \arcsin(1/\bar{c}_p)$, $H(\bar{x},\bar{t}) = F(\bar{x},\bar{t}) - G(\bar{x},\bar{c}_p)/(\bar{c}_p^2 - \bar{t}^2)^{\frac{1}{2}}$, $\bar{c}_p = c_p/c$, $\bar{x} = x/c$, $\{\bar{\sigma}^\infty\} = [N^\infty/t\sigma_s,\ 6M^\infty/t^2\sigma_s]$, $[B] = \begin{bmatrix} 1 & 0 \\ 0 & \gamma \end{bmatrix}$, $\gamma = (1+\nu)/3(3+\nu)$, ν = Poisson's ratio $F(\bar{x},\bar{t}) = (1/\pi)\ln|((\bar{c}_p^2 - \bar{x}^2)^{\frac{1}{2}} + (\bar{c}_p^2 - \bar{t}^2)^{\frac{1}{2}})/((\bar{c}_p^2 - \bar{x}^2)^{\frac{1}{2}} - (\bar{c}_p^2 - \bar{t}^2)^{\frac{1}{2}})|$, and

$$(\pi\bar{\sigma}_M^\infty/2 - \int_0^1 \bar{\sigma}_M(\bar{x})/\sqrt{\bar{c}_p^2 - \bar{x}^2}d\bar{x}\ )^2/s^2 + 2|\pi\bar{\sigma}_B^\infty/2 - \int_0^1 \bar{\sigma}_B(\bar{x})/\sqrt{\bar{c}_p^2 - \bar{x}^2}\ d\bar{x}|/3s = 1 \tag{12}$$

Differentiateing the equation (11) and (12), we get respectively[9]

$$d\{\bar{q}\} = (4\sigma_s c/t)[B](I(\bar{x})d\{\bar{\sigma}^\infty\} - \int_0^1 H(\bar{x},\bar{t})d\{\bar{\sigma}(\bar{t})\}d\bar{t} + L_1(\bar{x})d\bar{c}_p \tag{13}$$

and

$$(\pi\bar{\sigma}_M^\infty/2 - \int_0^1 \bar{\sigma}_M(\bar{x})/(\bar{c}_p^2 - \bar{x}^2)^{\frac{1}{2}}\ d\bar{x})(\pi d\bar{\sigma}_M^\infty/2 - \int_0^1 d\bar{\sigma}_M(\bar{x})/(\bar{c}_p^2 - \bar{x}^2)^{\frac{1}{2}}d\bar{x}) + (s/3)(\pi d\bar{\sigma}_B^\infty/2 - \int_0^1 d\bar{\sigma}_B(\bar{x})/(\bar{c}_p^2 - \bar{x}^2)^{\frac{1}{2}}d\bar{x}) + L_2 d\bar{c}_p = 0 \tag{14}$$

Substituting equation (10) into equation (13), gives

$$(Et/4\sigma_s c)[B]^{-1}[S_t^{ep}]d\{\bar{\sigma}\} + \int_0^1 H(\bar{x},\bar{t})d\{\bar{\sigma}\}d\bar{t} - L_1(\bar{x})dc_p = I(\bar{x})d\{\bar{\sigma}^\infty\} \tag{15}$$

Thus, the equations (14) and (15) constitute the basic incremental equations for this problem.

When the stress $\bar{\sigma}_e$ takes the form of equation (8), from equation (9), the constitutive relation of line-spring is

$$\{\bar{q}\} = \{\bar{q}^e\} + \{\bar{q}^p\} = [S^{ep}]\{\bar{\sigma}\} \tag{16}$$

where $[S^{ep}] = [2\sigma_s(1-\nu^2)/E][C] + \beta_1\zeta h_3(\zeta,n)\ \bar{\sigma}_e^{n-1}\ [D_t]$. Substituting equation (16) into equation (11), gives

$$(Et/4\sigma_s c)[B]^{-1}[S^{ep}]\{\bar{\sigma}\} + \int_0^1 H(\bar{x},\bar{t})\ \{\bar{\sigma}(\bar{t})\}d\bar{t} = I(\bar{x})\ \{\bar{\sigma}^\infty\} \tag{17}$$

the resulting equation (17) and (12) are the basic equations in total deformation expression for this problem.

Now, we come to the formulation of J-integral and tearing modulus T_J.

For the case of fixed force

$$J = \frac{\partial \bar{U}}{\partial a} \qquad (18)$$

where $\bar{U}$ is the complementary strain energy. In the present case, it becomes $\bar{U} = \int (t\delta d\sigma_M + t^2\theta d\sigma_B/6) = t^2\sigma_s \int [\bar{q}] d\{\bar{\sigma}\}$. Substituting equation (16) into the integral and the above equation consecutively into Eq.(18), gives

$$J = (t\sigma_s^2/E)(1-\nu^2)[\bar{\sigma}][C']\{\bar{\sigma}\} + (\alpha/(n+1)(1.455)^n)\bar{\sigma}_e^n B(\zeta,n)) \qquad (19)$$

where $[C'] = \begin{bmatrix} \alpha'_{MM} & \alpha'_{MB} \\ \alpha'_{BM} & \alpha'_{BB} \end{bmatrix}$, $\alpha'_{\lambda\mu} = g_\lambda(\zeta_e)g_\mu(\zeta_e)$, $B(\zeta,n) = A'\bar{\sigma}_e + \frac{1}{3}(n+1)Ak'(\zeta)\bar{\sigma}_s$,

$A = \zeta h_3(\zeta,n)/(\sqrt{(1-\zeta)^2+\zeta^2}-\zeta)^n$. When $\{\bar{\sigma}\}$ is known, we can calculate the J-integral from equation (19) and hence tearing modulus T_J.

NUMERICAL RESULTS

(1) Thin plate with a surface crack, subjected to remote uniform uniaxial N^∞. The calculated stress distribution $\bar{\sigma}_M(\bar{x})$ of the line-spring are given in Fig. 2. In this figure, the difference between the calculated stress distributions from Rice's linear and our nonlinear line-spring models indicates the plastic effects. Plasticity, as a rule, lowers the stresses. The normalized $J \sim \bar{\sigma}_M^\infty$ curves for semi-elliptical surface cracks of constant length 2c=6t and various depth ratio a/t are given in Fig. 3. The related results obtained by Parks[3] are also included in the same figure for comparison. It is apparent that the agreement is fairly good, but our results are somewhat lower than those from Parks.

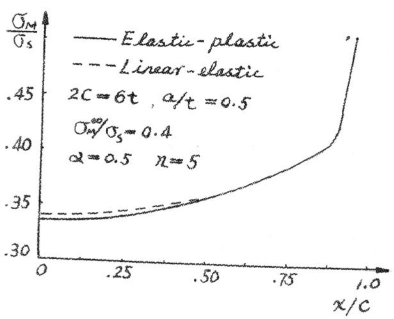

Fig. 2

(2) Cylindrical shell with a long longitudinal surface crack on its inner surface, subjected to a uniform internal pressure p. When the longitudinal surface crack is comparatively long, the problem becomes a plane strain one with a single line-spring element. Similar to the procedure steps taken by Parks[3], the numerical results are given in Fig.4 and Fig.5. The effect of depth ratio a/t on the J∼p curves with R/t=10, R=mean radius of cylindrical shell and t=wall thickness of shell is given in Fig.4. The effect of radius-thickness ratio R/t on the J∼p curves, is given in Fig. 5. The results obtained by Parks[3] are also included in these Figs. It can be seen that in Parks' curves there is an abrupt transition from partial yielding to complete yielding of ligament, which is rather unnatural.

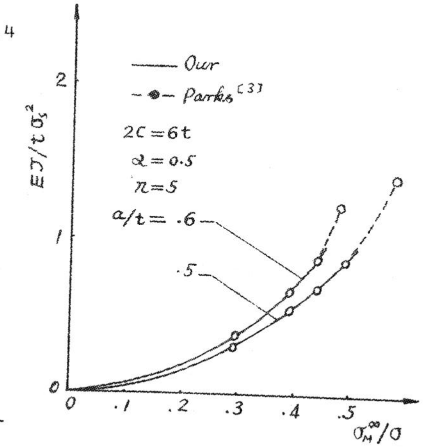

Fig. 3

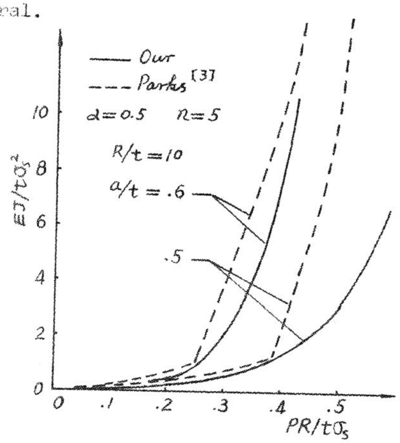

Fig. 4

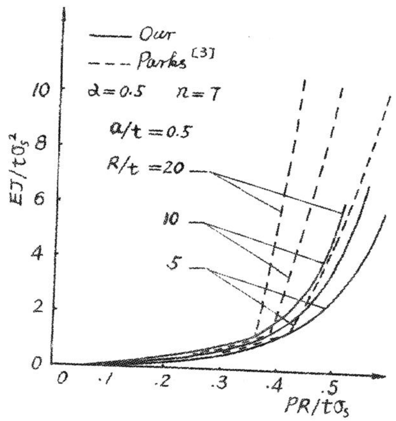

Fig. 5

CONCLUSION

We have developed a nonlinear line-spring model for surface crack problem of hardening materials based on the fully plastic solution of edge crack strip. The model is quite suitable for the computation of J-integral and tearing modulus T_J. Numerical results are compared with those by linear elastic model and Parks' elastic-plastic model. It is shown that the present model is easy to handle and yields reasonable results.

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