ON THE NUMERICAL EVALUATION OF C*-INTEGRALS FOR CREEP FRACTURE ANALYSIS

Liu Yuanjie(刘元杰)
Zhengzhou Research Institute of Mechanical Engineering, China.
T.R. Hsu and Z.H. Zhai
Department of Mechanical Engineering, University of Manitoba, Winnipeg. Manitoba, Canada.

INTRODUCTION

The concept of using an energy rate (or power) line integral, C* for the characterization of crack growth in a solid due to creep was first introduced in the early Seventy's[1]. It is simply a modification of the J-integral[2] where strain and displacement components are replaced by their rates. The power integral as expressed in [1] is considered to be path-independent such as the case for the J-integral. The use of the C*-integral has also been extended to characterize the crack growth in a solid under steady state power law conditions[3] and the description of the crack tip behaviour at different stages of creep deformation[4,5].

This paper presents the formulation of a new power integral C^*_g which covers the entire range of creep deformation coupled with general thermomechanical loading conditions. It can be used in conjunction with the tearing modulus for the prediction of complex crack growth behaviour including the initial and the subsequent growth of cracks at elevated temperature due to creep as demonstrated in a separate publication. Numerical evaluation of the C* and C^*_g integrals has been performed by means of a finite element analysis on an experimental case described in Ref. [6]. It can be shown that better agreement with the experimental results have been achieved by using C^*_g than by the C*-integral. Both C* and C^*_g-integral appeared to show fairly good path-independence, except the presence of large plastic region enclosed by the path of integration.

DERIVATION OF C^*_g-INTEGRAL

The generalized power integral, C^*_g can be derived by a similar energy balance concept as described in [7] with a virtual time-independent crack extension method and the following assumptions:

(1) The material is assumed to be initially isotropic and homogeneous. A single through crack is treated in a solid of uniform thickness;

(2) A fracture process region exists near the crack tip as shown in Fig.1. As discussed by Broberg[8,9], the size and shape of this region are independent of the solid geometry and the loading conditions. Void initiation, growth and linkage are to take place in this region, thus invalidate the use of the continuum mechanics theory;

(3) Balance of energy production and dissapation during the creep fracture process is to be satisfied despite of the presence of the fracture process region. Similar assumption was also made in [7];

(4) The fracture process region is considered to be an energy sink during the creep energy dissipation process. Since the distribution of creep energy dissipation in a cracked solid is not uniform, severe concentrations of energy dissipation will occur in the fracture process region, thus causing a creep energy dissipation flux into this region. The intensity of the energy flux flowing into this region characterizes the singularity of the creep energy dissipation rate near the crack tip.

Based on the above assumptions, it is possible to derive this integral on the basis of the equilibrium and energy rate balance equations:

$$\sigma_{ij,j} + F_i = 0 \qquad (1)$$

and

$$\int_s T_i \dot{U}_i \, ds - \int_A (-F_i \dot{U}_i + \sigma_{ij} \dot{\varepsilon}_{ij}) dA = 0 \qquad (2)$$

in which T_i is the traction acting on the surface s and F_i is the body force components in the volume V and area A with unit thickness. The creep energy dissipation flux into the fracture process

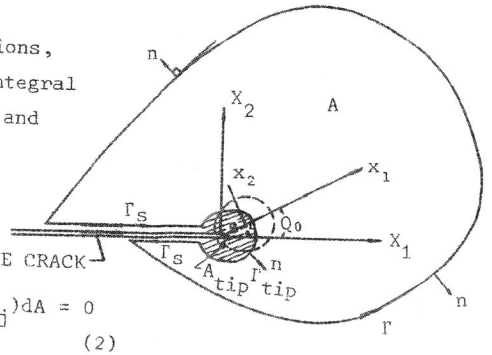

Fig.1 Configuration and coordinates of line crack

region in a cracked solid can be expressed as:

$$\dot{P}(a) = \int_{\Gamma+\Gamma_s} T_i \dot{U}_i d\Gamma - \int_A [-F_i \dot{U}_i + \sigma_{ij}\dot{\varepsilon}_{ij}]dA \qquad (3)$$

where T_i is the traction acting on the contour Γ and crack surface Γ_s as illustrated in Fig.1, $\dot{U}_i$, $\dot{\varepsilon}_{ij}$ are the respective rates of displacement and strain components. It has been proven that $\dot{P}(a)$ is a path independent integral which represents creep energy dissipation rate into the fracture process region.

The total strain rate components in Eq. (3) can be expressed in the following form by the partition theory:

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}^e_{ij} + \dot{\varepsilon}^c_{ij} + \dot{\varepsilon}^p_{ij} \qquad (4)$$

where $\dot{\varepsilon}^e_{ij}$, $\dot{\varepsilon}^c_{ij}$ and $\dot{\varepsilon}^p_{ij}$ are the respective elastic, creep and plastic strain rate components. Eq. (3) can thus be expressed in the following form:

$$\dot{P}(a) = \int_{\Gamma+\Gamma_s} T_i \dot{U}_i d\Gamma - \int_A [-F_i \dot{U}_i + \sigma_{ij}\dot{\varepsilon}^e_{ij} + \sigma_{ij}\dot{\varepsilon}^c_{ij} + \sigma_{ij}\dot{\varepsilon}^p_{ij}]dA \qquad (5)$$

Considering the energy balance of the material in A and A_{tip} during virtual crack extention Δa, we assume all changes of the field quantities, such as $\Delta\dot{u}$, $\Delta\dot{\varepsilon}^e_{ij}$, $\Delta\dot{\varepsilon}^c_{ij}$ and $\Delta\dot{\varepsilon}^p_{ij}$ are attritable to changes in crack length, then the following relation is obtained:

$$\Delta\dot{P}(a) = \frac{d\dot{P}(a)}{da}\cdot\Delta a$$

$$= \int_{\Gamma+\Gamma_s} T_i \Delta\dot{U}_i d\Gamma - \int_A [-F_i \Delta\dot{U}_i + \sigma_{ij}\Delta\dot{\varepsilon}^e_{ij} + \sigma_{ij}\Delta\dot{\varepsilon}^c_{ij} + \sigma_{ij}\Delta\dot{\varepsilon}^p_{ij}]dA \qquad (6)$$

and hence:

$$C^*_g = \frac{d\dot{P}(a)}{da} = \int_{\Gamma+\Gamma_s} T_i \frac{d\dot{U}_i}{da} d\Gamma - \int_A [-F_i \frac{d\dot{U}_i}{da} + \sigma_{ij}\frac{d\dot{\varepsilon}^e_{ij}}{da} + \sigma_{ij}\frac{d\dot{\varepsilon}^c}{da} + \sigma_{ij}\frac{d\dot{\varepsilon}^p_{ij}}{da}]dA \qquad (7)$$

A proper transformation of the C^*_g-integral in Eq. (7) between the fixed and moving frames is necessary in order to describe a growing crack.

Refering to the coordinate systems given in Fig. 1, the following relations can be used for such transformation:

$$x_1 = X_1\cos\theta_0 + X_2\sin\theta_0 - a$$

$$x_2 = -X_1\sin\theta_0 + X_2\cos\theta_0 \qquad (8)$$

$$U_i = U_i(x_1, x_2, a) \qquad (9)$$

in which (x_1, x_2) and (X_1, X_2) represent the respective instantaneous and the original coordinate systems.

From the "creep-elastic analogue theory",

$$\frac{\partial\dot{W}_c(\dot{\varepsilon}^c_{ij})}{\partial\dot{\varepsilon}^c_{ij}} = \sigma_{ij} \qquad (10)$$

By using the Gauss' theorem, Eq. (7) can be expressed as:

$$C^*_g = \sum_{m=1}^{2}\left\{\int_{\Gamma+\Gamma_s-\Gamma_{tip}} \dot{W}_c(\dot{\varepsilon}^c_{ij})n_m d\Gamma - \int_{\Gamma+\Gamma_s} T_i\frac{\partial\dot{U}_i}{\partial x_m} d\Gamma + \right.$$

$$\left. + \int_A [-F_i\frac{\partial\dot{U}_i}{\partial x_m} + \sigma_{ij}\frac{\partial\dot{\varepsilon}^*_{ij}}{\partial x_m}]dA\right\} \times \begin{cases}\cos\theta_0 & \text{for } m = 1\\ \sin\theta_0 & \text{for } m = 2\end{cases} \qquad (11)$$

in which $n_m (m = 1,2)$ are outward normals to the contour Γ with respect to x_1 and x_2 coordinates, and $\dot{\varepsilon}^*_{ij} = \dot{\varepsilon}^e_{ij} + \dot{\varepsilon}^p_{ij}$

For Mode I, i.e. opening mode crack extension with $\theta_0=0$, Eq. (11) reduces to:

$$C^*_g = \int_{\Gamma+\Gamma_s-\Gamma_{tip}} \dot{W}_c n_1 d\Gamma - \int_{\Gamma+\Gamma_s} T_i\frac{\partial\dot{U}_i}{\partial x_1} d\Gamma + \int_A (-F_i\frac{\partial\dot{U}_i}{\partial x_1} + \sigma_{ij}\frac{\partial\dot{\varepsilon}^*_{ij}}{\partial x_1})dA \qquad (12)$$

NUMERICAL EVALUATION OF C-INTEGRALS

Since the fracture process region is normally negligibly small, the C^*_g-integral in Eq. (12) cna be simplified to be:

$$C^*_g = \int_{\Gamma+\Gamma_s} (\dot{W}_c n_1 - T_i\frac{\partial\dot{U}_i}{\partial x_1})d\Gamma + \int_A \sigma_{ij}\frac{\partial\dot{\varepsilon}^*_{ij}}{\partial x_1} dA \qquad (13)$$

with the absence of the body force. If creep strain rate $\dot{\varepsilon}^c_{ij}$ is dominant

during deformationa, the contributions of elastic and plastic strain rate, $\dot{\varepsilon}^{*}_{ij}$, can be neglected.

The expression of C-integral neglected 2nd term in Eq. (13) is identical to that given in [1] as shown below:

$$C^{*} = \int_{\Gamma+\Gamma_s} [\dot{W}_c n_1 - T_i \frac{\partial \dot{U}_i}{\partial x_1}] d\Gamma \qquad (14)$$

Numerical values of C^{*}_{g} and C^{*} in Eqs. (13) and (14) were computed by a finite element stress analysis code, TEPSA developed in the senior author's laboratories. General description of this code can be found in [10]. The case used for the evaluation involves the growth of a line crack in a thin plate made of 304 stainless steel. Detail description of this experiment can be found in [6]. Fig. 2 shows the dimensions and the loading conditions for the cracked plate.

Pertinent material properties used in the numerical evaluation are tabulated as follows:

Young's modulus:	140,000 MPa
Shear modulus of elasticity:	53,800 MPa
Poisson's ratio:	0.3
Plastic modulus:	700 MPa
Yield strength:	103 MPa
Creep law:	$\dot{\varepsilon}^{c} = 1.37 \times 10^{-18} \sigma_n^{7.1}$/hr.

with the nominal stress, $\sigma_n < 176.5$ MPa

Fig. 3 shows the region for the finite element model with four selected contour regions, Γ_1 to Γ_4 as indicated.

Two element sizes were used near the crack for the computations; Type A elements were of 0.4 mm ×1 mm with a total number of 154 elements whereas Type B elements were of 0.3 mm × 0.6 mm with a total of 201 elements.

Numerical values for C^{*}_{g} in Eq. (13) and C^{*} in Eq. (14) by the four selected contours are illustrated in Fig. 4 and 5 respectively, whereas the corresponding values by empirical formula C^{*}_{exp} are shown in Fig. 6. The values of C^{*}_{exp} were computed by the following expression [6]:

$$C^{*}_{exp} = \frac{\alpha-1}{\alpha+1} \sigma_{net} \dot{V} \qquad (15)$$

in which α is the stress power in material's creep law (7.1 in this case), σ_{net} is the net cross-sectional stress and $\dot{V}$ is the calculated crack face opening velocity.

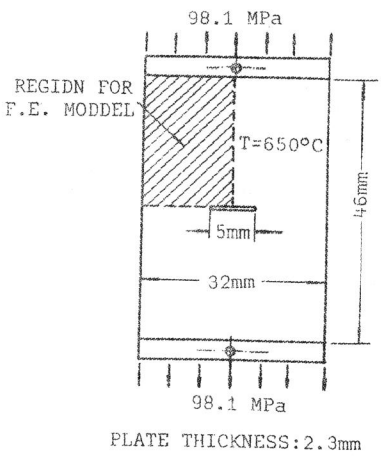

Fig. 2 Dimensions and loadings on a cracked plate specimen

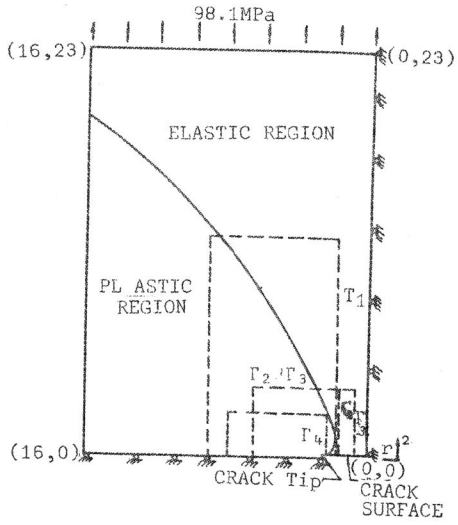

Fig. 3 Close contours for C*-Integrals

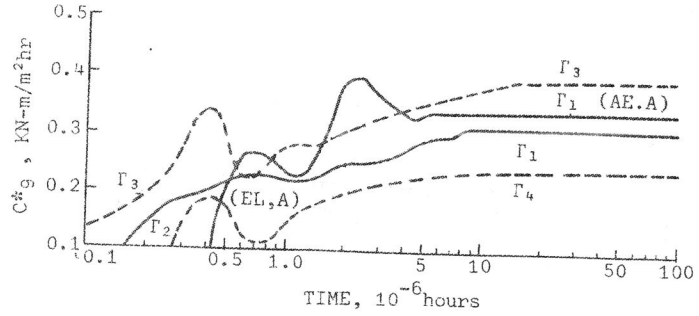

Fig. 4 Numerical values for C^{*}_{g}-Integral

Fig. 7 shows the correlations of C^{*}_{g} and C^{*} with the corresponding results by C^{*}_{exp}. Results shown in this figure were computed by integrations over the contour Γ_2 using both types of elements. The bold lines are the results obtained by the type A elements whereas the thin lines represent the results by type B elements.

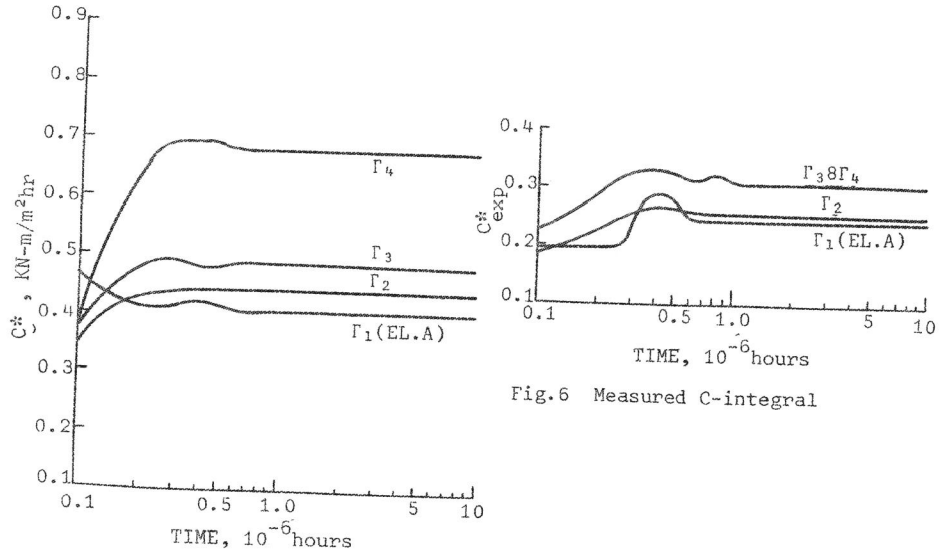

Fig.5 Numerical values of C*-integral

Fig.6 Measured C-integral

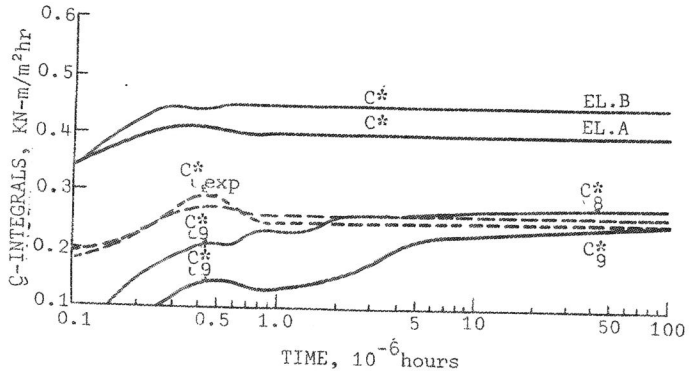

Fig. 7 Correlation of C*-integrals

DISCUSSION OF RESULTS

A generalized power line integral C^*_g which can characterize the crack growth in a solid subject to thermoelastic-plastic-creep loadings has been derived. Numerical evaluation of this integral, along with those of the established C* integral have been performed and the results have been correlated with the results by C^*_{exp}. Four distinct contours, Γ_1 to Γ_4 were selected for the numerical evaluation as shown in Fig. 3. As can be observed from this figure, Γ_1 encompassed evenly on both elastic and plastic regions, whereas Γ_4 is entirely in the plastic region.

The numerical results of C^*_g and C* in Figs. 4 and 5 clearly indicate fairly good path-independence of the integrals except the case with the Γ_4 contour. One possible explanation is that the C* or C^*_g, like J, is considered to be valid only for a small plastic deformation, the selection of a contour such as Γ_4 which involves an entire plastic region had indeed led to inaccurate results.

Results shown in Fig. 7 indicate a closer correlation between C^*_g and C^*_{exp} than that by C* in the present case.

Anobvious shortcoming of the C^*_g-integral is its numerical instability exhibited at the early stage of the creep deformation as shown in Fig. 4. The numerical values of C*-integral have shown a more stable behavior than C^*_g in computation.

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