A FINITE ELEMENT ANALYSIS OF NONPLANAR CRACK GROWTH IN A LINEAR ELASTIC AND IN AN ELASTIC-PLASTIC MATERIAL

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1. INTRODUCTION

In the past twenty years the inclined crack shown in Fig.1a has aroused considerable interest because it presents a mixed-mode problem involving in general nonplanar crack extension in a direction indicated by the orientation angle α. The greater majority of the investigations were linear elastic applicable to very brittle materials but more recently mixed mode finite element analyses have been carried out for non-linear elastic materials exhibiting Rice-Rosengren-Hutchinson (RRH) type singularities [1] and on elastic-plastic materials under uniaxial [2] and biaxial [3] loading.

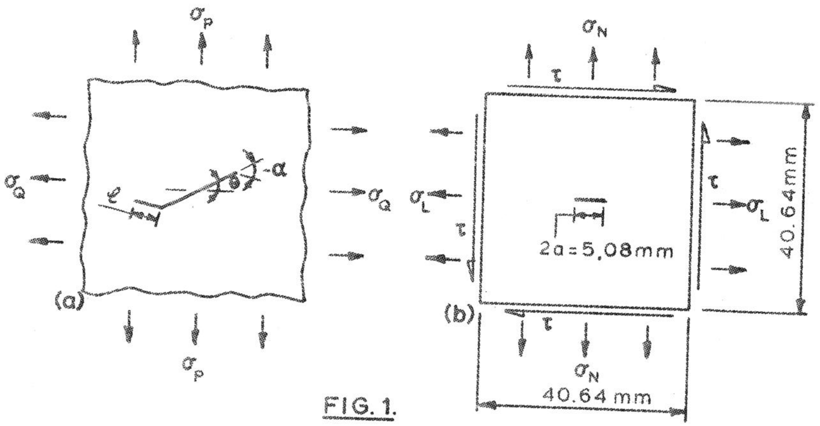


FIG. 1.

An inclined crack (a) in a biaxial principal stress field and (b) a center-cracked plate loaded by an equivalent stress system.

Unfortunately, lack of space prohibits any historical account but more extensive lists of references are contained in the ones mentioned here. Experimental studies on mixed mode loadings have also been undertaken, e.g. the ones with implications on fatigue crack growth threshholds [4].

It is convenient to analyse the centre-cracked plate (CCP) in Fig. 1b with an equivalent applied stress system to that in Fig.1a. The relations between the stresses in the two systems can be found in [2]. The loads on the crack have also been characterized by $K_I = \sigma_N K^* \sqrt{a}$ and $K_{II} = \tau K^* \sqrt{a}$ where a is the half-crack length and $K^* = \sqrt{\pi}$ for the infinite plane, e.g. by Bilby and Cardew [5] who examined a crack with a small kink of length ℓ emanating from the tip of the main crack in a linear elastic material. They used the following formulae to evaluate the Mode I and Mode II stress intensity factors, $k_1(\alpha)$ and $k_2(\alpha)$, respectively, at the tip of the kink

$$k_i(\alpha) = K_{i1}(\alpha)\, K_I + K_{i2}(\alpha)\, K_{II}, \quad i = 1,2 \qquad (1)$$

where K_{ij} are quadrature coefficients.

Equations (1) apply strictly when (ℓ/a) is infinitesimal, in which case the lateral stress σ_L, which does not appear in the equation, has no effect on the value of $k_i(\alpha)$. However, when (ℓ/a) is finite there is a 'T-effect' which can be characterized by the stress $\sigma_T = \sigma_L - \sigma_N$ in the case of the CCP. Using the parameter $\bar{\lambda} = (2\ell/a)^{\frac{1}{2}}$ Bilby et al calculated values of $k_1(\alpha)$ and $k_2(\alpha)$ for different values of α and $\bar{\lambda}$ in the uniaxial loading case, $\sigma_Q = 0$.

1.1. Griffith Energy Release and Crack Separation Energy Rates.

In a linear elastic material the crack separation energy rate $G^\Delta(\alpha)$ associated with a small discrete crack extension Δa in the direction α, is equal to Griffith's energy release rate $g(\alpha)$ when the kink extends in its own plane.

$$G^\Delta(\alpha) = g(\alpha) = G_I^\Delta(\alpha) + G_{II}^\Delta(\alpha) = k_1^2(\alpha)/E' + k_2^2(\alpha)/E' \qquad (2)$$

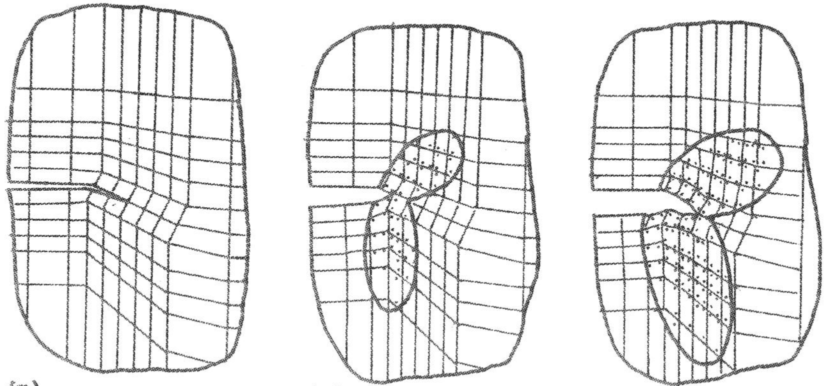
where $E' = E/(1-\nu^2)$. Noting that for coplanar crack extension $G_I^\Delta(0) = K_I^2/E'$ and $G_{II}^\Delta(0) = K_{II}^2/E'$ and using equation (2), equation (1) can be re-written

$$[G_I^\Delta(\alpha)]^{1/2} = K_{11}(\alpha)[G_I^\Delta(0)]^{1/2} + K_{12}(\alpha)[G_{II}^\Delta(0)]^{1/2}$$
$$[G_{II}^\Delta(\alpha)]^{1/2} = K_{21}(\alpha)[G_I^\Delta(0)]^{1/2} + K_{22}(\alpha)[G_{II}^\Delta(0)]^{1/2} \quad (3)$$

When the material is elastic-plastic $G^\Delta(\alpha)/g(\alpha)$ decreases as the ratio $(\Delta a/r_p)$ decreases, where r_p is the crack tip plastic zone size. In a non-hardening material $G^\Delta(\alpha)/g(\alpha)$ vanishes in the limit as $(\Delta a/r_p)$ tends to zero.

2. THE ANALYSES

The analyses were carried out on a similar plate to the one in reference [2] which contains also a description of the crack tip node release procedure used in the calculation of $G^\Delta(\alpha)$. Three consecutive nonplanar crack growth steps of length $\Delta a = 0.254$ mm were also applied here. Fig. 2 shows the region near the crack tip.



(a) (b) (c)

FIG. 2.

The area near the crack tip (a) in the linear-elastic material after two crack growth steps, (b) in the elastic-plastic material before crack extension and (c) after two crack growth steps. The crack opening displacements are exaggerated by a factor of 50.

2.1 The Elastic Analyses

Two cases were considered with the following loading patterns:

Case	θ	σ_P	σ_Q	σ_N	σ_L	τ	K_{II}/K_I	σ_T/σ_N
1	45^o	1.0	.4142	.7071	.7071	.2929	.4142	0
2	25^o	.8284	0	.7071	.1213	.2929	.4142	-.8285

The last two columns show that K_{II}/K_I is the same in both cases but in the first case $\sigma_T = 0$, i.e. there is no T-effect. Fig.3 shows the normalized values $[k_1^2(\alpha) + k_2^2(\alpha)]^{1/2}/K_{Io}$ against α for the two cases, where σ_o, K_{Io} and G_o are the values of σ_p, K_I and G^Δ at incipient yielding when $\theta = \alpha = 0$. Fig. 3 also shows the values obtained by the procedure of Bilby et al for the cases $\bar{\lambda} = 0$ and $\bar{\lambda} = 0.55$.

2.2 Elastic-Plastic Analyses

Elastic-plastic analyses were carried out for $\theta = 25^o$ and $\alpha = -25^o$. Fig.4 gives values of G^Δ/G_o calculated during the second crack growth step against the normalized uniaxially applied load $(\sigma_p/\sigma_o)^2$. Note the crack profiles in Figs. 2b and 2c corresponding to the load $(\sigma_p/\sigma_o)^2 = 6.75$ and the extension of the crack tip plastic zone as the crack advances, in Fig. 2c.

Although equations (3) apply strictly to linear elastic materials, using arguments concerning continuity of macroscopic behavioural patterns during the transition from linear elastic to elastic-plastic responses, it was suggested [2] that equations (3) could also be used to give working estimates of $G^\Delta(\alpha)$ in elastic-plastic materials when r_p is not large. The estimated values of $G^\Delta(\alpha)$ from equations (3), requiring the calculation of $G^\Delta(0)$ only, are also shown on Fig. 4.

3. CONCLUSIONS

3.1 Values of $[k_1^2(\alpha) + k_2(\alpha)^2]^{1/2}/K_{Io}$ obtained from linear elastic G^Δ calculations for nonplanar crack extension were in

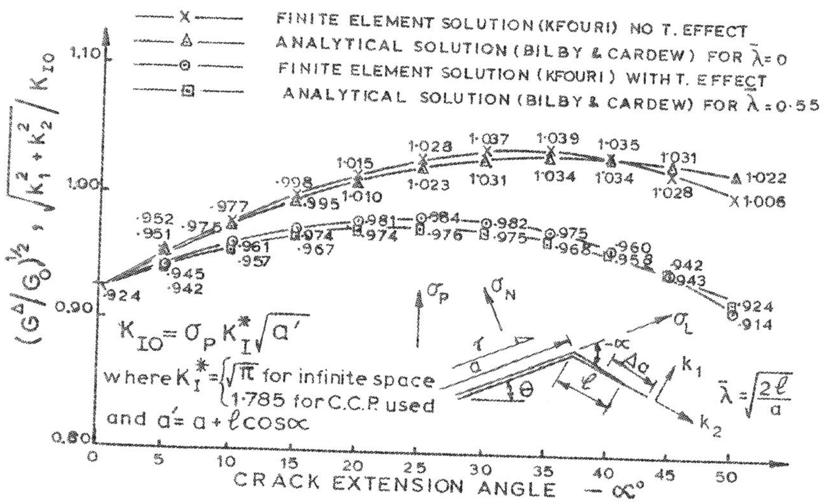


FIG. 3.

Normalized stress intensity factors at the tip of the kinked crack in a linear elastic material against the orientation angle α.

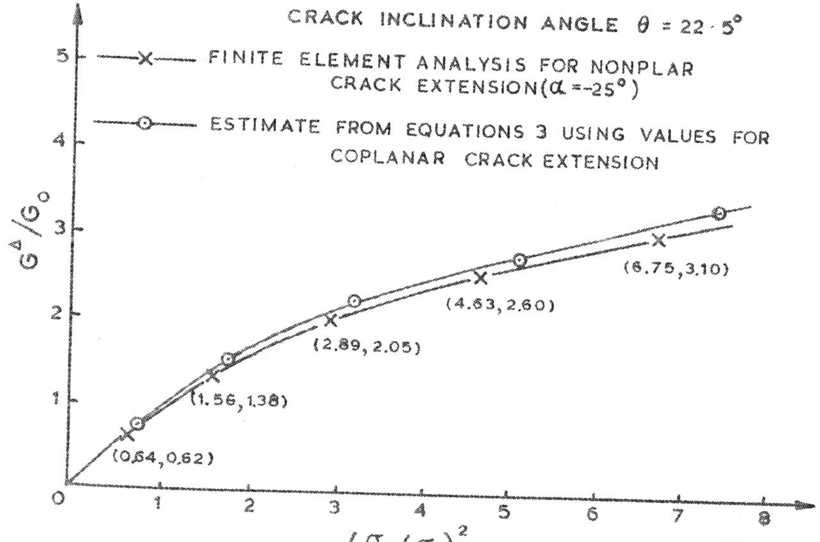


FIG. 4.

Values of G^{Δ}/G_o for nonplanar crack extension in an elastic-plastic material against the load $(\sigma_p/\sigma_o)^2$.

good agreement with theoretical values obtained by using the procedure of Bilby et al for the two cases considered, namely in the presence and absence of a T-effect.

3.2 The T-effect caused by the lateral stress σ_T has a significant influence on the values of $k_1(\alpha)$ and $k_2(\alpha)$ when ℓ/a is not very small.

3.3 Estimates of $G^{\Delta}(\alpha)/G_o$ for an elastic-plastic material, obtained by using equations (3) requiring the evaluation of $G^{\Delta}(0)$ for coplanar crack growth only are in reasonably good agreement with the results of the finite element analyses for nonplanar crack extension.

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