

## ON THE THREE DIMENSIONAL CRACK GROWTH ANALYSIS

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### I. INTRODUCTION

J integral concept has been extended to describe the stable crack growth problems, so-called J controlled growth<sup>[1]</sup>. Many experimental and analytical studies have been carried out on this problem in these several years<sup>[2,3]</sup>. By using the finite element method, crack growth is well simulated by releasing the nodal reaction force at the crack tip and creating the new crack surfaces in the two dimensional space.

However, in the three dimensional problems, the same method is not available because the crack front configuration depends largely on the mesh pattern and it is difficult to get smooth crack front.

Then, in this paper, a new method, proposed by Atluri et al.<sup>[4]</sup> is used to simulate the three dimensional crack growth. In the new method, the nodal points at the crack front are translated along the crack growth direction, and the crack front configuration keeps its smooth line.

In this paper, at first the new method is briefly introduced. Then the two dimensional problem is analyzed by both new and conventional methods, and the results are compared with each other. Three dimensional CT specimens are analyzed elasto-plastically, and crack growth analyses are carried out. The results are compared with those of experiments and good agreements are obtained.

### II. THE NEW METHOD TO SIMULATE THE CRACK GROWTH.

The finite element simulation of crack growth is carried out by the following three steps: (1) geometrical change in the crack surface boundary, (2) translation of the crack tip singularities to the advanced

crack tip, and (3) release of nodal reaction forces on the newly created crack surface. The nodal reaction forces are expressed by the next equation including the numerical error by the translation.

$$\{R_k\} = \int_V \sigma_{ij} B_{ijk} dV - \int_{S_\sigma} F_k dS \quad (1)$$

where  $\epsilon_{ij} = B_{ijk} u_k$ ,  $u_k$  denotes displacement vector, and  $S_\sigma$  means the surface on which the surface traction  $F_k$  is subjected.  $\{R_k\}$  is released gradually by several steps because unloading occurs during the release process. Then  $\{R_k\}$  is recalculated from new stress-strain states, and the same procedure is repeated until the norm of  $\{R_k\}$  become nearly zero. During these iteration, the error by the translation is canceled. Basically, this method is same as the conventional one though this method involves the iteration process to check the equilibrium. Fig.1 shows the differences of mesh pattern between both methods.

Fig.2 shows the two dimensional model of center cracked tension specimens. After elasto-plastic analyses, the crack growth is simulated by both new and conventional methods. The material constants are: Young's modulus  $E=206$  GPa, Poisson's ratio  $\nu=0.3$ , Yield stress  $\sigma_y=0.49$  GPa, and hardening ratio  $H=0.981$  GPa.

Fig.3 shows the J- $\Delta a$  curves of both methods. It is shown that both results agree very well. Fig.4 shows the changes of the crack opening profiles during crack growth. When  $\Delta a=1$ mm, both results agree well, but for  $\Delta a=2$ mm, a little differences are observed between both results. It may be due to the differences of the mesh patterns for both methods.

### III. Three dimensional crack growth analyses of CT specimens.

#### 1. Elastic analyses.

Fig.5 shows the size and mesh pattern of a quarter part of 1 CT specimen which is used in the following analyses. In Fig.6(a)-1, the hatched region means the initial crack surfaces, and (a)-2 shows the distribution of the three dimensional J value<sup>[5]</sup> along the crack front. The dotted line means the mesh dividing line. In Fig.6(a), the J value becomes the maximum at the center of the plate, so at the center of the plate, the crack growth occurs and the new configuration of the crack

front is shown in Fig.6(b)-1. Fig.6(b)-2 shows the new distribution of the J value and again the crack growth occurs at the maximum position of J value. The same procedures are repeated until the J value becomes nearly uniform along the crack front, as shown in Fig.6(c)-1 and 6(c)-2.

Fig.7 shows the real configuration of the crack front obtained by fatigue test. The configurations for both experiment and analyses agree qualitatively. So it is estimated that in the fatigue crack growth, the crack grows so as to uniform the J value distribution along the crack front.

The same analyses are carried out for 25% side grooved CT specimen. In Fig.8 (a) and (b), the initial and final configurations of analyses are shown and in Fig.9 the result of experiment is shown. It is noted that in this case, the results also agree qualitatively.

## 2. Elasto-plastic analyses.

The CT specimen, shown in Fig.6, is analyzed elasto-plastically until the maximum J value becomes nearly  $100 \text{ kJ/m}^2$ , which is considered to be the critical J value of A533B steel. The crack front configuration and the corresponding distribution of J value are shown in Fig.10(a)-1 and (a)-2. The same procedure as mentioned in elastic analyses are carried out and finally, the crack front configuration is obtained as shown in Fig.10(b)-1 and (b)-2. Also in this case, the tunnel effect is recognized and at the center of the plate, the crack length becomes the maximum.

## IV. CONCLUDING REMARKS.

By employing a new method to simulate the crack growth, in which the distance of the crack growth is independent to the mesh pattern, it is shown that the three dimensional crack growth problem can be treated well. As shown in this paper, with the evaluation technique of the three dimensional J integral, the J controlled growth problem can be discussed in the three dimensional space.

## REFERENCES

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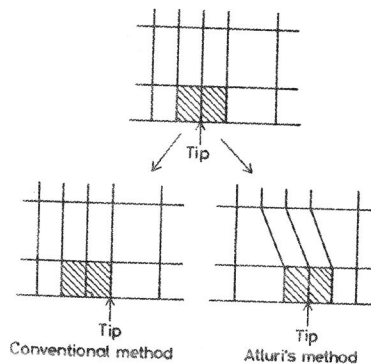
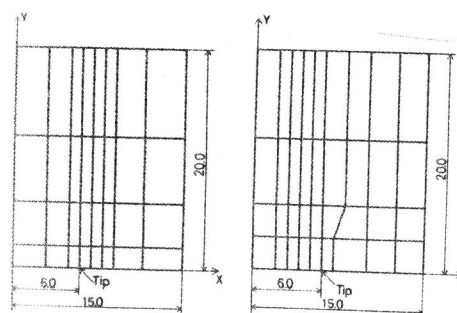


Fig.1 Differences of mesh pattern between both methods.



(a) For conventional (b) For this method.

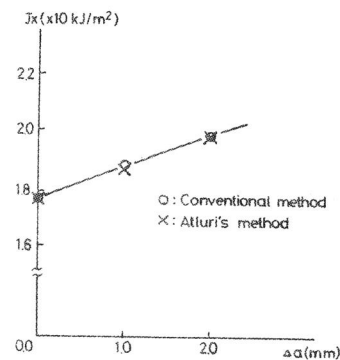


Fig.3 J- $\Delta a$  curves of both methods.

Fig.2 Mesh patterns of CCT specimen.

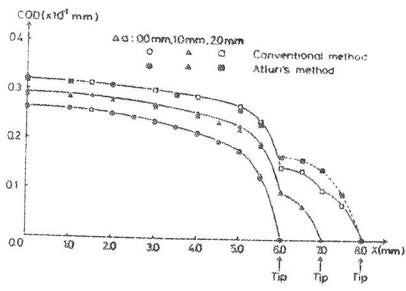


Fig.4 Crack opening profiles of both methods.

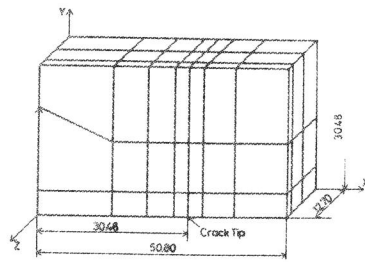
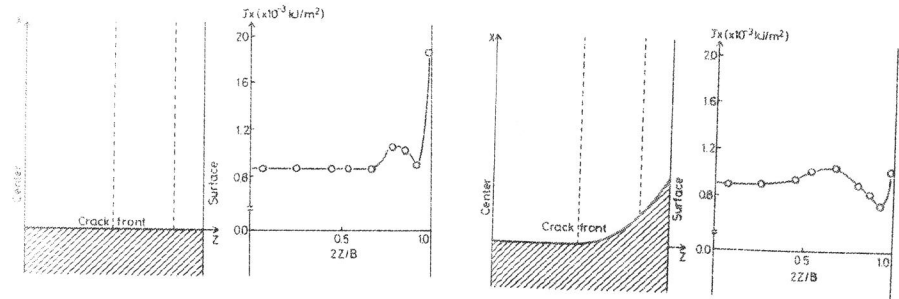


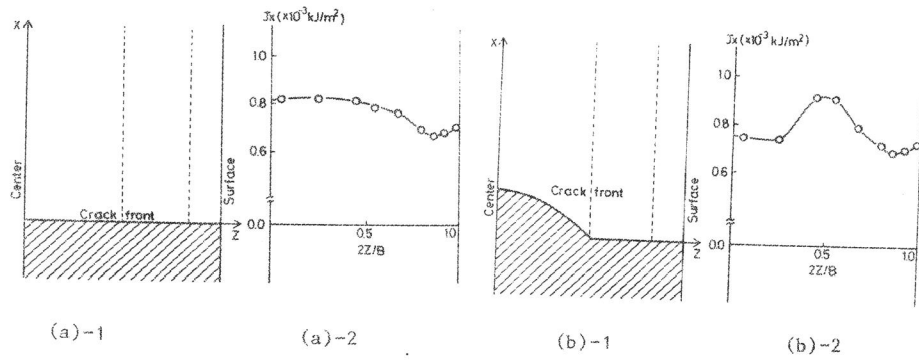
Fig.5 Size and mesh pattern of 1 CT specimen.



(a) Initial state.

(b) Final state.

Fig.8 Crack growth of 25% side grooved CT specimen.

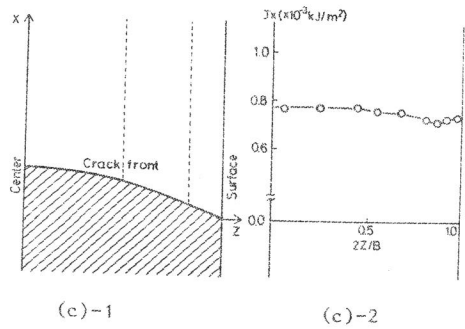


(a)-1

(a)-2

(b)-1

(b)-2



(c)-1

(c)-2

Fig.6 Crack front configurations and J distributions of 1 CT specimen in elastic states.

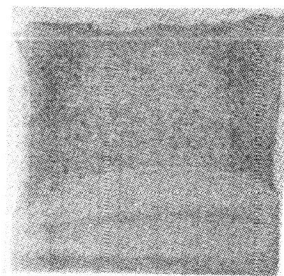


Fig.7 Fatigue crack front of 1 CT specimen.

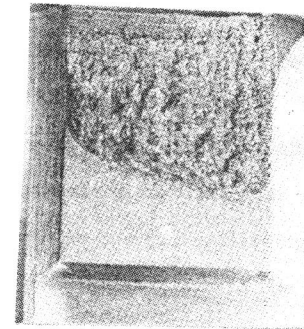
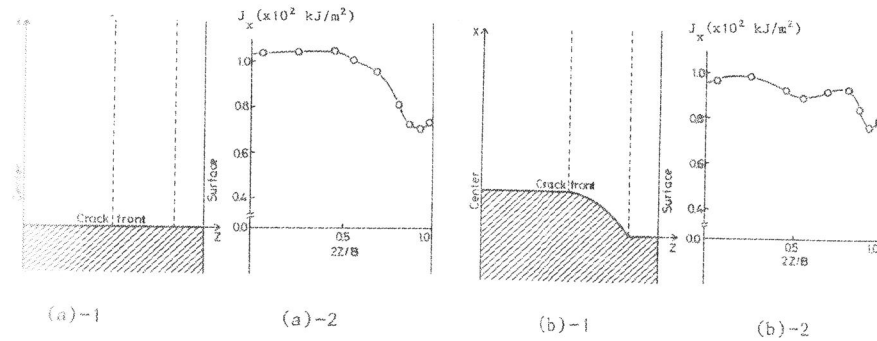


Fig.9 Fatigue crack of 25% side grooved CT specimen.



(a)-1

(a)-2

(b)-1

(b)-2

Fig.10 Crack growth of 1 CT specimen in elasto-plastic states.