Analysis on Arithmetic and Application of Rigidity Distribution for Simply Supported Structure

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Abstract Bridges under life–long service often suffered from invisible diseases which could cause the bridge stiffness decay (rigidity) and pose great threatening to human lives. This paper focused on studying the real stiffness distribution in a simple beam. The relationship between stiffness and deflection in a bridge segment has been established which is based on the material mechanics, numerical modeling, and reciprocal displacement theorem. The conversion of the deflection curve at mid span resulted from three axle vehicle and single unit load has been developed so that by measuring the deflection caused by low speed vehicle passing over the bridge, the real stiffness distribution of the bridge could then be obtained and the assessment of the bridge damage could be carried out. Compatible software was developed in accordance with the proposed stiffness calculation method. The software was found worked well in predicting the stiffness distribution and the test results supported that the proposed prediction method is applicable. The test results showed that the stiffness distribution in longitudinal direction varied significantly, which is due to the different degrees of damage. The analysis showed that it is not good to use the overall stiffness to evaluate the damage of the whole beam.

Key words: simple supported beam, longitudinal stiffness, invisible damage, damage assessment

1. Introduction

In recent decades, as the developing of our country, the number of highway bridges increased significantly, which in turn increased the number of bridges under life–long service. Due to the climate change, environment varies, temperature drops and wind and mechanical erodes, bridge aging becomes gradually a serious problem. Cyclic traffic loading could cause bridge elements to lose its mechanic properties and evolve into fatigue damage. Some bridges currently in service suffered from concrete crack, rebar erosion, and pre–stress relaxation, which could result in the overall stiffness decay and reduce its engineering performance. When the bridge stiffness decays, the bearing capacity would be greatly reduced which prohibited its service life and threatens the human lives[1]. Currently, there are two different ways to evaluate the bridge stiffness distribution, namely the direct way and indirect way. The indirect way is that to look over the outside of the bridge, for example, conduct a walk tour throughout the bridge, look for cracks, damage points and find out weak areas, which is very objective and cannot give a very accurate evaluation. The direct way is to carry out load test and measure the real stress strain response under different loadings. The load test can reflect the real stiffness of the bridge; however, it cannot indicate the damage degree of a bridge and show where the damage area is[2]. Therefore, to check the overall quality of a bridge, and evaluate the performance of the bridge and each segment has great significance in transportation engineering.

The assessment of the bridge stiffness and damage has been studied by many researchers. Zhao et al. [3] proposed to use bridge structure bilateral difference response model to analyze the integrity of the bridge in longitudinal and transverse directions, so that to study the stiffness in longitudinal directions. Yuan et al.[4] studied the erosion of Cl^- under freezing–thaw cycles. They have proposed a method to predict the expiration life of Cl^- under freezing–thaw cycles. Y.M. et al.[5] have proposed method to calculate the rotation when the top of the beam has cracks, and to predict the service life of the beam under such situation. Yao et al. [6] studied the bearing capacity of beams that suffered from damage in longitudinal directions. Shan et al.[7] studied the detection of damage location and damage degree of highway bridges under the vibration of vehicles moving. Z. H. et al.[8] studied the bearing capacity of bridges under service by using the dual coding genetic algorithm. Yuan et al.[9] proposed a method to calibrate the stiffness matrix which based on the assumption that the finite element mass matrix and the stiffness matrix were asymmetry, and the mass matrix was given.

The previous studies have advanced the state of the art of the knowledge of stiffness decay damage of bridges; however, there are no effective methods in determining the damage inside the bridge (ex. The invisible concrete cracks, the rebar corrosion, and pre–stress relaxation). Those common diseases could result in heavily decay of the stiffness of the bridge elements, thus influencing the service life and posing failure the bridge. This paper focused on a simple beam structure to study the real stiffness distribution at each bridge segment. The stiffness prediction is based on the measured deflection from the mid span, and used proposed method to calculate the real stiffness distribution, so that the stiffness decay degree inside the bridge could be evaluated, the location of the damage area could be found, and the damage degree could be estimated.

2. The relationship between deflection influence line and stiffness in bridge segment

2.1 Finite element equation of single beam element

Simple bridge beam does not have axle force (the traffic loading cannot cause axle force). For a beam without axial force, the four degree of freedoms are: the vertical displacement at both sides of the beam, v_1 *and* v_2; and the rotation angles at both sides, θ_1 *and* θ_2 (Fig. 1).

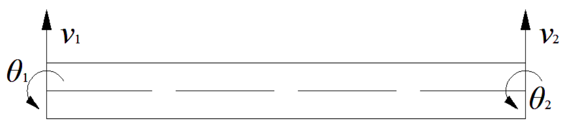


Figure 1.beam element without of axial force

According to finite element theory[9],the solution equation for a rigid beam element is:

$$K^e \cdot q^e = P^e \tag{1}$$

where: K^e is the stiffness matrix without axial force [10], K^e is the parameters in the matrix as shown in Eq.(2), E is the elastic modulus of the beam, I is the bending moment of inertia, and l is the length of the beam.

$$K^e = EI\begin{bmatrix} \frac{12}{l^3} & \frac{6}{l^2} & \frac{-12}{l^3} & \frac{6}{l^2} \\ \frac{6}{l^2} & \frac{4}{l} & \frac{-6}{l^2} & \frac{2}{l} \\ \frac{-12}{l^3} & \frac{-6}{l^2} & \frac{12}{l^3} & \frac{-6}{l^2} \\ \frac{6}{l^2} & \frac{2}{l} & \frac{-6}{l^2} & \frac{4}{l} \end{bmatrix} \tag{2}$$

q^e is the displacement matrix of the rigid beam at each nods, $q^e = \begin{bmatrix} v_1 & \theta_1 & v_2 & \theta_2 \end{bmatrix}^T$

P^e is the stress matrix, $P^e = \begin{bmatrix} P_1 & M_1 & P_2 & M_2 \end{bmatrix}^T$

2.2 Establish the relationship between deflection influence line and stiffness in bridge segment

Take simple beam as an example; divide the bridge into N segments in longitudinal direction. The deflection influence line could be achieved by superposition of the deflection resulted from each unit load that applied at the mid span of each bridge segment. From structure mechanics point of view, according to the reciprocal displacement theorem that in a rigidity beam, a force applied to point i, which caused the displacement at point j is equal to the same force that applied to j that caused the displacement that happened at i. Similarly, the unit force that applied in the mid－span of the N segment, where the resulted deflection is equal to the unit force that applied in the mid span where the resulted deflection happened at the N segment. Therefore, when the deflection influence line is known at mid span, the real stiffness is easily achieved. Because there are no element nods at the mid span, each bridge segment is subdivided into two sections, thus the overall segments become to 2N, and the computational model is shown in Fig. 2.

For the bridges under service, it is convenient to use moving vehicles passing through the bridge to collect the mid span deflection to predict the stiffness distribution (model 1),the conversion relationship between the moving vehicles and the single unit weight is shown in section 2. For the beams in laboratory and precast beams in field, the stiffness could be calculated based on the mid span loading condition (model 2).

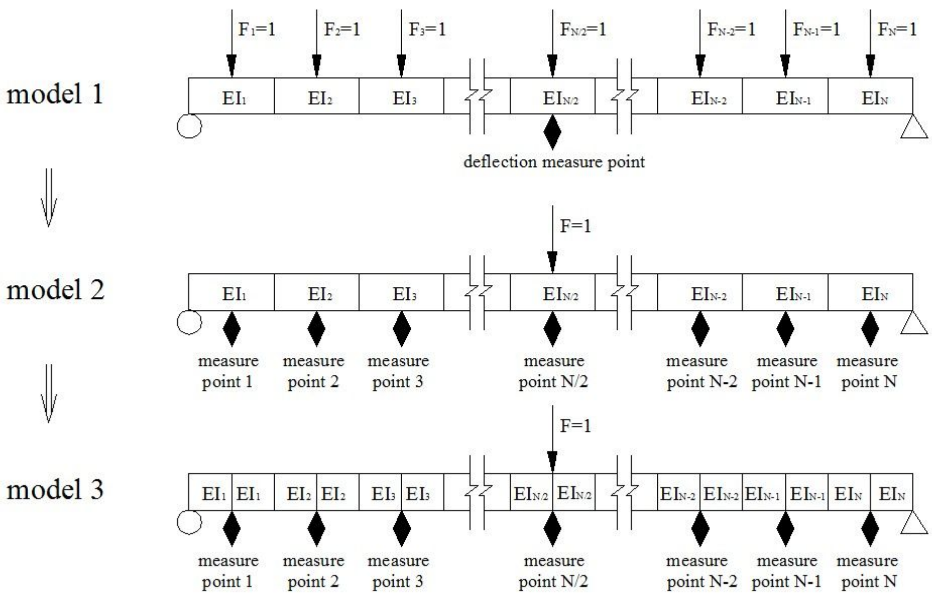


Figure 2.Computational Models

The relationship between load, deflection, and stiffness in model 3 are presented below:

$$\left.\begin{aligned}
&EI_1\left(\frac{12}{l_1^3}v_1+\frac{6}{l_1^2}\theta_1-\frac{12}{l_1^3}v_2+\frac{6}{l_1^2}\theta_2\right)=P_1\\
&EI_1\left(\frac{6}{l_1^2}v_1+\frac{4}{l_1}\theta_1-\frac{6}{l_1^2}v_2+\frac{2}{l_1}\theta_2\right)=M_1\\
&EI_i\left(-\frac{12}{l_i^3}v_i-\frac{6}{l_i^2}\theta_i+\frac{12}{l_i^3}v_{i+1}-\frac{6}{l_i^2}\theta_{i+1}\right)+EI_{i+1}\left(\frac{12}{l_{i+1}^3}v_{i+1}+\frac{6}{l_{i+1}^2}\theta_{i+1}-\frac{12}{l_{i+1}^3}v_{i+2}+\frac{6}{l_{i+1}^2}\theta_{i+2}\right)=P_{i+1}\\
&EI_i\left(\frac{6}{l_i^2}v_i+\frac{2}{l_i}\theta_i-\frac{6}{l_i^2}v_{i+1}+\frac{4}{l_i}\theta_{i+1}\right)+EI_{i+1}\left(\frac{6}{l_{i+1}^2}v_{i+1}+\frac{4}{l_{i+1}}\theta_{i+1}-\frac{6}{l_{i+1}^2}v_{i+2}+\frac{2}{l_{i+1}}\theta_{i+2}\right)=M_{i+1}\\
&EI_{2N}\left(-\frac{12}{l_{2N}^3}v_{2N}-\frac{6}{l_{2N}^2}\theta_{2N}+\frac{12}{l_{2N}^3}v_{2N+1}-\frac{6}{l_{2N}^2}\theta_{2N+1}\right)=P_{2N+1}\\
&EI_{2N}\left(\frac{6}{l_{2N}^2}v_{2N}+\frac{2}{l_{2N}}\theta_{2N}-\frac{6}{l_{2N}^2}v_{2N+1}+\frac{4}{l_{2N}}\theta_{2N+1}\right)=M_{2N+1}
\end{aligned}\right\} \quad (3)$$

where: EI_i is the stiffness of each bridge segment(i=1～2N), v_i is the vertical deflection at each

element nod(i=1～2N+1), θ_i is the rotation angle at each element nod(i=1～2N+1), P_i is the vertical load at each element nod(i=1～2N+1), M_i is the bending moment at each element nod(i=1～2N+1), l_i is the length of each element(i=1～2N), in summary, there are 12N+4 parameters.
In Eq. (3) there are some boundary conditions:

$$\left.\begin{aligned} & v_1 = v_{2N+1} = 0 \\ & P_{N+1} = 1 \\ & P_j = 0 \\ & M_{\mathrm{i}} = 0 \\ & l_i = \frac{L}{2N} \\ & EI_i = EI_{i-1} \end{aligned}\right\} \tag{4}$$

where: L is the span of bridge($i=1,2,3...2N$), P_j is the vertical load at each element nod when $j \neq N+1$ and $j \in i$.

2.3 Solution of the deflection stiffness equation

Maple is the most popular tool in Math and Engineering practice worldwide. Maple is also a good tool for symbolic operations, and very helpful in studying the mathematics models. In this research, therefore, Maple was adopted to solve the Eq. (3) and Eq. (4), and all the parameters (12N+4) in the equations. It is found that Maple could offer an analytic solution for those equations. The stiffness EI_i (i=1～2N)in each bridge section could be represented as follows:

$$EI_i = EI_{i+1} = \frac{L^3}{96N^4 \cdot B_i} \tag{5}$$

where: B_i is the i row in matrix $\mathbf{B}$ $(i=1,3,5...2N-1)$,the matrix $\mathbf{B}$ could be obtained by Eq. (6).

$$\mathbf{B} = \mathbf{A}^{-1} \cdot \mathbf{f} \tag{6}$$

In Eq. (6), $\mathbf{f}$ is the deflection at the mid span of each bridge segment, which is similar to the displacement matrix $v_i(i=2,4,6...2N)$, however, matrix $\mathbf{A}$ is very complex, and the element in matrix $\mathbf{A}_{i,j}$ could be categorized into two groups according to their parity:
(1)When N is even:

$$\mathbf{A}_{i,j} = \begin{cases} (24j^2 - 24j + 8)(2N - 2i + 1) & (i > j \text{ and } j \leq \dfrac{N}{2}) \\ (-24m^2 + 24m + 24mN - 12N - 8)(2N - 2i + 1) & (N > j > \dfrac{N}{2} \text{ and } i > j \text{ and } m = N - j + 1) \\ (-24m^2 + 24m + 24mN - 12N - 8)(2m - 1) - 6mN + 5N & (i = j > \dfrac{N}{2} \text{ and } m = N - i + 1) \\ \mathbf{A}_{N+1-i,N+1-j} & (i = j \leq \dfrac{N}{2} \text{ or } j > \mathrm{i}) \end{cases} \tag{7}$$

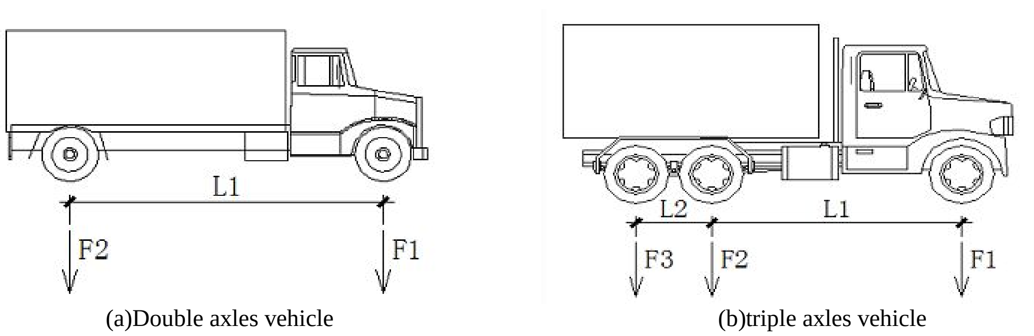
(2)When N is odd:

$$
\mathbf{A}_{i,j} = \begin{cases}
(24j^2 - 24j + 8)(2N - 2i + 1) & (i > j \ and \ j < \dfrac{N}{2}) \\
(-24m^2 + 24m + 24mN - 12N - 8)(2N - 2i + 1) & (N > j > \dfrac{N+1}{2} \ and \ i > j \ and \ m = N - j + 1) \\
(-24m^2 + 24m + 24mN - 12N - 8)(2m - 1) - 6mN + 5N & (i = j > \dfrac{N+1}{2} \ and \ m = N - i + 1) \\
\mathbf{A}_{N+1-i,N+1-j} & (i = j < \dfrac{N}{2} \ or \ j > \mathrm{i}) \\
6N^2 - 3N & (i = N \ and \ j = \dfrac{N+1}{2}) \\
(6N^2 - 3N)(2N - 2i + 1) & (\dfrac{N+1}{2} < i < N \ and \ j = \dfrac{N+1}{2}) \\
2N(3N^2 - 3N + 1) & (i = j = \dfrac{N+1}{2}) \\
\mathbf{A}_{N+1-i,j} & (i < \dfrac{N+1}{2} \ and \ j = \dfrac{N+1}{2})
\end{cases} \tag{8}
$$

Based on Eq. (5) to Eq. (8), the stiffness EI_{i} (i=1～2N) of each bridge segment could be obtained from the known deflection values, hence the real stiffness of the whole bridge could be determined.

3. The Deflection Influence Line in Mid－Span

From section 1, it is known that when the mid span deflection influence line is given, the real stiffness distribution could be determined. However, it is hard to use a unit concentrated force to pass through the bridge, usually, the mobility of the load is similulated as a two axle or three axle vehicle passing over the bridge (speed at 5km/h), and measure the deflection at the mid span. The vehicle size and models are shown in Fig. 3. The conversion relationship of the resulted deflection curve in the mid span between common two axles or three axles vehicle and the unit load are achieved and showed in Eq. (9) to Eq. (12).



(a)Double axles vehicle (b)triple axles vehicle

Figure 3.Vehicle Models and Sizes

(1)Double axle vehicle

For the vehicle model in 3 (a), the single concentrated load F and vehicle length L after conversion are shown from Eq. (9) to Eq. (10).

$$F = F_1 + F_2 \tag{9}$$

$$L = \frac{F_2 L_1}{F_1 + F_2} \tag{10}$$

(2)Triple axle vehicle

For the vehicle model in 3 (b), the single concentrated load F and vehicle length L after conversion are shown from Eq. (11) to Eq. (12).

$$F = F_1 + F_2 + F_3 \tag{11}$$

$$L = \frac{F_2 L_1 + F_3(L_1 + L_2)}{F_1 + F_2 + F_3} \tag{12}$$

When the multiple axles loads have been converted to single concentrated unit load, use the linear relationship between concentrated load and unit load, the deflection at mid span caused by the vehicles could be converted into deflection influence line, so that to calculate the real stiffness distribution in the whole beam.

4. Case Study and the Development of the Compatible Software

When the deflection values D_1 to D_4 are known in model 1 or model 2 in Fig. 2, the real stiffness of a bridge in four segments at length of L could be obtained as presented below:
Based on Eq. (7) when N=4:

$$\mathbf{A} = \begin{bmatrix} 36 & 88 & 56 & 8 \\ 40 & 236 & 168 & 24 \\ 24 & 168 & 236 & 40 \\ 8 & 56 & 88 & 36 \end{bmatrix} \tag{13}$$

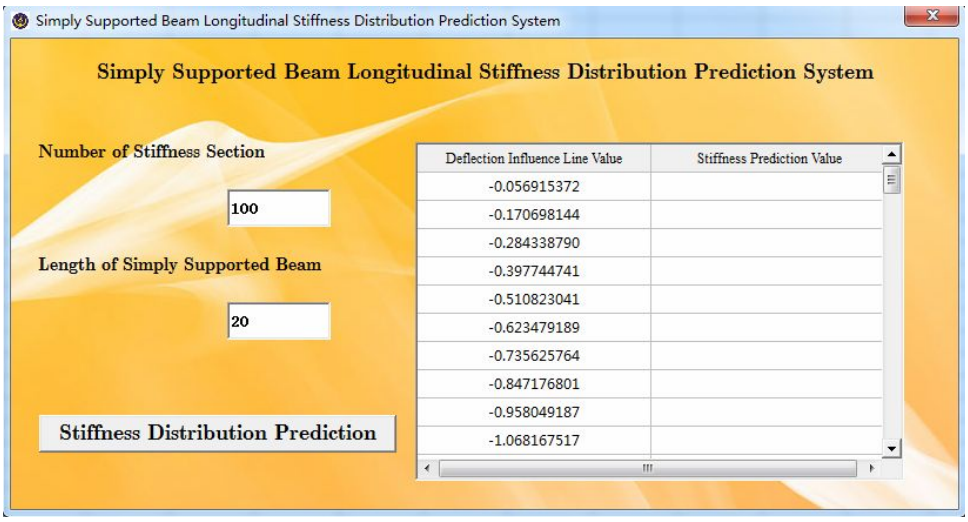
$$\mathbf{B} = \mathbf{A}^{-1} \cdot \mathbf{f} = 10^{-5} \begin{bmatrix} 4802D_1 - 1990D_2 + 308D_3 - 83D_4 \\ -949D_1 + 1263D_2 - 749D_3 + 201D_4 \\ 201D_1 - 749D_2 + 1263D_3 - 949D_4 \\ -83D_1 + 308D_2 - 1990D_3 + 4802D_4 \end{bmatrix} \tag{14}$$

From Eq. (5), the stiffness are shown below, for example, when i=1:

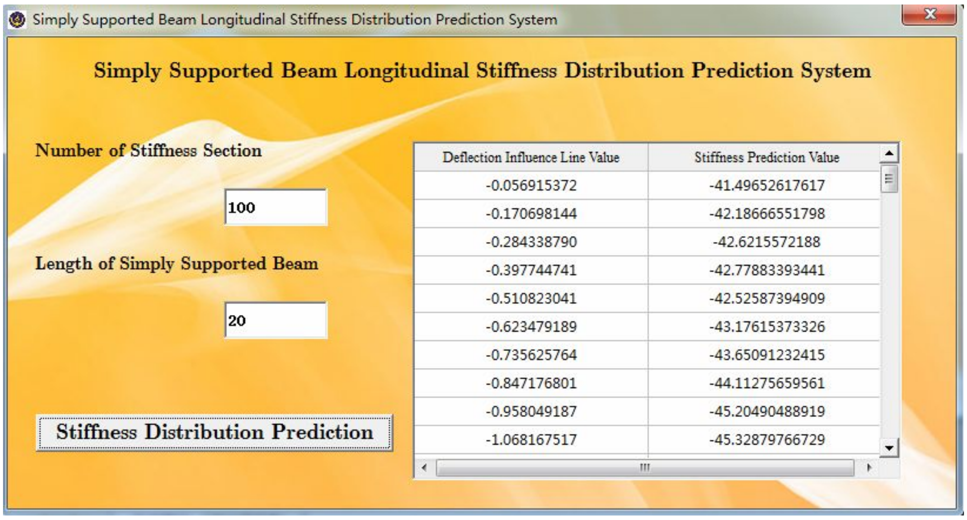
$$EI_1 = EI_2 = \frac{L^3}{96 \times 4^4 \cdot B_1} = \frac{L^3}{1180.1D_1 - 489.1D_2 + 75.7D_3 - 20.4D_4} \tag{15}$$

The Eq. (15) is the stiffness prediction equation that at the number of 4 bridge segments, the results were matched well with the published results in literature[10].

When N is large, the matrix $\mathbf{A}$ and $\mathbf{A}^{-1}$ are complex and hard to manipulate. In order to make it simple to carry out the stiffness prediction, windows based software was developed in accordance with the proposed equations and simple beam model. When the number of bridge segments, the length of the simple beam, and the influence line have been given, the real stiffness distribution could then be easily calculated, as shown in Fig. 4.



(a) Pretreatment



(b) post processing

Figure 4.the user interface

An old simple beam bridge, 20m in length, was adopted in the case study. The bridge was divided into 100 segments; the mid span deflection is tested by moving a two axle vehicle passing through the bridge at a low speed. The deflection influence line was calculated and shown in Fig. 5, and the stiffness distribution is shown in Fig. 6.

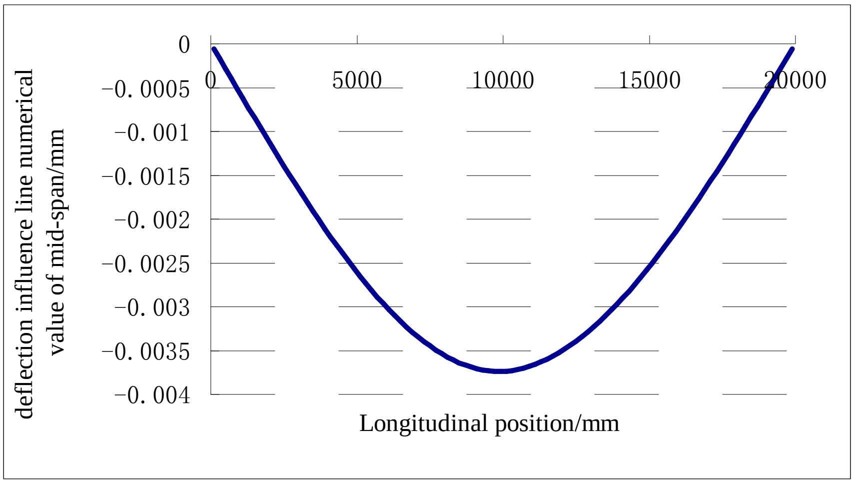


Figure 5.deflection influence line at the mid span

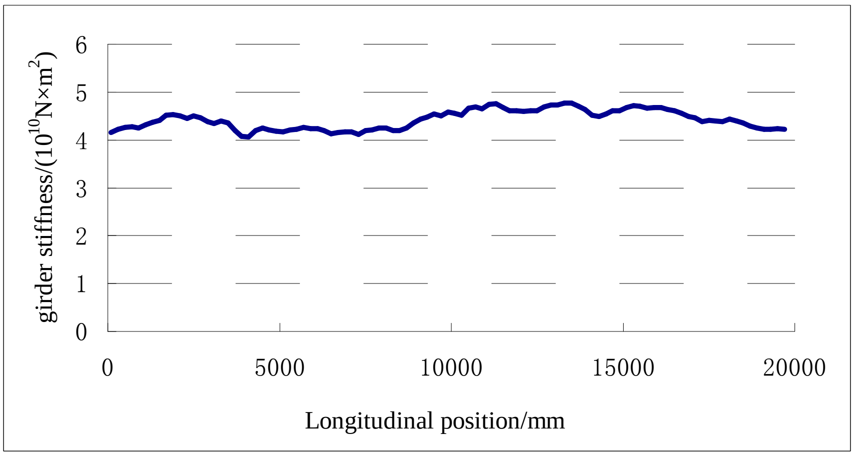


Figure 6.stiffness distribution curve

The calculation results showed that the proposed prediction methods worked pretty well, the bridge stiffness distribution could be calculated from the mid span deflection influence line. From the stiffness distribution curve the following features could be observed, the minimum stiffness is 4.09, maximum is 4.71, and the difference is 14%; significant stiffness decay happens at L/4, however, the stiffness decay was slightly at 3L/4. According to the stiffness distribution curve, the real working condition of the bridge could be evaluated, and thus helps the damage detection and bridge reinforcement work.

5. Conclusion

(1)This research is focused on the real stiffness distribution of simple beam structure in bridge engineering. The real stiffness distribution prediction equations were achieved based on the relationship between stiffness and deflection. The analysis results showed that there were analytical solutions for the stiffness distribution prediction equations, and it could be achieved from by measuring the deflection influence line in the mid span of each bridge segment.

(2)In order to simplify the measurement of the deflection in the field, the conversion method between two axles (three axles vehicles) and single unit concentrated loading condition was developed, so that to improve the measurement of the real deflection data in filed and help the prediction of the actual stiffness distribution.

(3)When calculating large numbers of bridge segments, for example, N is very large, the direct calculation work would be very heavy, therefore, a compatible software was developed to simplify the calculation work. A case study was carried out to check the workability of the software. The calculation results showed that the stiffness in each segments were slightly different. The developed software was found to actually predict the stiffness change of each segment in a bridge, estimate the decay area and decay degree, and help to improve the bridge and reinforcement work.

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Reference

[1] HE Dong–dong, SHAN Cheng–lin, Discussion on the evaluating method of bearing strength to existing bridge. Guangdong Highway Communications, 1 (2009) 36–39.

[2] Xu Wenping, Yang Zi, Study of Appraisal Standard for Carrying Capacity of Existing Concrete Bridges. Urban Roads Bridges & Flood Control, 10 (2010) 174–176.

[3] Zhao Hu, Pu Qian–hui, Shi Zhou, Study on Identification of Bridge Component Damage Based on Transverse Modal Difference. Railway Standard Design, 7 (2010) 77–81.

[4] YUAN Cheng–fang, NIU Di–tao1, GAI Qing–shan, SUN Cong–tao, Research on chloride penetration life prediction of reinforced concrete bridge with consideration of the damage caused by freeze–thaw cycles. Journal of Xi'an University of Architecture & Technology (Natural Science Edition), 42 (2010) 256–260.

[5] YANG Ming, HUANG Qiao, WANG De–jun, Equivalent bending stiffness of simply supported preflex beam bridge with variable cross–section. Journal of Harbin Institute of Technology, 17 (2010) 13–17.

[6] YAO Xiao–fei, XU Yue, FU Ying–chun, FENG Zhuo–de, Experimental study on ultimate bearing capacity of damaged concrete T–beam bridge systems. Journal of Guangxi University (Natural Science Edition), 34 (2009) 144–149.

[7] SHAN Deshan, LI Qiao, Damage Detection of Bridge Structures Based on Moving Vehicle Caused Vibration. Journal of Southwest Jiaotong University, 44 (2009) 60–65.

[8] Zhao Hongmei, Wang Huali, Jin Jun, Beam shaping with limited amplitude weight values for

satellite active phased array antenna. Engineering Sciences, 8 (2010) 62–66.
[9] Yuan Yongxin, Jiang Jiashang, A direct updating method for the stiffness matrix. Journal of Jiangsu University of Science and Technology (Natural Science Edition). 24 (2010) 193–196.
[10] WANG Ling–bo, HE Shuan–hai, ZHAO Yu, JIANG Pei–wen, A comprehensive evaluation method of damage in in–service girder bridges based on the finite element method. Journal of Guangxi University (Natural Science Edition), 36 (2011) 94–100.