Propagation of tensile planar cracks in highly heterogeneous media: A numerical study

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Abstract: Effect of highly heterogeneous fracture properties on tensile penny-shape planar crack propagation governed by Irwin's criterion is investigated numerically by taking into account the large crack front deformations induced by the high toughness contrasts. To compute the variations of stress intensity factor (SIF) along the crack front arising from its progressive deformation, perturbation approach based on Bueckner-Rice weight function theory is used iteratively. Effective fracture toughness is obtained from the local toughness map for a few examples. It is shown that when Irwin's criterion is satisfied all along the crack front, the effective toughness is equal to the mean value of the local ones along the crack front. This value depends on the shape of the front, and so is different from the total mean value of the toughness in the plane. In the examples studied here, the weak toughness zones are favored by the crack front deformations, so that the effective toughness is lower than its spatial average.
Keywords: Tensile planar crack; Heterogeneous medium; Effective failure properties; Perturbation method; Linear elastic fracture mechanics.

Quantifying the effective fracture properties of a planar crack propagating in a heterogeneous material is a key issue in material science. Since the crack propagation results from the interplay of local material properties with long range elastic interactions, the problem is not trivial and can not, in general, be reduced to take the spatial mean value of the fracture properties. Two regimes shall be distinguished [1]: For slightly fluctuating maps of local toughness, the elasticity of the crack front dominates over the destabilizing effects of heterogeneities, and the motion of the front is smooth (weak pinning). For materials with stronger heterogeneities with larger gradient of toughness, the crack front can jump abruptly from one equilibrium position to the other (strong pinning) [3]. In the weak pinning regime, it has been shown in the limit of a first order approach that the effective macroscopic toughness can be obtained by averaging the local toughnesses, contrary to the strong pinning case where the effective macroscopic toughness is *larger* than the average local toughness [1].

Here, we address these questions in the context of highly heterogeneous local fields of toughness for which the first order approach can not be used. In particular, we take into account the effect of large crack front deformations induced by high toughness contrasts, and investigate the relationship between local toughness map and macroscopic effective toughness. In this paper, we will limit our study to the weak pinning quasistatic regime. We consider the case of a circular embedded crack propagating under remote mode I loading in an axisymmetric toughness map (see § 1). We solve the problem by using an incremental method [4, 5], based on Rice's perturbation approach [6, 7]. This method is presented in § 2 and then applied in § 3 to the resolution of our problem.

1 Problem definition

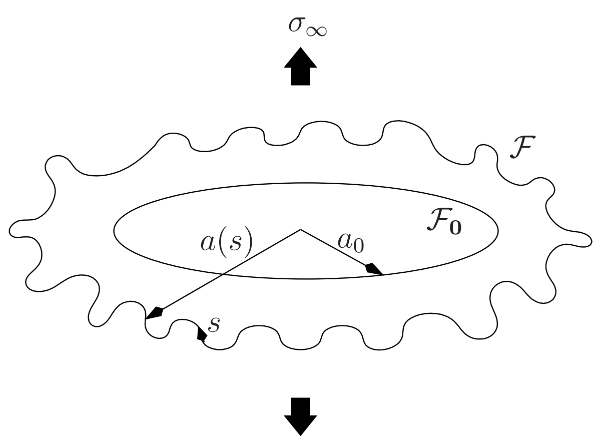


Figure 1: A tensile planar crack in an infinite body under uniform stress σ_∞

Consider a planar penny shape crack $\mathcal{F}$ of initial radius a_0, embedded in an infinite isotropic elastic medium with heterogeneous fracture toughness properties and loaded in pure mode I through some uniform remote stress applied at infinity σ_∞ (see Fig. 1). We assume a quasistatic propagation of the crack front, so that the crack advance at a point M of the front is governed by Irwin's criterion:

$$\begin{cases} K(M) < K_c(M) & : \quad \text{no crack advance} \\ K(M) = K_c(M) & : \quad \text{possible crack advance,} \end{cases} \tag{1}$$

where $K(M)$ is the SIF and $K_c(M)$ the toughness at point M. Let us denote $\overline{K}_c$ the average material toughness, $\kappa_c (= \Delta K_c / \overline{K}_c)$ its relative contrast and $\eta(M)$ the toughness fluctuations. With these notations, we have:

$$K_c(M) = \overline{K}_c \left[1 + \kappa_c \eta(M)\right] \tag{2}$$

We suppose that the remote loading σ_∞ adapts in order to stay in the quasistatic regime and to ensure crack propagation at least, on some part of the front. This implies that at each moment:

$$\max_{M \in \mathcal{F}} \frac{K(M)}{K_c(M)} = 1 \tag{3}$$

Under the assumption of quasistatic propagation, the problem is to find, for a given toughness map $K_c(M)$, the successive positions of the crack front and the corresponding loading σ_∞. From them, one can obtain the SIF along the crack front and consequently its mean value. In the homogeneous case ($\kappa_c = 0$), the problem can be solved analytically. One has at each moment $K = \overline{K_c}$ all along the front, the successive positions of the crack front are circles of radius a and the corresponding loading follows $\sigma_\infty = \frac{\overline{K_c}}{2}\sqrt{\frac{\pi}{a}}$. In the heterogeneous case, the crack front deforms. In an homogenization process, we shall replace it by an equivalent circular crack of

- radius a_m given by the mean value of the crack extension a:

$$a_m = \frac{1}{L}\int_{\mathcal{F}} a(M(s))\mathrm{d}s \tag{4}$$

- SIF K_m given by the mean value of K:

$$K_m = \frac{1}{L}\int_{\mathcal{F}} K(M(s))ds \tag{5}$$

where s is the curvilinear abscissa and L is total length of the crack front.
We will see in the following that in the cases investigated here, K_m tends to a constant stationary value after some transient propagation regime. As a result, this stationary value will be used to define the macroscopic effective toughness K_c^{eq} of the heterogeneous media studied here. Alternatives would have been to define K_c^{eq} as the maximum value of K_m during crack propagation or its mean value during propagation. However, we would like to define an effective toughness as a quantity that does not depend of the initial geometrical configuration of the crack, so the value of K_m in the stationary regime seems the most appropriate definition.
The aim of the paper is to discuss the influence of the toughness map $K_c(M)$ on this effective toughness. In this paper, we consider it periodical and axisymmetric given by:

$$K_c(M) = \overline{K}_c\left[1 + \kappa_c \cos(k\theta(M))\right] \text{ where } \theta(M) \text{ denotes the polar angle of } M, \tag{6}$$

and discuss the influence of toughness contrast κ_c and spatial wavenumber k on K_m, hence on K_c^{eq}. This choice of K_c allows to obtain a weak pinning regime and to focus, as wanted, on the influence of the large deformations of the crack front on the effective toughness.

2 Numerical Procedure

A characteristic feature of this problem is that the shape of crack is determined by the variation of SIF and material properties. In general, neither the distribution of SIF, nor the geometry of the crack are known a priori and must be determined as part of the solution. An appealing perturbative technique for solving such problems is provided by the studies of Rice [6], who has developed a linear scheme for calculating the variation in SIF due to small changes in the crack geometry. For large deformations of the front, Bower and Ortiz [4] followed by Lazarus [5], developed a powerful method based on the iteration of the linear scheme. The efficiency of this method arises from the need for the sole 1D meshing of the crack front. In the sequel, we extend the numerical notations and procedures developed by Lazarus [5]. For dimensional reasons, we can introduce the dimensionless SIF $\widehat{K}$ by writing:

$$K = \sigma_\infty\sqrt{a_0}\,\widehat{K} \tag{7}$$

This quantity depends only on the crack shape.
We start from the initial situation of a crack of radius a_0 for which $\widehat{K} = \frac{2}{\sqrt{\pi}}$. We then use a regularization of Irwin's criterion to obtain the crack front displacement $\delta a(s)$ by a Paris' type law [5]:

$$\delta a(s) = \delta a_{max} \left(\frac{\widehat{K}(M(s))/K_c(M(s))}{\max_{M \in \mathcal{F}} \widehat{K}(M(s))/K_c(M(s))} \right)^{\beta} \text{ with } \beta \gg 1. \tag{8}$$

where δa_{max} corresponds to the maximum crack advance during a numerical step. The corresponding loading is obtained by introducing the definition 7 of $\widehat{K}$ in equation 3:

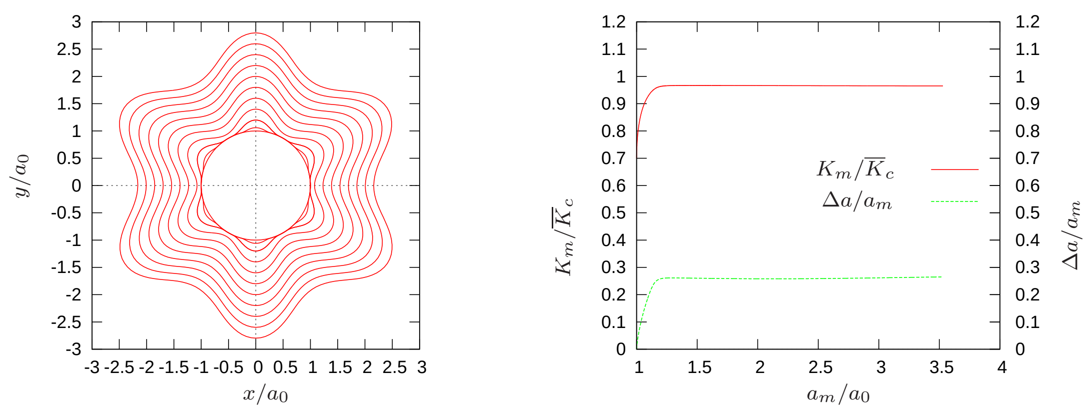
$$\frac{\sigma_\infty \sqrt{a_0}}{\overline{K}_c} = \left[\max_{M \in \mathcal{F}} \frac{\widehat{K}(M)}{1 + \kappa_c \eta(M)} \right]^{-1} \tag{9}$$

Subsequently, Rice's formulae (see Refs. [6, 5]) are used for updating the dimensionless SIF $\widehat{K}$ corresponding to the advance $\delta a(s)$ and the whole step (determination of δa, updating of $\widehat{K}$) is reitirated as long as necessary.

3 Results

The previous procedure is applied to the toughness map given by Eq. (6). In § 3.1, the propagation in the case of a given value of k and κ_c is studied. In §3.2, the influence of those parameters on the mean quantities K_m and a_m is considered.

3.1 Propagation for a given toughness map



(a) Successive crack front positions (b) Evolution of key quantities during propagation

Figure 2: Case $k = 6$ and $\kappa_c = 0.3$

As a typical example, the results for $k = 6$ and $\kappa_c = 0.3$ are shown in Fig. 2. On Fig. 2(a), successive equilibrium positions of the crack fronts are plotted. The propagation is continuous without jumps, showing that the pinning is weak. One can notice that first the crack front deforms from a circular crack to a k-petals flower shape, which then remains the same. The process is as follows. In the initial stage, the crack is circular, so that only the points where K_c is minimum propagate. Then, more and more points reach the threshold and propagate at the same time. Finally, the crack front attains and stays in a shape for which all of its points satisfy $K = K_c$ (stationary regime) so that K_m is equal to the mean value of K_c along the crack front:

$$K_m = \frac{1}{L} \int_{\mathcal{F}} K_c(M(s))\mathrm{d}s \tag{10}$$

To quantify the moment where the crack shape becomes stationary, we introduce the amplitude Δa of $a(s)$, the value of $\Delta a/a_m$ remaining constant for a given shape. The evolutions of $\Delta a/a_m$ and of the normalized mean SIF $K_m/\overline{K}_c$ as a function of the mean radius a_m are plotted on Fig. 2(b). It can been seen that both quantities increase until a plateau is reached. The plateau corresponds to the stationary regime. Once in this stationary regime, the effective toughness K_c^{eq}, whether it is defined as the maximum or mean value of K_m, corresponds to the value of this plateau, hence to the mean value of K_c along the crack front. It shall be noticed that $K_c^{\mathrm{eq}} < \overline{K}_c$. Physically, it is due to the fact that the length of the crack front which is in the weaker zone is higher than in the stronger one, so that the mean value of K_c along the crack front is lower than the mean value $\overline{K}_c$ of K_c in the whole plane. This result is specific to the circular geometry, and it is linked to the dependance of the SIF on the crack size. In next section, we discuss the influence of k and κ_c on the value of K_c^{eq}.

3.2 Influence of the geometrical parameters of the toughness field

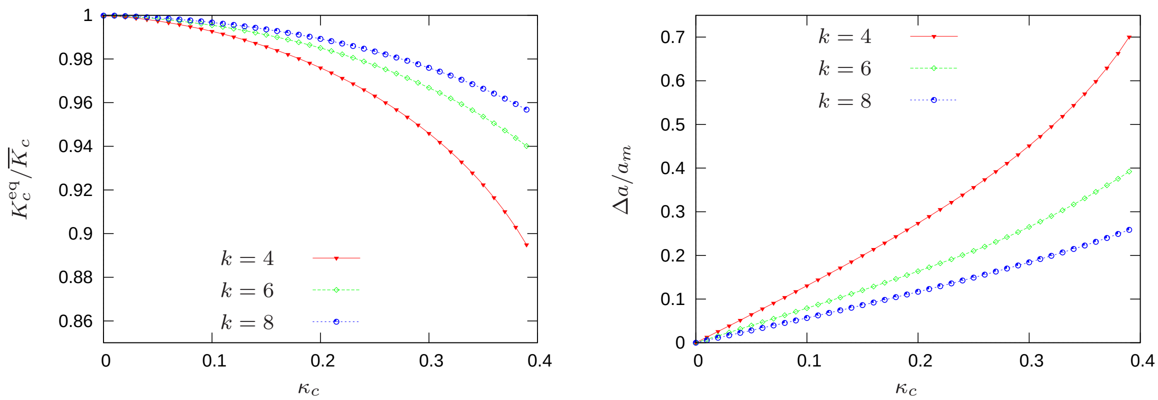
Figure 3 shows the effect of the toughness contrast κ_c and toughness spatial repartition k on the normalized effective toughness $K_c^{\mathrm{eq}}/\overline{K}_c$ and on the crack front deformation $\Delta a/a_m$.

For a given values of k, $K_c^{\mathrm{eq}}/\overline{K}_c = 1$ for $\kappa_c \ll 1$ and decreases, whereas $\Delta a/a_m$ increases, with κ_c. Physically, it looks obvious, that in case of higher contrast, the crack front deforms more, hence propagates more in weaker regions and therefore, the mean toughness along the front is decreasing as the contrast increases. For small $\kappa_c \ll 1$, it remains equal to one, as linear theory predicts [1, 8].

Now for a given value of κ_c, $K_c^{\mathrm{eq}}/\overline{K}_c$ increases, whereas $\Delta a/a_m$ decreases with the heterogeneity wavenumber k, that is when the number of defects increases along the crack front. Physically, it is due to the fact that when k increases, the amplitude of the deformation has less space to develop, and so the front becomes more straight.

4 Conclusion

In this paper, we defined the effective toughness K_c^{eq} of a heterogeneous field of toughness as the stationary mean value of the SIF along the crack front. In order to focus on the effect of the large crack front deformations on this effective toughness, we studied numerically the case of a circular crack propagating in an axisymmetric infinite toughness map. This allows us to reach a stationary crack front shape regime in which Irwin's threshold is reached at each point of the front. In this



(a) Normalized mean toughness versus κ_c

(b) $\frac{\Delta a}{a_m}$ versus κ_c

Figure 3: Influence of mapping parameter on the stationary regime

regime, we find that the SIF K_m averaged along the front reaches a plateau that is equal to the mean value of K_c along the crack front. Since the crack front deforms due the heterogeneities, this mean value is different from the mean value $\overline{K}_c$ of K_c in the whole plane. It depends on the crack front deformations, which themselves depend on the local toughness values. In the case studied here, the deformations are more important in the weak part of the toughness map so that K_c^{eq} is lower than $\overline{K}_c$, the ratio $K_c^{\text{eq}}/\overline{K}_c$ decreasing with increasing toughness heterogeneity or with decreasing the number of obstacles. The next step is to extend this study to the case of strong pinning with large crack front deformations. Defining the equivalent toughness from the macroscopic loading required to make the crack propagate, and not only from the local values of SIF along the front might then become crucial.

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