Evaluation of Strength Statistics of Quasibrittle Structures Based on Mean Size Effect Analysis

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Abstract This paper presents a new method to indirectly determine the probability distribution function of strength of quasibrittle structures. Based on the finite weakest link model, which relates the probability distribution of the structural strength to the size dependence of mean strength, it is shown that that the cumulative distribution function of structural strength can be directly determined from the parameters of the mean size effect curve. A comprehensive experimental set of tests, which includes both strength histograms and mean size effect tests on specimens of asphalt mixture at low temperature, is used to verify the proposed method. The predicted strength histograms obtained with this method are found to be in very good agreement with the experimental histograms of asphalt mixture specimens of different sizes, confirming the validity of the newly proposed formulation.

Keywords Strength Statistics, Histogram Testing, Scaling, Weakest Link Model

1. Introduction

Most engineering structures, such as buildings, bridges, are designed for a very low failure probability (less than 10^{-6}) [1]. Determining the design strength corresponding to such low failure probability directly from histogram testing is prohibitive and, therefore, we need to rely on probabilistic models for indirectly determining the statistics of structural strength.

Simple statistical models can be used for describing the strength distribution in the case of perfectly ductile and brittle structures. For ductile structures, the peak load is equal to the weighted sum of random strengths of material elements along the failure surface. Therefore, according to the Central Limit Theorem, the cumulative distribution function (cdf) of strength of ductile structures follows the Gaussian distribution. On the other hand, the failure of brittle structures is triggered by the failure of one material element whose size is negligible compared to the structure size. Based on the infinite weakest link model (WLM), the strength cdf of brittle structures follows the Weibull distribution [2, 3]. Both Guassian and Weibull distribution are two-parameter probability distribution functions for which the statistical parameters can be easily obtained by histogram testing involving a limited number of specimens.

This is not true for structure made of quasibritte materials, which are brittle heterogenous materials such as concrete and asphalt mixture (at low temperature). For this type of structures the size of inhomogenieties is not negligible compared to the structure size. By limiting our focus to quasibrittle structures for which the peak load is reached as a macro-crack initiates from one representative volume element (RVE) [2-4], it is possible to statistically model this class of structures as a finite chain of RVEs [2-4]. Therefore, the cdf of strength of each RVE must be known for calculating the strength cdf of the entire structure. Recent studies [3, 4] showed that the strength cdf of one RVE can be described by a Weibull distribution grafted on the left tail of a Gaussian cdf. At structural level, the finite WLM shows that the strength cdf consists of an upper part, which can be calculated as a finite chain of Gaussian elements, and a lower segment that follows the Weibull distribution [2-4] resulting into an intricate size effect on the mean structural strength. Determining such a type of strength distribution through histogram testing requires a large number of specimens, which could be very costly and time consuming, due to material and labor.

This paper presents an alternative method to determine the strength cdf of quasibrittle structures, which is anchored at the analysis of the mean size effect curve. The proposed method is experimentally verified by a comprehensive set of tests on the asphalt mixture at a low temperature.

2. Theoretical background

The weakest link model schematizes a structure as finite chain of RVEs. Based on the joint probability theorem, which provides a mathematical expression of the WLM, the failure probability of the entire structure, P_f, made of N RVEs can be obtained according to Eq. 1:

$$P_f(\sigma_N) = 1 - \prod_{i=1}^{N} \left[1 - P_1(s_i \sigma_N)\right] \tag{1}$$

where P_1 is the cdf of strength of one RVE having characteristic size l_0, $\sigma_N = cP_{max}/bD$ is the nominal strength of the structure, P_{max} is the maximum load that the structure can sustain, D is the structure characteristic size, b is the width of the structure, c is a constant such that σ_N represents the maximum elastic principal stress in the structure and s_i dimensionless stress field such that $\sigma_N s_i$ corresponds to the maximum elastic principal stress at the center of i^{th} RVE. Based on atomistic fracture mechanics and a statistical multi-scale transition model [3,4] it was recently demonstrated that the failure cdf of one RVE can be approximated by a Weibull cdf grafted on the left tail of a Gaussian cdf (core) at a point with a probability of about 10^{-4}—10^{-2}. The grafted cdf of strength of one RVE can be expressed as [2-4]:

$$P_1(\sigma) = 1 - \exp[-(\sigma / s_0)^m] \approx \langle \sigma / s_0 \rangle^m \qquad (\sigma_N \le \sigma_{gr}) \tag{2a}$$

$$P_1(\sigma) = P_{gr} + \frac{r_f}{\delta_G \sqrt{2\pi}} \int_{\sigma_{gr}}^{\sigma_N} \exp\left\{-\left[\frac{(\sigma' - \mu_G)^2}{2\delta_G^2}\right]\right\} d\sigma' \qquad (\sigma_N > \sigma_{gr}) \tag{2b}$$

where σ is the maximum elastic principal stress at the center of the RVE, m is the Weibull modulus, s_0 is scale parameter of the Weibull tail, $\langle x \rangle = \max(x, 0)$, μ_G and δ_G are the mean and the standard deviation of the Gaussian core. P_{gr} is the grafting probability between the Gaussian and the Weibull parts of the distribution, σ_{gr} is the grafting stress and r_f is a scaling factor ensuring that Weibull-Gaussian grafted cdf is normalized: $P_1(\sigma \to \infty) = 1$. Six statistical parameters, μ_G, δ_G m, s_0, r_f and σ_{gr} are used to describe the failure distribution of one material RVE; however, due to normalization and continuity conditions, only four of these parameters are independent, and suffice to define $P_1(\sigma)$.

The strength distribution of the entire structure can be calculated by mean of the WLM together with Eqs. (2a) and (2b). The mean structural strength for structures of different sizes can be obtained according Eq. 3:

$$\bar{\sigma}_N = \int_0^1 \sigma_N \mathrm{d}P_f = \int_0^\infty [1\text{-}P_f(\sigma_N)]\mathrm{d}\sigma_N \tag{3}$$

However, a closed form does not exist for Eq. 3, and, therefore, a numerical solution is needed to determine the effect of structure size, D, on the mean strength for geometrically similar specimens. Based on asymptotic matching, Bažant and co-workers [2, 5] proposed an approximate expression for the size dependence of the mean strength:

$$\bar{\sigma}_N = \left[C_1 / D + \left(C_2 / D\right)^{rn/m}\right]^{1/r} \tag{4}$$

where m is the Weibull modulus, n is the number of dimensions to be scaled (n = 1, 2 and 3). C_1, C_2, and r can be determined using the following asymptotic conditions for small and large-size limits: $[\bar{\sigma}_N]_{D \to l_m}$, $[d\bar{\sigma}_N / dD]_{D \to l_m}$ and $[\bar{\sigma}_N / D^{n/m}]_{D \to \infty}$, where l_m represents the small size limit of the structure.

3. Relation between mean size effect curve and strength distribution of one RVE

In this section a method to relate the statistical parameters, μ_G, δ_G m, s_0, of $P_1(\sigma)$ to the properties of the size effect curve of mean structural strength is proposed as an alternative to calibration through histogram testing. The mean size effect (Eq. 4) is determined by five parameters: n, m, r, C_1 and C_2. The scaling dimension, n, is known a priori, while m is the Weibull modulus, which is also one of the four statistical parameters of $P_1(\sigma)$. Therefore, the remaining three parameters μ_G, δ_G and s_0, must be determined from the three statistical parameters r, C_1 and C_2.

Since rn/m << 1, the large-size limit of Eq. 4 can be rewritten as $\bar{\sigma}_N = (C_2/D)^{n/m}$. In such a case the RVE size becomes negligible compared to the structure size. Therefore, the classical extreme value statistics can be applied. Since the strength cdf has a power-law tail, the resulting strength cdf of the entire structure must follow the Weibull distribution [2, 3, 6, 7]. Therefore, the mean strength of very large structures can be calculated as [2-4, 8]:

$$\bar{\sigma}_N = s_0 \Gamma\left(1+1/m\right)\left(D_0/D\right)^{n/m} \tag{5}$$

where $D_0 = l_0\left[\int_V \langle s(\xi)\rangle^m \mathrm{d}V(\xi)\right]^{-1/m}$, $\xi = \mathbf{x}/D$ is the normalized coordinate, and $\Gamma(x)$ is the Eulerian gamma function. By equating Eq. 5 with the large size expression of Eq. 4, it is possibile obtaining the Weibull scaling parameter s_0 in terms of the parameter C_2:

$$s_0 = \left(C_2/D_0\right)^{n/m}\Gamma^{-1}\left(1+1/m\right) \tag{6}$$

In order to use Eq. 6, the RVE size l_0 must be known a priori. Based on a recent study [2] the RVE size can be estimated as 2 times of size of the material inhomogeneities, which can be determined through digital image analysis [9] or from the aggregates gradation curve as was done in this study. At the small size limit two asymptotic conditions can be written for relating μ_G and δ_G to r and C_1:

$$\bar{\sigma}_N\Big|_{D\to l_m} = \int_0^\infty \prod_{i=1}^{N_m}[1-P_1(s_i\sigma_N)]\mathrm{d}\sigma_N = \left[\frac{C_1}{l_m}+\left(\frac{C_2}{l_m}\right)^{rn/m}\right] \tag{7}$$

$$\frac{\mathrm{d}\bar{\sigma}_N}{\mathrm{d}D}\bigg|_{D\to l_m} = -\int_0^\infty \frac{\mathrm{d}P_f}{\mathrm{d}D}\mathrm{d}\sigma_N = -\frac{1}{r}\left[\frac{C_1}{l_m^2}+\frac{nr}{ml_m}\left(\frac{C_2}{l_m}\right)^{rn/m}\right]\left[\frac{C_1}{l_m}+\left(\frac{C_2}{l_m}\right)^{rn/m}\right]^{1/r-1} \tag{8}$$

where N_m is the number of RVEs in the structure at the small-size limit and l_m is the smallest specimen size which makes physical sense. Recent studies showed that for beams under three-point bending, which is commonly used in the laboratory testing, the minimum depth of the beam for the WLM to be valid is about 4 RVEs [10]. By using the logarithm of the WLM (Eq. 1) and integrating over the entire structure, the left hand side of Eq 8 can be rewritten as:

$$-\int_0^\infty \frac{\mathrm{d}P_f}{\mathrm{d}D}\mathrm{d}\sigma_N = \frac{n}{D}\int_0^\infty \prod_{i=1}^{N_m}[1-P_1(s_i\sigma_N)]\sum_{i=1}^{N_m}\ln[1-P_1(s_i\sigma_N)]\mathrm{d}\sigma_N \tag{9}$$

At the small size limit, N_m is usually small (i.e. 4 RVEs) and, therefore, it is expected that the Weibull tail of the strength cdf is very short. Hence, the cdf of strength can be entirely approximated by the Gaussian core; $P_1(\sigma)$ can be replaced by a Gaussian distribution, which has a mean μ_G and standard deviation δ_G. Eqs. 7 and 8 can then be reformulated as:

$$\int_0^\infty \prod_{i=1}^{N}[1-\Phi_G(s_i\sigma_N,\mu_G,\delta_G)]\mathrm{d}\sigma_N = \left[\frac{C_1}{l_m}+\left(\frac{C_2}{l_m}\right)^{rn/m}\right]^{1/r} \tag{10}$$

$$\frac{n}{l_m}\int_0^\infty \prod_{i=1}^{N_m}[1-\Phi_G(s_i\sigma_N,\mu_G,\delta_G)] \sum_{i=1}^{N_m}\ln[1-\Phi_G(s_i\sigma_N,\mu_G,\delta_G)]\mathrm{d}\sigma_N = \\ = -\frac{1}{r}\left[\frac{C_1}{l_m^2}+\frac{nr}{ml_m}\left(\frac{C_2}{l_m}\right)^{rn/m}\right]\left[\frac{C_1}{l_m}+\left(\frac{C_2}{l_m}\right)^{rn/m}\right]^{1/r-1} \tag{11}$$

where $\Phi_G(s_i\sigma_N,\mu_G,\delta_G)=(2\pi)^{-1/2}\int_{-\infty}^{s_i\sigma_N}\exp[-(x-\mu_G)^2/2\delta_G^2]\mathrm{d}x$.

By solving the system of Eqs. 10 and 11, together with Eq. 6 and the Weibull modulus m, it is possible to determine the four parameters of the strength cdf of one RVE (μ_G, δ_G m, s_0) from the mean size effect curve. This permits us to calculate the strength distribution of structures of any size and geometry through the WLM.

4. Material and testing

A comprehensive set of experiments, including both mean strength tests on asphalt mixture specimens of different sizes and strength histogram testing at a low temperature, was used to verify the proposed method for indirectly determining the strength distribution from the mean strength curve. The asphalt mixture used for the experimental phase was prepared with a blend of aggregates, consisting of taconite aggregates (55% of MIN TAC tailings and 10% of ISPAT tailings) and pit sand (35%), and a PG 64-34 asphalt binder (7.4% by weight) [11]. The nominal maximum aggregate size was 4.75mm. Based on the sieve size analysis, the dimension of the asphalt mixture RVE was estimated to be twice the size of the material inhomogeneities, which for asphalt mixture corresponds to the average aggregate size. For the particular asphalt mixture considered in this study, we estimated an average aggregate size to be 1.22 mm and an RVE volume V_0 to be 14.4 mm^3.

Since the objective of this study is to derive the strength distribution from the mean size effect curve, a large size range is needed. Therefore, size effect tests were performed by using three-point bend (3PB) beams (Fig. 1).

3PB beam (C)

3PB beam (B)

3PB beam (A)

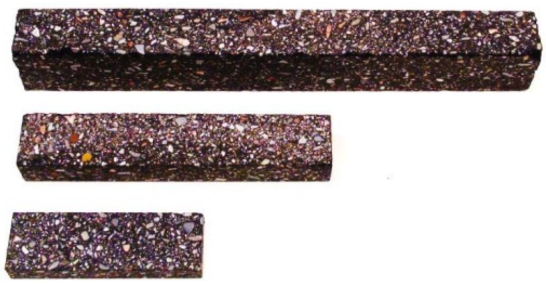

Figure 1. Beam specimens

Nevertheless, the sizes of the climatic chamber and of the testing machine limited the size of the beam specimens. Therefore, an alternative type of test was required to achieve a sufficiently large range of sizes to fully verify the ductile-to-brittle transition of material behavior as structure size increases. For this reason, mean strength tests were also conducted on prismatic specimens in direct tension (DT) configuration. Such an approach was selected because by varying test type and geometry both stress field and failure probability change. Therefore, given the same mean strength, it is possible to convert the dimensions of a structure for a specific stress field and geometry into a different structure with different stress field and geometry [2]. Since Weibull distribution governs the cdf of very large structures, the size of the DT specimen was set as the largest possible. This ensures that the DT specimen consists of a large number of RVEs and thus the classical Weibull statistics can directly be used to calculate the size of the equivalent three-point bend beam.

Specimens were obtained from twenty-six slabs of asphalt mixture (size 380mm by 200mm) compacted at target air voids of 7% by mean of a Linear Kneading Compactor (LKC). Asphalt mixture beams for 3PB tests present thickness to span ratio equal to 1 : 6 and size ratio 1 : $\sqrt{3}$: 3; since 2D scaling was used for beams, a constant width b = 40 mm was selected. The forth specimen type used for DT tests was prepared by cutting one-size asphalt mixture prisms. Table 1 presents detailed information on the specimens used. The thickness of the beams, D, and the width of the prism were assumed as characteristic dimensions for the three-point bend and DT specimens, respectively (Figure 2a and 2b). The dimensions of the specimens were chosen based on the dimensions of the compacted slabs and on the limitation imposed by the climatic chamber and loading frame.

Table 1. Specimen details and mean strength results

Specimen ID	Replicates #	Dimensions (L x D x b)	Test Type	Mean/Histogram	Mean Strength $\overline{\sigma}_N$ (MPa)
A	12	100 x 16.7x 40 mm	3PB	Mean	14.3
B	28	173 x 28.9 x 40 mm	3PB	Histogram	12.4
C	30	300 x 50 x 40 mm	3PB	Histogram	11.4
D	7	255 x 55 x 55 mm	DT	Mean	8.2

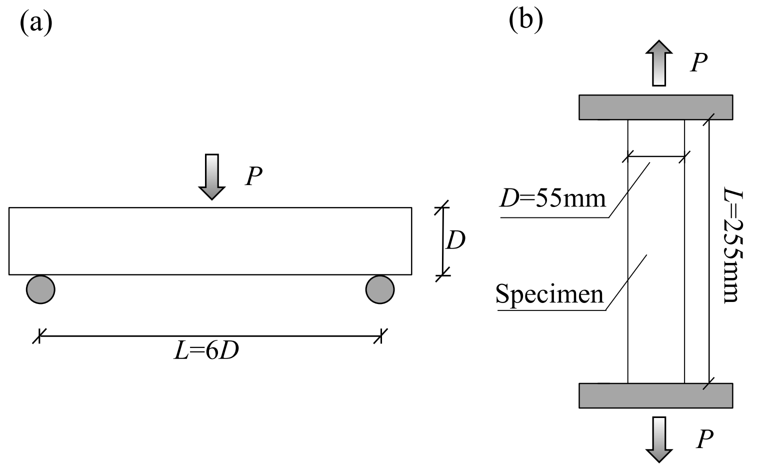

Figure 2. Schematics of (a) three-point bend test and (b) direct-tension test

Testing temperature was set to T=-24°C (low PG+10°C), which is close to the glass transition temperature of the asphalt mixture used. Together with very short tests duration this reduces the viscoelastic effect of the binder component. The desired testing temperature was achieved through a controlled flow of nitrogen inside the climatic chamber of the MTS device used for testing; a conditioning time of three hours was imposed to all the type of specimens before testing.

DT tests were performed by gluing the specimens to a set of plates with an epoxy compound, and then attached to loading frame through a set of screws. Since only the peak load is of interest for this study, both 3PB and DT tests were conducted in load-control mode. In order to minimize the loading rate effect and achieve a similar loading rate of the fracture process zone (FPZ), a time to failure of about 5 minutes was set for all the specimens. Consequently, different loading rates were set for each specimen type; for this reason, a rate calibration was initially performed by testing four to six additional specimens at each testing condition.

5. Size effect analysis

The nominal strength of three-point bend σ_N^B and direct tension σ_N^T were calculated as $\sigma_N^B = (3P_{\max}L)/2bD^2$ and $\sigma_N^T = P_{\max}/bD$, where P_{max} is the peak load, L is the length of the beam, D is the scaling dimension (the thickness of the beam or the width of the DT prism) and b is the depth of the beam for three-point bend (40mm) or the depth of the prism (55mm) for DT specimens. Table 1 presents the mean nominal strength for all the specimens and the number of replicates used. In order to calculate the probability distribution of nominal strength of beam series B and C (Table 1), the strength values were ranked in an ascending order, i = 1, ...,N, where i is the rank and N is the total number of test specimens, and the strength cdf was next calculated according to the midpoint position method [12] as $P_f(\sigma_N^B) = (1-0.5)/N$. The resulting strength histogram is shown in Fig. 3 on the Weibull scale.

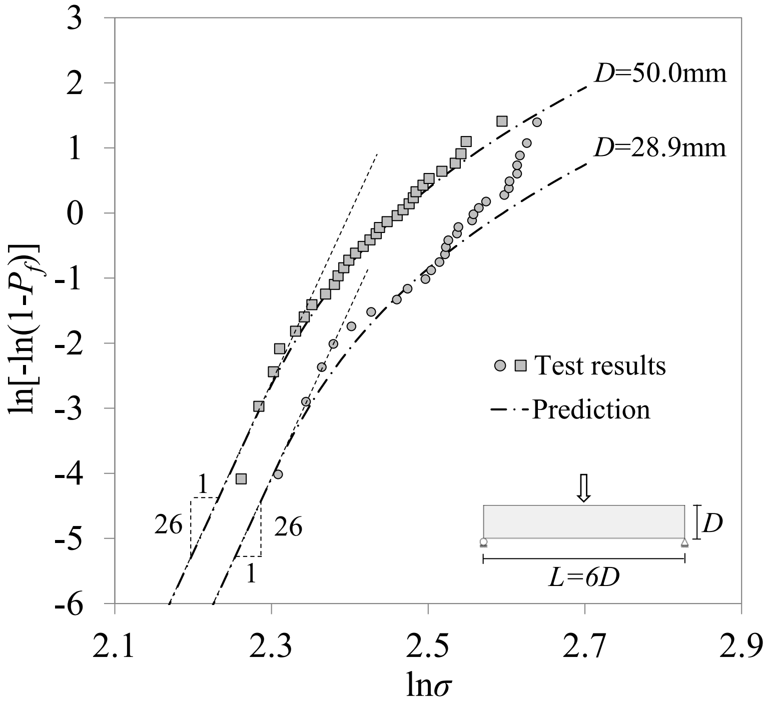

Figure 3. Experimental strength histograms and WLM predictions

The experimental strength histogram presents a trend which is common to many other quasibrittle materials such as concrete and engineering ceramics [13, 14], and is composed of two segments separated by a kink point: the lower segment follows a straight line (i.e. a Weibull distribution) while the upper portion is curved, according to the finite WLM. The experimental histograms indicate that the strength distribution strongly depends on the structure size: as the size increases, the Weibull portion becomes more and more dominant. The Weibull modulus, m, of the lower part of the two histograms can be obtained by fitting; this value is identical and equal to 26, which implies that the Weibull modulus does not vary with the structure size.

The mean strength of DT specimens needs to be converted into the corresponding equivalent 3PB beam to be next used for plotting the mean strength size effect curve. The DT specimen used in this investigation has a volume of 756,000 mm^3 (Table 1), which consists of almost $5.25 \cdot 10^4$ RVEs (V_0=14.4 mm^3). Such a large number of RVEs ensures that the failure cdf of DT specimens is fully governed by the Weibull distribution, and, therefore, by the two parameters m and s_0. The mean strength of DT specimens, σ_N^T, and the mean strength of the equivalent 3PB beams, σ_N^B, can be written as:

$$\bar{\sigma}_N^T = N_T^{-1/m} s_0 \Gamma\left(1+1/m\right) \quad \text{and} \quad \bar{\sigma}_N^B = N_{eq,B}^{-1/m} s_0 \Gamma\left(1+1/m\right) \tag{12}$$

where N_T is the number of RVEs in the direct tension specimen and the equivalent number of RVEs in the 3PB beam is obtained as: $N_{eq,B} = \dfrac{b}{l_0^3}\int_V [(\sigma(\mathbf{x})/\sigma_N^B)]^m \, \mathrm{d}V(\mathbf{x})$, where V is the volume of the beam. The equivalent characteristic size, D_{eq} (thickness), of the three-point bend beam, which has a nominal strength equal to $\bar{\sigma}_N^T$, is obtained by equating Eqs. 12:

$$D_{eq} = \sqrt{[(m+1)^2 N_T V_0]/(3b)} = 2143 \text{ mm} \tag{13}$$

Therefore, by simply selecting a different loading mode it is possible to achieve significantly different equivalent structure sizes, which allows obtaining size effect curve for a much wider size range. At the same time, based on the measured mean strength of DT specimens, and by using Eq. 12 for DT, we can directly estimate the value of the Weibull scaling parameter s_0 = 12.68 MPa. For beam specimens with 2D scaling (i.e. n = 2), we can consider that the damage is throughout the entire beam thickness. Therefore, the effective size of the RVE can be obtained as $l_0 = \sqrt{V_0/b} = 0.6$ mm; based on this condition and on Eq. 6, the following value of C_2 can be calculated: $C_2 = (m+1)/\sqrt{3} \cdot l_0 s_0^{m/2}\left[\Gamma\left(1+1/m\right)\right]^{m/2} = 1.55 \cdot 10^{15}$.

Fig. 4 shows the mean size effect curve of structural strength for 3PB beams and DT specimens (with their equivalent 3PB size). The experimental mean strength curve presents a pattern that is typical to that observed in other quasibrittle structures such as concrete beams [5, 15].

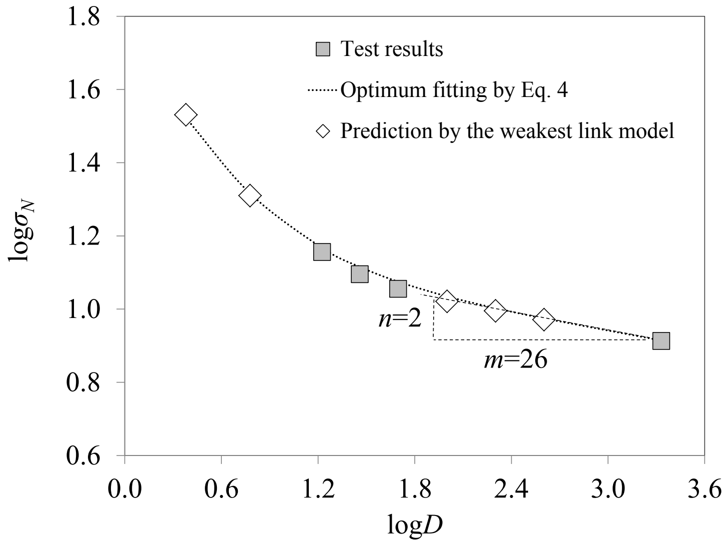

Figure 4. Mean strength curve

However, it is clear that the four mean strength data points obtained experimentally are not sufficient to determine the large-size asymptote of the size effect curve. For this reason, the Weibull modulus, m, was determined from the lower portion of the strength histogram as shown earlier in this paper. Therefore, $C2$ was calculated from the DT specimen Weibullian mean. If we tested DT specimens of two different sizes, m could be calculated easily by fitting the linear portion of the mean strength curve at the large size limit and, hence, obtain s_0 directly from Eq. 6.

Parameters C_1 and r in Eq. 4 can be finally determined by non-linear fitting of the experimental mean strength data: the following values were obtained: C_1 = 49.49 and r = 1.01 which agrees well with the analytically derived size effect on the modulus of rupture of three-point bend beams [16]. By knowing C_1 and r, and solving Eqs. 10 and 11, it is possible to obtain the remaining two parameters of the Gaussian core, μ_G, δ_G, of the of the RVE strength cdf. Since for this computation we are interested to the small size limit, the smallest beam size which can be used has a characteristic size $D = 4l_0$ (Fig. 5). According to the elastic stress field two layer of RVE are subjected to tensile stress. The stress at the center of the RVEs upper-layer is half of the stress at the center of the RVEs bottom-layer. Moreover, along the beam span there is a significant decay of stress moving away from the two central RVEs, “C”, of 9.5% and 26% for RVEs “L” and “A”, respectively. Given the large value of the Weibull modulus, it is reasonable to only include the bottom-layer RVEs “C” and “L” for the calculation of μ_G, δ_G.

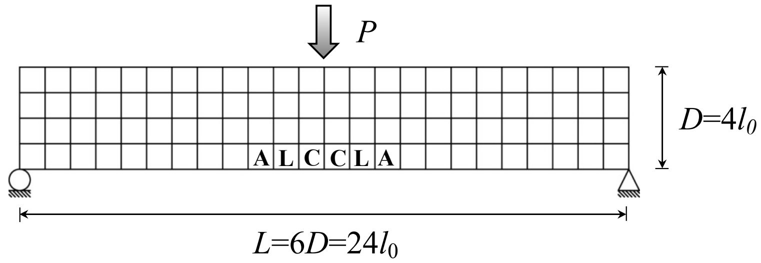

Figure 5. Small size limit beam

Therefore, Eqs. 10 and 11 can be rewritten as:

$$\int_0^\infty \prod_{i=1}^{2}[1-\Phi_G(s_i\sigma_N,\mu_G,\delta_G)]^2 \mathrm{d}\sigma_N = \left[\frac{C_1}{4l_0}+\left(\frac{C_2}{4l_0}\right)^{2r/m}\right]^{1/r} \tag{14}$$

$$\begin{aligned}&\frac{1}{l_0}\int_0^\infty \left\{\prod_{i=1}^{2}[1-\Phi_G(s_i\sigma_N,\mu_G,\delta_G)]^2 \sum_{i=1}^{2}\ln[1-\Phi_G(s_i\sigma_N,\mu_G,\delta_G)]\right\}\mathrm{d}\sigma_N = \\ &= -\frac{1}{r}\left[\frac{C_1}{16l_m^2}+\frac{2r}{4l_0 m}\left(\frac{C_2}{4l_0}\right)^{2r/m}\right]\left[\frac{C_1}{4l_0}+\left(\frac{C_2}{4l_0}\right)^{2r/m}\right]^{1/r-1}\end{aligned} \tag{15}$$

where s_1 = 0.718, and s_2 = 0.656. The system of Eqs. 14 and 15 can be numerically solved, yielding μ_G = 45.24 MPa and δ_G = 14.82 MPa, which, can be easily demonstrated, are the only couple of values satisfying Eqs. 14 and 15.

With knowing μ_G, δ_G m and s_0 the strength distribution of one RVE (Eqs. 2a and 2b) can be evaluated. With the finite WLM (Eq. 1), it is possible predicting the strength cdf of beams series B and C. Fig. 3 shows that the predicted strength cdf matches very well the measured strength histograms for both beam sizes. This indicates the statistical parameters of strength cdf can be calibrated from the mean size effect curve. By using the WLM we can further calculate the strength distribution and the mean strength of beams with other different sizes; Fig. 4 indicates that the mean structural strength predicted from the WLM lies on the size effect curve represented by Eq. 4, supporting the validity of this expression in providing a good approximation of the exact size effect curve calculated from the finite WLM.

In previous studies [2-4] it was shown that the finite WLM could correctly describe the deviation of the strength histogram of quasibrittle structures of a single size from the two-parameter Weibull distribution. Based on the strength histograms of structures of two sizes, this study provides a further validation of the finite WLM; by properly selecting specimen sizes and number of replicates the size effect on the grafted strength distribution was clearly demonstrated (Fig. 3). The good agreement between the predicted and measured strength distributions of beams of two sizes indicates that the recently proposed finite WLM can well capture the size effect on the strength cdf of quasibrittle structures.

6. Conclusions

In this paper an analytical and experimental demonstration on the possibility of indirectly determining the strength cdf of quasibrittle from the mean size effect curve was presented. This method provides a valid alternative to the conventional histogram testing, it requires a smaller number of specimens, and it is less prone to experimental errors. The size effect tests on the strength histogram of asphalt mixture indicates that the probability distribution of structural strength strongly depends on the structure size, and such dependence can be well explained by the finite weakest link model.

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