A strategy to evaluate the properties of circular ring dielectric actuators

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Abstract Dielectric elastomer actuator can be used to adjust the properties of many optical elements. For example, place a diffractive transmission grating on the center of a circular ring actuator. The deformation of the actuator under voltage can change the grating period. The circular ring actuator is an annular dielectric elastormer coated with soft electrodes sandwiched between an elastic circle and an elastic annulus. When subject to a voltage, the active dielectric region will expand and compress the central elastic circle as well as the outer elastic annulus. The compressive deformation of the central elastic circle changes the grating period. We study the properties of circular ring actuators based on the ideal dielectric model combined with the Arruda-Boyce hyperelastic constitutive law and obtain the relationship between the applied voltage and the grating period. The strategy presented here is generic and the results may contribute to the use of dielectric actuators in optical devices.

Keywords Dielectric elastomer actuator; Tunable grating; Ideal dielectric

1. Introduction

Adaptive optical elements are of great importance in a wide range of scientific applications [1]. For the active dynamic tuning of the optical elements, a variety of methods has been proposed, ranging from change the refractive index of a material [2] to rely on the acousto-optic effect[3]. But so far, these methods are not so successful in the tuning of optical elements.

Now the tunable diffraction grating is based on relatively stiff materials and can only achieve a limited spatial tuning grating. A promising approach for combining rapid response time, good optical quality and large actuation amplitude is the use of dielectric actuators [4-6], which is known as "artificial muscles". When a DE is used as an actuator, it can achieve a large voltage-induces strain and can be driven at acoustic frequencies [7].

Here, we investigate a pre-stretched polymer film of VHB4910 which is fixed by a rigid frame and partially covered with high conductive carbon grease as electrodes. Then a polymeric diffractive structure is bonded onto the actuator, similar to the previous work[8-10]. our work is investigate the previous structure with new theory analysis method. The results can be used in the calculation of the tunable transmission grating.

2. Circular ring actuator analysis

The schematic arrangement of a circular ring dielectric actuator is shown in Fig. 1. A circular ring actuator is made from pre-stretched elastomer with a soft electrode annulus which is sandwiched by two electrode-less regions. When subject to a voltage, the active dielectric region will expand and compress the central elastic circle as well as the outer elastic annulus. The inner passive region deforms from its reference configuration with initial radius A, to its current configuration with radius a. The active region evolve from its reference configuration with inner radius A and outer radius B to its current configuration with inner radius and outer radius b, respectively. The outer passive region evolves from its reference configuration with inner radius B and outer radius C to its current configuration with inner radius b and outer radius fixed, respectively. All kinetic variables are expressed in the reference polar coordinates R, Θ, Z systems, which are denoted by the subscript 1, 2 and 3.The inner passive region, middle active region and outer passive region, are denoted by superscript 1, 2 and 3, respectively.

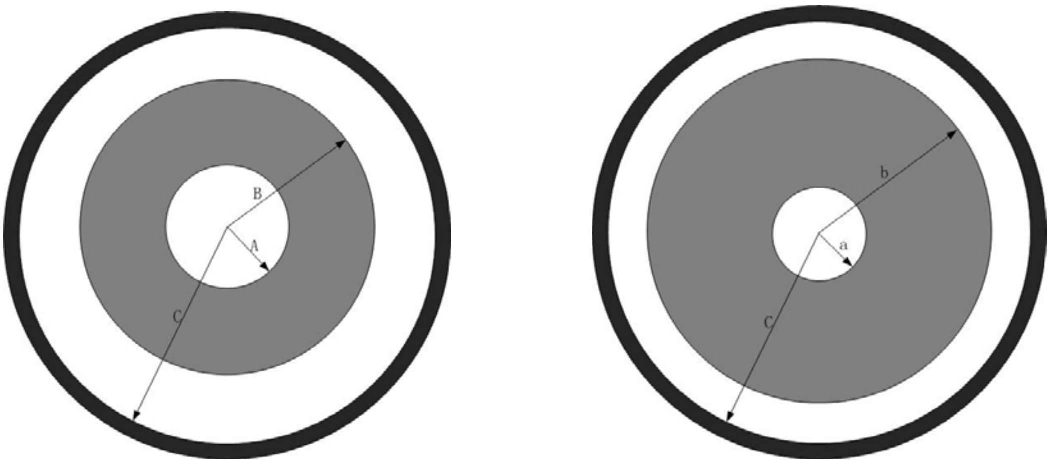

Figure 1. The schematic of a typical experimental arrangement of a circular ring dielectric actuator. The central gray area is the dielectric actuator, and outer black area is the rigid frame to apply pre-stretch.

Here the strain energy of an ideal dielectric material is rewritten in terms of stretches and nominal electric field, i.e.,

$$W(\lambda_1,\lambda_2,\tilde{E}) = W_0(\lambda_1,\lambda_2) + W_{ele}(\lambda_1,\lambda_2,\tilde{E}), \tag{1}$$

For the two passive regions, the material laws are only given the first term of the Eq. (1). Based on the elastic theory, the geometric relationship can be obtained readily as

$$\lambda_1 = \frac{\partial r}{\partial R}, \quad \lambda_2 = \frac{r}{R} \tag{2}$$

where λ_1 and λ_2 are radial and hoop stretches, respectively. The equilibrium equation is

$$\frac{\partial s_1}{\partial R} + \frac{s_1 - s_2}{R} = 0 \tag{3}$$

We use the Arruda-Boyce material to express the hyper-elastic part $W_0(\lambda_1,\lambda_2)$, and it gives

$$W_0(I) = \mu\left[\frac{1}{2}\left(I-3\right) + \frac{1}{20N}\left(I^2-9\right) + \frac{11}{1050N^2}\left(I^3-27\right) + \ldots\right] \tag{4}$$

where $I = \lambda_1^2 + \lambda_2^2 + \lambda_1^{-2}\lambda_2^{-2}$ is the first principle invariant. We use nominal stress and nominal electric field as well as electric displacement as mechanical and electrostatic measures. The nominal stresses of the material are given by the following material laws

$$s_1 = \frac{\partial W}{\partial \lambda_1} = \frac{\partial W_0}{\partial I}\frac{\partial I}{\partial \lambda_1} + \frac{\partial W_{ele}}{\partial \lambda_1} \tag{5}$$

$$s_2 = \frac{\partial W}{\partial \lambda_2} = \frac{\partial W_0}{\partial I}\frac{\partial I}{\partial \lambda_2} + \frac{\partial W_{ele}}{\partial \lambda_2} \tag{6}$$

Taking derivative of Eq. (5) with respect to R and substituting Eq. (5) and (6) into Eq. (2) gives the following ODE

$$\frac{\partial \lambda_1}{\partial R} = \frac{1}{R} g_1(\lambda_1,\lambda_2)(\lambda_2-\lambda_1) + \frac{1}{R} g_2(\lambda_1,\lambda_2) \tag{7}$$

Another equation can be deduced by invoking Eq. (2) and gives

$$\frac{\partial \lambda_2}{\partial R} = \frac{1}{R}(\lambda_1-\lambda_2) \tag{8}$$

The $g_1(\lambda_1,\lambda_2)$ and $g_2(\lambda_1,\lambda_2)$ functions in Eq. (7) are expressed as

$$g_1(\lambda_1,\lambda_2) = \frac{\dfrac{\partial^2 W_0(I)}{\partial I^2}\dfrac{\partial I}{\partial \lambda_1}\dfrac{\partial I}{\partial \lambda_2} + \dfrac{\partial W_0(I)}{\partial I}\dfrac{\partial^2 I}{\partial \lambda_1 \partial \lambda_2} + \dfrac{\partial^2 W_{ele}}{\partial \lambda_1 \partial \lambda_2}}{\dfrac{\partial^2 W_0(I)}{\partial I^2}\left(\dfrac{\partial I}{\partial \lambda_1}\right)^2 + \dfrac{\partial W_0(I)}{\partial I}\dfrac{\partial^2 I}{\partial \lambda_1^2} + \dfrac{\partial^2 W_{ele}}{\partial \lambda_1^2}} \tag{9}$$

$$g_2(\lambda_1,\lambda_2) = \frac{\dfrac{\partial W_0(I)}{\partial I}\left(\dfrac{\partial I}{\partial \lambda_2} - \dfrac{\partial I}{\partial \lambda_1}\right) + \left(\dfrac{\partial W_{ele}}{\partial \lambda_2} - \dfrac{\partial W_{ele}}{\partial \lambda_1}\right)}{\dfrac{\partial^2 W_0(I)}{\partial I^2}\left(\dfrac{\partial I}{\partial \lambda_1}\right)^2 + \dfrac{\partial W_0(I)}{\partial I}\dfrac{\partial^2 I}{\partial \lambda_1^2} + \dfrac{\partial^2 W_{ele}}{\partial \lambda_1^2}} \tag{10}$$

For the passive region, the inner region's deformation are equal biaxial and $\lambda_1 = \lambda_2 = \lambda$ and

$s_1 = s_2 = s$. The nominal stress $s_{1,2}$ is given by

$$s_{1,2}^1 = \frac{\partial W_0(I)}{\partial I} 2\left(\lambda - \lambda^{-5}\right) \tag{11}$$

For the active region, the stresses are given by Eq. (5) and Eq. (6):

$$s_1^2 = \frac{\partial W_0}{\partial I}\left(2\lambda_1 - 2\lambda_1^{-3}\lambda_2^{-2}\right) - \varepsilon \tilde{E}^2 \lambda_1 \lambda_2^{\,2} \tag{12}$$

$$s_2^2 = \frac{\partial W_0}{\partial I}\left(2\lambda_2 - 2\lambda_1^{-2}\lambda_2^{-3}\right) - \varepsilon \tilde{E}^2 \lambda_1^{\,2} \lambda_2 \tag{13}$$

And the outer passive region' stresses can be obtained in the same way:

$$s_1^3 = \frac{\partial W_0(I)}{\partial I}\left(2\lambda_1 - 2\lambda_1^{-3}\lambda_2^{-2}\right) \tag{14}$$

$$s_2^3 = \frac{\partial W_0(I)}{\partial I}\left(2\lambda_2 - 2\lambda_1^{-2}\lambda_2^{-3}\right) \tag{15}$$

The stresses given by Eq. (11) ~ (15) should satisfy the following force balance requirement

$$s_1^1(R=A) = s_1^2(R=A) \tag{16}$$

$$s_1^2(R=B) = s_1^3(R=B) \tag{17}$$

The stretches across the interface should also fulfill the following boundary conditions

$$\lambda_2^1(R=A) = \lambda_2^2(R=A) \tag{18}$$

$$\lambda_2^2(R=B) = \lambda_2^3(R=B) \tag{19}$$

Eq. (7) and (8) together with the boundary conditions (16) ~ (19) constitute the nonlinear ODEs that should be calculated numerically. We use a shooting technique to solve these ODEs by iterating a assumed radial stretch until the computed hoop stretch on the outer boundary equals to the prescribed pre-stretch, and the computed stress on the inner boundary fulfill the force balance equation, Eq. (16) and (17).

Based on the above-mentioned procedure, the properties of the circular ring dielectric actuator can be evaluated. In a simulation example, we fixed A=4mm, B=9mm, C=10.5mm, λ_p =1.2 and $\tilde{E}/\sqrt{\mu/\varepsilon}=0.5$.Fig. 2 shows the deformation of the circular ring actuator under a pre-stretch and a dimensionless nominal electric field. It is noted that the middle active region's deformation is unhomogeneous and the inner radius deforms greater than the others which results the inner passive region is compressed more obvious. Also the radical's deformation is incontinuous along the radius which is unknown before the calculation. Fig. 3 shows the dimensionless nominal stress versus radius R. From this figure, we know the distribution of the nonimal stress along the radius and the radius R=A and R=B are likely to suffer loss of tension when the voltage is increasing.

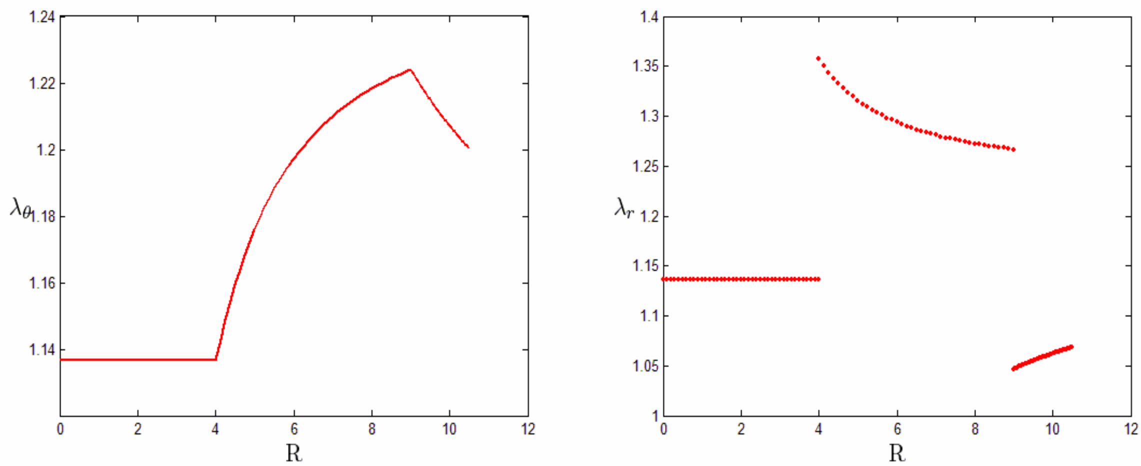

Figure 2. The radius R versus hoop and radial stretches

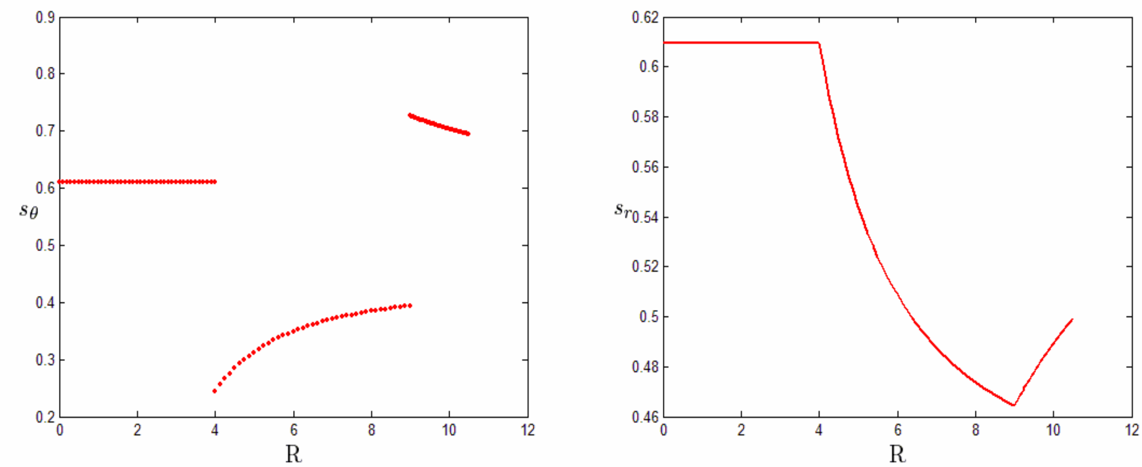

Figure 3. The radius R versus hoop and radial dimensionless nominal stress

3. Applications in tunable transmission grating

Tunable transmission grating is bonded onto circular ring actuator, as showed in Fig. 4. When the voltage is applied, the middle active region enables a compact implementation and results in a change of the grating period,

$$d = (1 - s_{center})d_0 \tag{20}$$

where d_0 is the grating period at $V = 0$, and s_{center} is the planar strain in the inner passive region. In the above simulation example, the grating period changes from $2\mu m$ to $1.9\mu m$

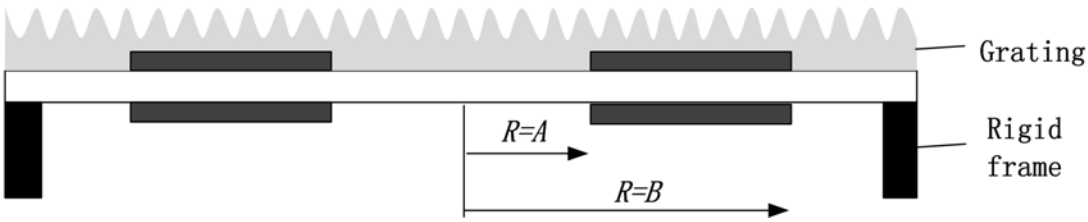

Figure 4. The tunable transmission grating based on circular ring dielectric actuators

Once the stretch in the inner passive region under a certain voltage is calculated, the grating period is also determined by Eq. (20).

Acknowledgements

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