# Fractional-order Modeling of Linear Viscoelastic Creep in Hami Melon


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**Abstract**: This paper describes our experimental testing of linear viscoelastic creep behaviors in Hami Melon. Experimental data shows that Hami Melon has complex viscoelastic property which can not be well described by the standard model. Consequently, this study develops a fractional derivative model to describe such complex viscoelastic creep behaviors of Hami Melon. The analytical creep function of the proposed fractional linear viscoelastic models is derived via the Boltzmann superposition principle and discrete inverse Laplace transform. And then such analytical solutions are used to fit the experimental data of viscoelastic Hami Melon. Our study shows that the present fractional linear viscoelastic model with merely three parameters is more efficient and accurate than the generalized Kelvin viscoelastic model of six parameters to describe the stress—strain constitutive relations of Hami Melon. It is noted that the present fractional model with adjustable parameters can also be used to describe creep damage.



**Keywords: Hami Melon; experiment; viscoelasticity; fractional derivative; creep**


## 1. Introduction

Hami Melon is famous for its delicious taste and aroma, and has become one of the most characteristic fruits in Xinjiang, China. However, a large amount of Hami Melon is often destroyed due to decay in storage and transport, which may be attributed to its high water content and maturates in hot summer. It has been recognized that biomaterials exhibit unique viscoelastic behaviours [1]. Such viscoelasticity has been investigated to minimize physical damage and improve textural quality of fruits. However, it is noted that the transient and dynamic measurements in most existing methods are restricted to small deformations within the linear viscoelastic range of specimens [4,5].These traditional linear models can not accurately describe complex viscoleastic behaviors of Hami Melon.

On the other hand, the characterisation of non-linear behaviour of apple flesh under stress relaxation and the basic homogeneous assumption have been studied by Lu and Puri [1]. The measured vibrations can be visualised with experimental modal analysis, used in the past with pineapples and melons [6]. However, the classical nonlinear models are mathematically complex and require some obscure parameters which are not easy to obtain from measurement data.

In recent decade, fractional derivative model has attracted great attention in the description of memory-dependent mechanics behaviours, such as dynamical behaviours of complex viscoelastic materials [7]. Fractional derivative viscoelastic models are presented by some researchers [7-14] and have been applied to a wide range of problems in bioengineering [15-19]. But little has so far been done on melon [6,15].

In this study, we employ the fractional-order Maxwell viscoelastic model to characterize the viscoelasticity of Hami Melon. Compared with the classical viscoelasitc model, our results show that the present fractional derivative model can characterize creep behaviour of Hami Melon with better accuracy and fewer parameters.

Among the objectives of this study are:

(1) to describe experiments for measuring the anisotropic creep properties of Hami Melon;

(2) to present a fractional linear viscoelastic model and determine its creep function;

(3) to make a comparison between the fractional and classical models in terms of our experimental data.

## 2. Experimental materials and methodology

### *2.1. Preparation of Hami Melon specimens*

Fresh Hami Melons for experiments were hand-harvested on 25 August 2010 from the same orchard in Hami city, Xinjiang, China and were selected in terms of uniformity and placed in cold storage (6°C to 7°C and 70–80%RH). Four hours before testing, melons were taken to equilibrate at room temperature (20°C). Each specimen was peeled and cut in half longitudinally. After having removed the central and near the hull (approximately 1.5cm of each side) parts, cylindrical axial and radial specimens (50mm diameter by 10mm height) were put into a sealed container for experimental measurements.

Anisotropic creep properties of melons were evaluated using cylindrical specimens taken in orthogonal radial and axial orientations, as shown in Fig. 1.

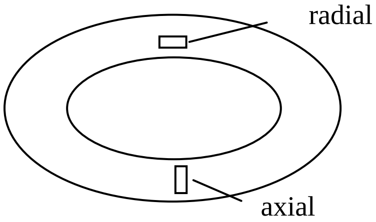


Figure 1. Location of specimens

### *2.2. Creep testing*

Creep tests, in which a shear stress is instantaneously applied to the specimen and then maintained constant, allow us to observe elastic, viscoelastic, and viscous flow behaviours, separately. Deformation and compliance increase with time. In the initial state of creep, the sample material behaves like a solid and subsequently like a fluid. Viscoelastic properties were characterized at 20°C in a TA.XT plus Texture Analyser made by Stable Micro Systems Ltd, UK. 0.06% strain value was selected for experiments to ensure linearity for all specimens.

Creep tests of melons were conducted by enforcing a constant shear stress 20Pa for duration of 60 seconds. An often-encountered problem in measuring the physical properties of fresh or minimally processed tissues is that they are usually alive and respiring, and can be dehydrated by high strain rate during measurements. Thus, the interpretation of creep behaviour in this paper ascribes considerable importance to the time scale over which creep occurs.

### *2.3. Analysis of data*

In this section, compliance data from creep experiments were fitted by a mechanical model consisting of one Maxwell model connected in series with two Kelvin models. It is noted that each Kelvin model has a spring and a dashpot in parallel as shown in Fig.2, which is described by the generalized Kelvin six parameters model [5],

$$J(t,\sigma_c) = (J_0) + \sum_{i=1}^{2}(J_i)(1 - e^{-t/\lambda_i}) + t/\eta_0 \ , \tag{1}$$

where $J(t,\sigma_c)=\varepsilon(t)/\sigma_c$ denotes the creep compliance, $\varepsilon(t)$ is the strain at instant $t$, and $\sigma_c$ is the constant stress; $J_0 = 1/E_0$ stands for the instantaneous compliance at $t$=0; $J_i = 1/E_i$ (i=1,2) means the retarded compliances; $\lambda_i = \eta_i \times J_i$ (i=1,2) represents the retardation times, and $\eta_i$ (i=1,2) is the coefficients of viscosity associated with the Voigt elements; $\eta_0$ denotes the coefficient of viscosity associated with Newtonian flow, and its inverse is the steady-state fluidity of the material. The parameters are optimally chosen by an exhaustive algorithm which results in the minimum errors between the fitting curve and the observed data.

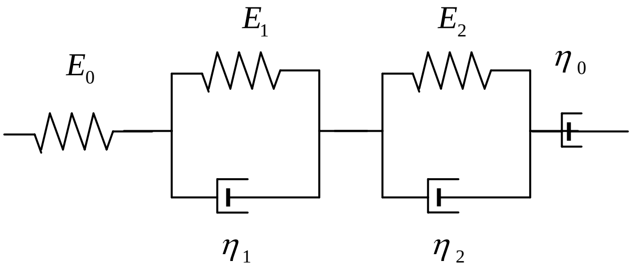


Figure 2**.** Generalized Kelvin model describing creep behaviours of

## *2.4. Fractional Maxwell modelling of creep*

The configuration of a fractional Maxwell model is shown in Fig.3 and consists of a spring and an Abel dashpot connected in series, which is characterized by replacing a Newtonian dashpot in the classical model with the fractional derivative Abel dashpot.

$$\sigma_E = E\varepsilon_E \ , \quad \sigma_V = \eta d^\alpha \varepsilon_V / dt^\alpha \quad (0 \le \alpha \le 1), \tag{2}$$

$$\varepsilon = \varepsilon_E + \varepsilon_V \ , \quad \sigma = \sigma_E = \sigma_V \quad , \tag{3}$$

where $\sigma$ and $\varepsilon$ denotes the stress and strain, respectively; $E$ represents elastic coefficients of the spring; $\eta$ is the viscous coefficient of the Abel dashpot; subscript $E$ means the spring, and subscript $V$ devotes the dashpot.

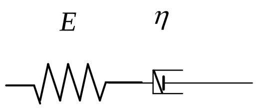


Figure 3**.** Fractional Maxwell

Fractional Maxwell model of constitutive equation is as follows

$$\frac{d^\alpha \sigma}{dt^\alpha} + \frac{1}{\tau}\sigma = E\frac{d^\alpha \varepsilon}{dt^\alpha} \ , \quad \tau = \frac{\eta}{E} \ , \tag{4}$$

where $\tau$ is the retard time. The fractional integral is defined by, see Ref. [9,20],

$$^{R}I^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)}\int_a^t (t-\tau)^{\alpha-1} f(\tau)d\tau (0 < \alpha < 1). \tag{5}$$

Applying the Laplace transformation to Eq. (4) yields

$$s^\alpha \bar{\varepsilon} = \frac{1}{E}s^\alpha \bar{\sigma} + \frac{1}{\eta}\bar{\sigma} \ . \tag{6}$$

Consequently, we have

$$J(s)=\eta^{-1}\sum_{k=0}^{\infty}(-1)^k(\frac{\eta}{E})^k s^{-\alpha k-\alpha-1} \ . \tag{7}$$

Applying the discrete inverse Laplace transformation, we obtain

$$J(t)=\frac{1}{E}+\frac{1}{\eta}\frac{t^{\alpha}}{\Gamma(1+\alpha)} \ , \tag{8}$$

where $\Gamma(g)$ denotes the Gamma function,

$$\Gamma(z)=\int_0^{\infty}e^{-t}t^{z-1}dt \ . \tag{9}$$

In particular, when $\alpha$ is equal to 1, Eq. (8) is reduced to the standard Maxwell model for creep compliance,

$$J(t)=\frac{1}{E}+\frac{1}{\eta}\frac{t}{\Gamma(1)}=\frac{1}{E}+\frac{t}{\eta} \ , \tag{10}$$

namely, Eq.(10) is the standard integer-order model [7].

## 3. Results and discussions

### *3.1. Classical viscoelastic model*

Creep compliance curves of Hami Melon specimens are illustrated in Fig. 4. Deformation of all samples is finite after 60s creep. Figs. 4(a) and 4(b) display the curves of radial and axial specimens, respectively. For the time scale of the experiments, the behaviours were described via the six parameters in the creep model Eq. (1). Table 1 provides the mechanical parameters that define creep behaviour of Hami Melon tissues. According to the interpretation in Mittal [21], $J_0$ represents those bonds of structural units that are stretched elastically when the stress is applied and characterizes instantaneous and complete recoveries when the stress is removed. The linear region of Newtonian compliance $t/\eta_0$ reflects those bonds that are ruptured during the shear creep, and its time required to deform is longer than the creep-recovery period.

As seen from Fig. 4, the viscoelastic creep compliances are significantly different between axial and radial specimens. The axial specimen has larger creep compliance than the radial one.

Table 1 displays the relatively larger standard deviations observed in the creep experiments for measured viscoelastic properties of melon tissues. Much of this variability can be attributed to physiological factors, i.e., anisotropism and non-homogeneity, which change their mechanical properties with age, moisture content, and locations around the melon and depth from which the specimen is taken [21,22].

Table 1 Creep compliance parameters of fresh Hami melon tissues in Eq. (1)

| specimen | $J_0$ (1/Pa)$\times10^{-5}$ | $J_1$ (1/Pa)$\times10^{-5}$ | $J_2$ (1/Pa)$\times10^{-5}$ | $\lambda_1$(s) | $\lambda_2$(s) | $\eta_0$ (Pa$\cdot$s)$\times10^{7}$ |
|---|---|---|---|---|---|---|
| radial | 0.5 | 0.41 | 0.21 | 18 | 3.1 | 5 |
| axial | 0.65 | 0.72 | 0.51 | 18.1 | 5.1 | 7 |

### *3.2. Fractional Maxwell model*

It is worth stressing that the curves displayed in Fig. 4 match quite well with the experimental creep data. This highlights the validity of the present fractional constitutive expression of Hami Melon viscoelasticity, which captures experimental data of creep tests by using merely the three parameters as shown in Table 2. The next experiment will examine if the fractional order constitutive model can depict the nonlinear gradual process of strain in creep.

Table 2. Creep compliance parameters for fresh Hami Melon tissues in Eq.(8)

| Sample | $E$(MPa) | $\eta$(MPa.s) | $\alpha$ |
|---|---|---|---|
| radial | 0.2 | 0.56 | 0.3 |
| axial | 0.14 | 0.49 | 0.38 |

*3.3. Comparisons*

We can see from Fig. 4(a) that compared with the classical viscoelasitc models, the fractional Maxwell model has the same level of accuracy in the fitting of experimental data but requires significantly fewer adjustable parameters. Fig. 4(b) shows that the classical model only fits well the elastic variation, initial small value of *J*, and then has a large departure from experimental data when viscosity comes into play. In stark contrast, the fractional model agrees pretty well with the experimental data for the whole viscoelastic duration with three parameters. In general, our results show that the present fractional derivative model can characterize creep behaviour of Hami Melon with better accuracy and fewer parameters.

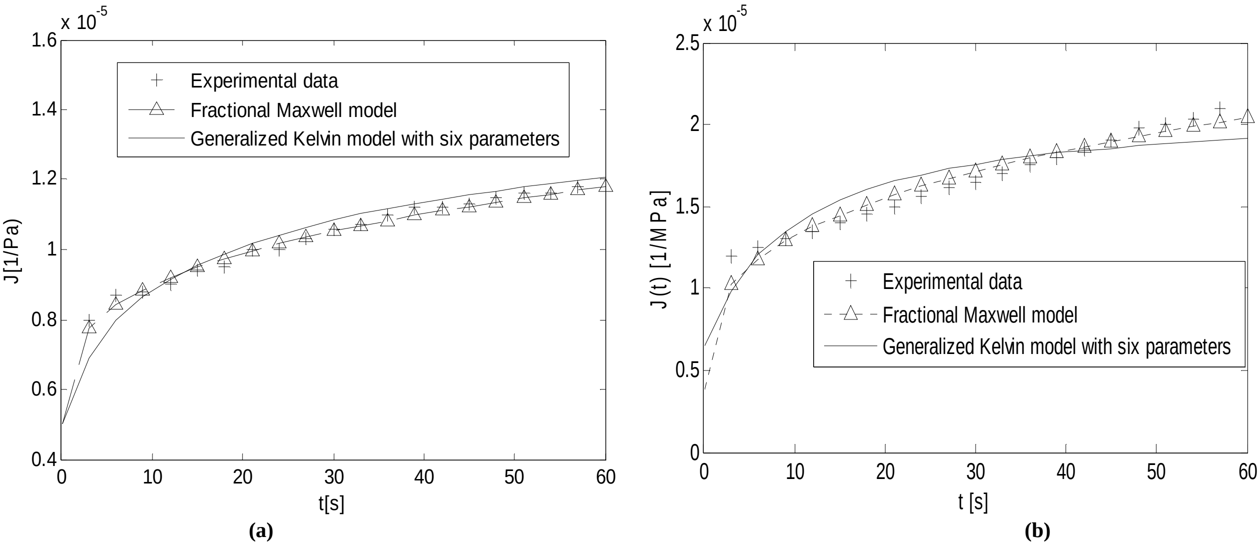


Figure 4. Creep compliance curve; a)radial; b)axial

## 4. Conclusions

This work shows the experimental results of creep tests of Hami Melon. All tests have been conducted in a range of small deformation, so that such creep can be reasonably treated as linear viscoelasticity. The standard linear and the present fractional derivative creep models have been investigated for comparisons in fitting our experimental data. The fractional derivative model not only reduces the computational effort in identification of coefficients, but also appears more promising in modeling of different loading conditions, with fewer parameters.

The creep tests for the specimens in different locations of Hami Melon has illustrated that the creep modulus is significantly influenced by the location and orientation of the specimens. The proposed fractional derivative model can accurately simulate the creep characteristics of different specimens, and the creep damage of Hami Melon in the transport and storage can be predicted. This study is very encouraging and more work is under way to apply the fractional model to damage behaviours of fruits, which will be reported in a subsequent paper.

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