On the Dynamic Behaviour of Surface-Bonded Piezoelectric Sensors/Actuators with Partially Debonded Adhesive Layers

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Abstract The current work presents the analytical study of the effect of the mechanical and geometrical properties of the adhesive layer on the coupled dynamic electromechanical behaviour of thin piezoceramic sensors/actuators bonded to elastic media. A sensor/actuator model with an imperfect adhesive bonding layer is proposed to simulate the two dimensional electromechanical behaviour of the integrated system. The analytical solution of the problem is provided to study the effect of the bonding layer upon the dynamic behaviour of the sensors/actuators under different loading frequencies. The interfacial debonding and its effect on the interlaminar strain and stress transfer mechanisms are discussed in detail. Numerical results obtained show that even for perfectly bonding cases the dynamic coupling between the sensors/actuators and the host structure is significantly affected by the material and geometric properties of the bonding layer. For imperfect bonding layers, debonding and loading frequency interact strongly to each other at certain frequencies, showing a significant coupling effect.

Keywords Piezoelectric, Electromechanical, Debonding, Sensor, Actuator

1. Introduction

Using piezoelectric sensors/actuators in smart structures for position control, vibration control, and damage detection has attracted significant attention from the research community [1-4]. Piezoelectric patches could be permanently bonded to host structures using proper adhesives. The adhesive layer transfers the load between the sensor/actuator elements and the host structure and significantly influences the efficiency of the sensing/actuating process. It is, therefore, an important issue to study the effect of the adhesive layer in such smart structures.

Beam, plate and shell elements have been extensively used [5-7] to study the global electromechanical behaviour of piezoelectric structures. To understand the load transfer through bonding layers in these structures, detailed local deformation near actuators/sensors must be studied. A one-dimensional actuator model has been developed to study the static local stress field near a thin-sheet piezoelectric element attached to an infinite elastic medium [8]. A similar analysis was also conducted based on a more general model which included both interfacial and normal stresses [9, 10]. The development of techniques of generating and collecting diagnostic elastic waves using piezoelectric actuators/sensors has showed the great potential of continuous monitoring of composite structures using distributed piezoelectric actuators/sensors [11]. The study showed very promising results for using integrated piezoelectric actuator/sensor systems in structural health monitoring. A special application of piezoelectric actuators/sensors is in the electromechanical impedance method, which was developed for identifying damage by monitoring the changes in the impedance of the structure [12-15]. Piezoelectric actuators/sensors are also used to generate wave propagation in smart structures for the health monitoring of the structures. To study the high-frequency dynamic behaviour of a piezoelectric thin-sheet actuator, a one-dimensional model was developed and used to evaluate the effect of geometric and loading parameters upon the load transfer between the actuators and the host structure [16, 17]. This actuator model is further modified to study problems with varying electric field distribution along the actuators [18]. Newly developed piezoelectric actuators can sustain a very high electric field without electric breakdown, and, therefore, can generate relatively high strain in the adhesive along the edge of the actuator, and may finally result in debonding [19, 20]. Existing studies on the electromechanical behaviour of piezoelectric actuators with imperfect bonding have been mainly confined to the global response of piezoelectric structures [21-23].

The current work reviews and presents the analytical study of the effect of the mechanical and geometrical properties of adhesive layers on the coupled dynamic electromechanical behaviour of thin piezoceramic sensors/actuators bonded to elastic media. A sensor/actuator model with an imperfect adhesive bonding layer is used to simulate the two dimensional electromechanical behaviour of the integrated system. The analytical solution of the problem is provided to study the effect of the bonding layer upon the dynamic behaviour of the sensors/actuators under different loading frequencies. The interfacial debonding and its effect on the interlaminar strain and stress transfer mechanisms are discussed. Numerical results obtained show that even for perfectly bonding cases the dynamic coupling between the sensors/actuators and the host structure is significantly affected by the material and geometric properties of the bonding layer. For imperfect bonding layers, debonding and loading frequency interact strongly to each other at certain frequencies, showing a significant coupling effect.

2. Formulation of the Problem

For surface-bonded piezoelectric thin sheets with the poling direction along the thickness, as either actuators or sensors, the axial deformation will play a dominant role. Since the thickness of the host medium is usually much larger than that of the piezoelectric elements, the host medium can be modelled as a semi-infinite medium. Accordingly, consider the plane strain problem of a thin piezoceramic sheet bonded to a homogeneous and isotropic elastic half plane through a thin bonding layer, as illustrated in Fig. 1. In the figure a represents the half length of the actuator/sensor, and the thicknesses of the actuator and the bonding layer are assumed to be h and h', respectively. Debonding at the bonding layer forms an interfacial crack, with the half length being d.

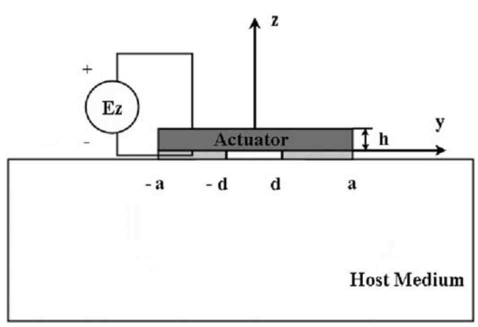

Figure 1. A surface piezoelectric element with debonding

For an actuator, a voltage is applied between the upper and lower electrodes of the piezoelectric element, which results in an electric field $E_z\,exp(-i\omega t)$ along the thickness of the actuator, where Ez is the magnitude of the electric field intensity, and ω is the angular frequency. For steady state problems, all field quantities contain the same temporal factor $\exp(-i\omega t)$, which is hereafter omitted for brevity.

2.1 Modelling of the piezoelectric elements

For a thin-sheet actuator/sensor, the axial stress and strain can be assumed to be uniform across the thickness. The normal stress between the actuator and the host medium is ignored because the upper surface of the thin actuator/sensor is traction free. The piezoelectric element can then be modelled as a one-dimensional element subjected to interfacial shear stress and electric field with the governing equation being

$$\frac{\mathrm{d}\sigma_y^a(y)}{\mathrm{d}y} + \frac{\tau(y)}{h} + \rho^a \omega^2 u_y^a = 0 \tag{1}$$

where $\sigma^a{}_y$, ρ^a, and $u^a{}_y$ are the axial stress, the mass density, and the axial displacement of the actuator, respectively. The axial stress of the actuator is related to the axial strain $\epsilon^a{}_y$ and the electric field E_z by the following general constitutive relation

$$\sigma_y^a(y) = E^a \epsilon_y^a(y) - e^a E_z \tag{2}$$

where E^a and e^a are the effective stiffness and piezoelectric constants [].

Based on this simplified model, axial the strsin and displacement in the piezoelectric element can be determined in terms of the interfacial shear stress (τ) and the applied electric field intensity (E_z) as

$$\begin{aligned}
\epsilon_y^a(y) &= \frac{e^a E_z}{E^a}\frac{\cos k^a y}{\cos k^a a} + \frac{\sin[k^a(a+y)]}{E^a h \sin 2k^a a}\int_{-a}^{a} \cos[k^a(\xi - a)]\tau(\xi)\mathrm{d}\xi \\
&\quad - \frac{1}{E^a h}\int_{-a}^{y} \cos[k^a(y-\xi)]\tau(\xi)\mathrm{d}\xi, \quad |y| < a, \\
u_y^a(y) &= \frac{e^a E_z}{E^a}\frac{\sin k^a y}{k^a \cos k^a a} - \frac{\cos[k^a(a+y)]}{k^a E^a h \sin 2k^a a}\int_{-a}^{a} \cos[k^a(\xi - a)]\tau(\xi)\mathrm{d}\xi \\
&\quad - \frac{1}{k^a E^a h}\int_{-a}^{y} \sin[k^a(y-\xi)]\tau(\xi)\mathrm{d}\xi, \quad |y| < a,
\end{aligned} \tag{3}$$

which represents the deformation of the piezoelectric element caused by the interfacial shear stress and the applied electric field.

2.2 The bonding layer

The bonding layer connects the piezoelectric element and the host medium by transferring shear stress. The shear stress in the bonding layer is given by its constitutive relation as

$$-\tau(y) = \frac{\mu^b - i\omega\gamma^b}{h'}[u^+(y) - u^-(y)] \tag{4}$$

where μ^b and γ^b are the shear modulus and the coefficient of viscosity of the bonding layer, respectively, and $u+$ and $u-$ are the longitudinal displacements of the upper and lower surfaces of the bonding layer, respectively.

2.3 The governing equations for debonded piezoelectric elements

When debonding occurs, the shear stress in the debonded part of the piezoelectric element will be zero. By solving the governing equations for the piezoelectric element and that for the elastic host medium, and satisfying the continuity conditions at all interfaces, the following governing equation for the entire system with interfacial debonding can be obtained,

$$
\begin{aligned}
\frac{e^a E_z}{E^a}\frac{\cos k^a y}{\cos k^a a} = &-\frac{1}{\pi\mu^h}\int_0^\infty\left[\frac{\beta k^2 s}{(2s^2-k^2)^2-4s^2\alpha\beta}+\frac{\lambda_0}{2}\right]\left\{\int_{-a}^{-d}\tau(\xi)\sin[s(y-\xi)]\mathrm{d}\xi\right.\\
&\left.+\int_d^a\tau(\xi)\sin[s(y-\xi)]\mathrm{d}\xi\right\}\mathrm{d}s+\frac{\lambda_0}{2\pi\mu^h}\left[\int_{-a}^{-d}\frac{\tau(\xi)}{y-\xi}\mathrm{d}\xi+\int_d^a\frac{\tau(\xi)}{y-\xi}\mathrm{d}\xi\right]\\
&-\frac{h'}{\mu^b-i\omega\gamma^b}\frac{\mathrm{d}\tau(y)}{\mathrm{d}y},\\
-\frac{\sin[k^a(y+a)]}{E^a h\sin 2ak^a}&\left\{\int_{-a}^{-d}\cos[k^a(\xi-a)]\tau(\xi)\mathrm{d}\xi+\int_d^a\cos[k^a(\xi-a)]\tau(\xi)\mathrm{d}\xi\right\}\\
+\frac{1}{E^a h}&\left\{\int_{-a}^{-d}\cos[k^a(y-\xi)]\tau(\xi)\mathrm{d}\xi+\int_d^y\cos[k^a(y-\xi)]\tau(\xi)\mathrm{d}\xi\right\},\quad d<y<a,
\end{aligned}
\tag{5}
$$

which represents the continuity condition across the adhesive layer. In addition, in the debonded part of the adhesive layer following continuity condition should be satisfied,

$$
\begin{aligned}
&-\frac{1}{\pi\mu^h}\int_0^\infty\left[\frac{\beta k^2 s}{(2s^2-k^2)^2-4s^2\alpha\beta}+\frac{\lambda_0}{2}\right]\int_{-d}^{d}\left\{\int_{-a}^{-d}\tau(\xi)\sin[s(y-\xi)]\mathrm{d}\xi\right.\\
&\left.+\int_d^a\tau(\xi)\sin[s(y-\xi)]\mathrm{d}\xi\right\}\mathrm{d}y\mathrm{d}s+\frac{\lambda_0}{2\pi\mu^h}\int_{-d}^{d}\left[\int_{-a}^{-d}\frac{\tau(\xi)}{y-\xi}\mathrm{d}\xi+\int_d^a\frac{\tau(\xi)}{y-\xi}\mathrm{d}\xi\right]\mathrm{d}y\\
&+\frac{h'}{\mu^b-i\omega\gamma^b}[\tau(-d)-\tau(d)]=\frac{2(\sigma_d+e^aE_z)\tan k^a d}{E^a k^a}.
\end{aligned}
\tag{6}
$$

from which the interfacial shear stress can be determined. In this equation, $\lambda_0=2(1-\nu^h)$, and

$$
\alpha=\begin{cases}\sqrt{s^2-K^2} & |s|>K,\\ -i\sqrt{K^2-s^2} & |s|<K,\end{cases}\qquad \beta=\begin{cases}\sqrt{s^2-k^2} & |s|>k,\\ -i\sqrt{k^2-s^2} & |s|<k,\end{cases}
$$

which are determined from the elastodynamic solution of the host medium. $K=\omega/c_L$, $k=\omega/c_T$ are two wave numbers with c_L and c_T being the longitudinal and transverse shear wave speed of the host medium.

2.4 The singular behaviour of the debonded piezoelectric elements

Interficial debonding forms interfacial cracks. In general, oscillating singularity will occur at the tips of the debonding. For the current piezoelectric element with a partially debonded layer, however, the well-known oscillating singularity disappears based on the model used and the corresponding governing equations, because the oscillating singularity occurs only at perfectly bonded interfaces.

3. Results and Discussion

The existence of adhesive layers could significantly affect the behaviour of the piezoelectric actuator/sensor [24-27]. The response of the piezoelectric element to an applied electric field (actuator) or an incident wave (sensor) has been studied under different geometric and loading conditions to evaluate the effect of the bonding layer upon the load transfer between the actuator/sensor and the host medium. In the following discussion, only the magnitude of the physical stress is presented. The numerical calculation is conducted by solving the governing equations using Chebyshev polynomials expansion of the shear stress τ. The convergence of the solution has been carefully evaluated.

The piezoelectric material considered is piezoceramics with its material property being given in Table 1 [28]. The properties of the bonding layer [3] and the host medium [29] are given in Table 2. The mass densities of the actuator and the host medium are assumed to be 2,700 kg/m^3 [28].

Table 1. Piezoelectric properties

Elastic	c_{11}	c_{12}	c_{13}	c_{33}	c_{44}
(Pa)	13.9×10^{10}	6.78×10^{10}	7.43×10^{10}	11.5×10^{10}	2.56×10^{10}
Piezoelectric	e_{31}	e_{33}	e_{15}		
(C/m^2)	-5.2	15.1	12.7		
Dielectric	λ_{11}	λ_{33}			
(C/Vm)	6.45×10^{-9}	5.62×10^{-9}			

Table 2. Properties of the host and the bonding layer

Host medium	
Young's modulus E^h (Pa)	2.74×10^{10}
Poisson's ratio ν^h	0.3
Bonding layer	
Shear modulus μ^b (Pa)	1.0×10^9

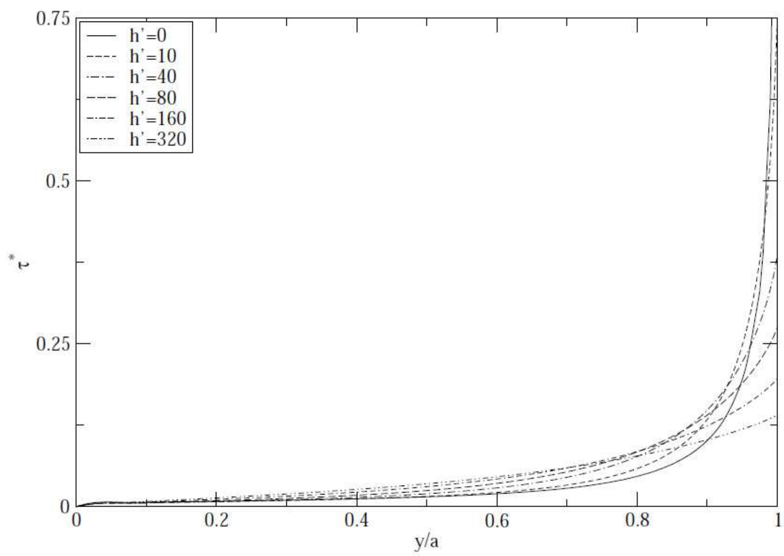

Figure 2 Shear stress distribution along the adhesive layer of an actuator

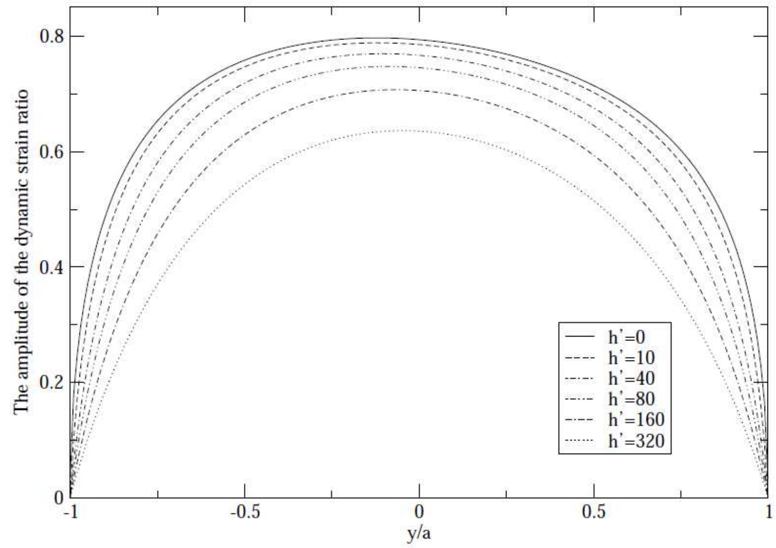

Figure 3 Axial strain distribution along a sensor

One major effect of the adhesive layer is that it reduces stress concentration at the end of an actuator. Figure 2 shows the shear stress distribution ($\tau = qE^aE_z$, $q=\pi E^h/(1-v^{h2})/E^a$) along an actuator with $a=1cm$ and $h=0.5mm$, for the case where the loading frequency is $ka=1.0$ and $\gamma_b = 0.0$ for different thicknesses of the bonding layer. Evaluating the six curves in this figure shows that, as the h' value increases from 0 to $320\mu m$, the stress concentration at the tip of the actuator, $y/a = 1.0$, decreases significantly. The adhesive layer also has a significant effect on the response of the piezoelectric element as a sensor. Figures 3 shows the amplitude of the normalized strain along the sensor caused by a normal incident wave for the case where $ka=1.0$ for different bonding layer thickness 0, 10, 40, 80, 160 and 320 μm. Significant decrease of the strain with increasing bonding layer thickness is observed.

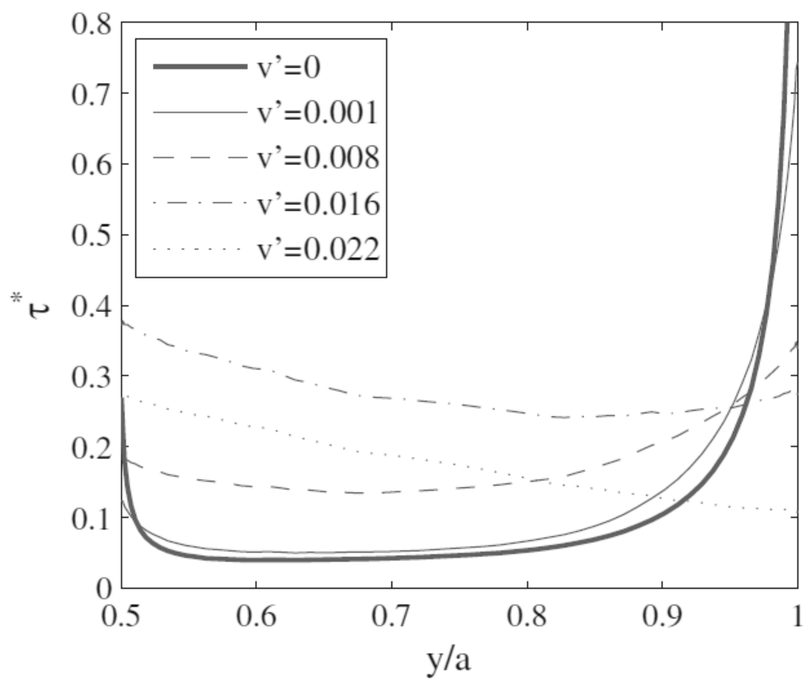

Figure 4 Shear stress distribution along a partially debonded adhesive layer of an actuator

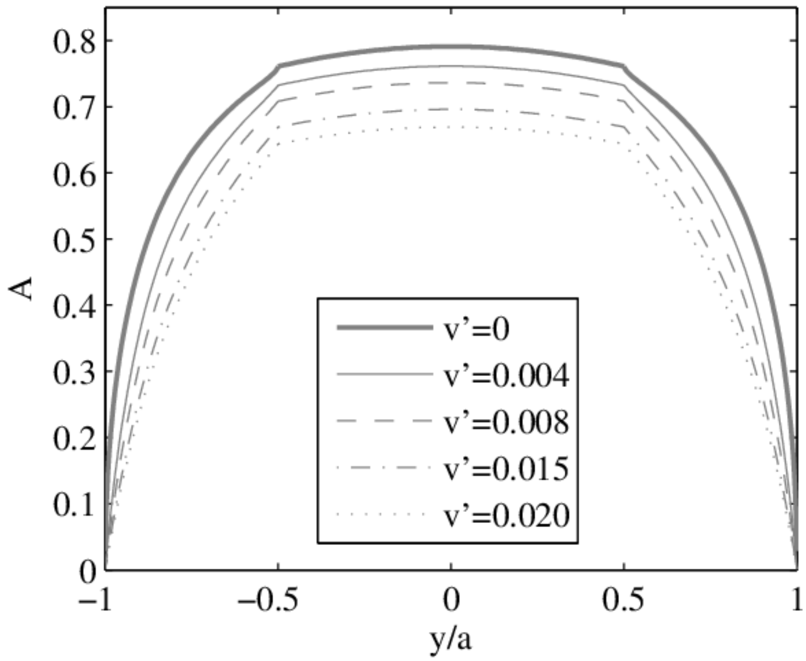

Figure 5 Axial strain distribution along a partially debonded sensor

When debonding occurs at the adhesive layer, the shear stress in the debonded region will disappear and the shear stress will be redistributed in the bonded region. Figure 4 shows the shear stress distribution in the bonded region of the adhesive layer for an actuator under the same geometric and loading conditions as that discussed in Figure 2, for the case where the debonding region is $-0.5a<y<0.5a$, for different thicknesses of the bonding layer ($v'=h'/a$). For very small layer thickness the shear stress shows square-root singularity at both the end of the actuator and the tip of the debonding. The existence of the adhesive layer will remove this singularity and generate stress concentration, which reduces with the increase of the layer thickness. Figure 5 shows the corresponding normalized axial strain distribution of a sensor in response to a normal incident wave. Significant decrease of the strain with increasing bonding layer thickness is observed. The adhesive layer also has a significant effect on the response of the piezoelectric element as a sensor. Figures 5 shows the amplitude of the normalized strain along the sensor caused by a normal incident wave for the case where $ka = 1.0$ for different bonding layer thickness $0, 10, 40, 80, 160$ and $320\ \mu m$. It is observed that debonding ($-0.5a<y<0.5a$) significant decreases the strain level in the sensor bot only in the debonded region but also in the bonded parts of the sensor.

Acknowledgements

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