

# Physical mechanisms of magnetostriction or electrostriction induced interfacial fracture in a bi-layered multiferroic smart composite

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**Abstract** The physical mechanisms of interfacial fracture in a multiferroic bimaterial under magnetostriction or electrostriction have been investigated by the methods of distributed dislocations and Green's function. The numerical results of the stress intensity factor are discussed and the physical mechanisms are then explained.

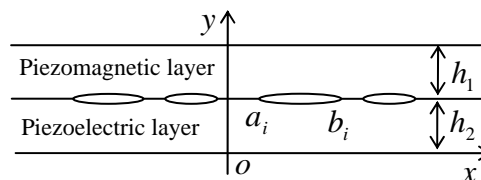
**Keywords** Magnetostriction, Electrostriction, Piezomagnetic stiffening, Piezoelectric stiffening, Interfacial fracture

## 1. Introduction

In layered multiferroic composites composed of alternate piezoelectric and piezomagnetic layers [1], interfaces are key regions to realize the magneto-electric coupling performance [2]. However, when these composites are in service, their interfaces might be damaged by magnetostriction or electrostriction. Therefore, it is significant to investigate the interfacial fracture behavior of layered multiferroic composites. In preceding work [2-3], we analyzed the idealized problems of a single interfacial crack. For practical composites, multiple cracks may simultaneously exist on their interfaces, which would affect the magneto-electric coupling behavior more seriously and also give more difficulties to fracture analysis. The present paper continues to study the more general problem of multiple interfacial cracks in a bi-layered multiferroic composite, and try to give theoretical explanation on the underlying physical mechanisms of magnetostrictive or electrostrictive interfacial fracture.

## 2. Problem formulatiuon and fracture analysis

Consider a bi-layered multiferroic composite with multiple interfacial cracks shown in Fig. 1. The two layers are poled along the  $z$  direction and isotropic in the  $xoy$  plane. Assume that it is loaded by in-plane magnetic field  $H_0$  or electric field  $E_0$  normal to the interface and surfaces, and the upper and lower surfaces are constrained mechanically. Then, only the anti-plane deformation is coupled with the in-plane magnetic/electric field, and the basic equations reduce to



**Fig. 1** Interfacial cracks in a multiferroic composite

$$\left. \begin{aligned} \tau_{kz}^{(1)} &= c_{44}^{(1)} \frac{\partial w_1}{\partial k} + h_{15} \frac{\partial \varphi}{\partial k}; & B_k &= h_{15} \frac{\partial w_1}{\partial k} - \mu_{11} \frac{\partial \varphi}{\partial k} \\ \tau_{kz}^{(2)} &= c_{44}^{(2)} \frac{\partial w_2}{\partial k} + e_{15} \frac{\partial \phi}{\partial k}; & D_k &= e_{15} \frac{\partial w_2}{\partial k} - \varepsilon_{11} \frac{\partial \phi}{\partial k} \end{aligned} \right\}, \quad k = x, y \quad (1)$$

The governing equations are [2]

$$\nabla^2 w_j = 0, \quad (j = 1, 2), \quad \nabla^2 \varphi = 0, \quad \nabla^2 \phi = 0. \quad (2)$$

For the magnetostriction (MS) case, the boundary and continuity conditions are

$$\tau_{yz}^{(1)}(x, h_1 + h_2) = \tau_{yz}^{(2)}(x, 0) = \tau_0, \quad \tau_{yz}^{(1)}(x, h_2) = \tau_{yz}^{(2)}(x, h_2) \quad (3)$$

$$B_y(x, h_1 + h_2) = B_y(x, h_2) = H_0, \quad D_y(x, h_2) = D_y(x, 0) = 0 \quad (4)$$

$$w_1(x, h_2) = w_2(x, h_2), \quad x \notin (a_j, b_j), \quad j = 1, 2, \Lambda, n \quad (5)$$

$$\tau_{yz}(x, h_2) = 0, \quad x \in (a_j, b_j), \quad j = 1, 2, \Lambda, n \quad (6)$$

where  $\tau_0$  is the mechanical constraining traction that can be determined by magnetoelastic analysis [2]. For the electrostriction (ES) case, Eq. (4) should be replaced by

$$B_y(x, h_1 + h_2) = B_y(x, h_2) = 0, \quad D_y(x, h_2) = D_y(x, 0) = D_0 \quad (7)$$

The methods of distributed interfacial dislocations, Green's function and Cauchy singular integral equation are employed to perform the fracture analysis [4]. For simplicity, the details of the theoretical derivation and numerical computation are omitted here.

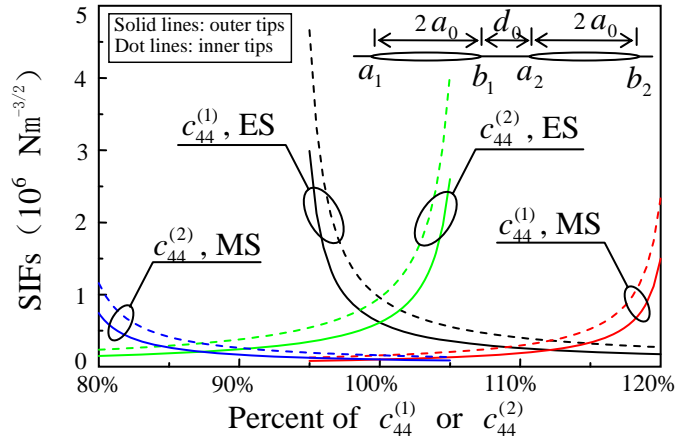
### 3. Numerical results and conclusion

Assume that the piezomagnetic and piezoelectric layers are CoFe<sub>2</sub>O<sub>4</sub> and BaTiO<sub>3</sub>, respectively, and their material constants are perturbed to reveal their effects on the stress intensity factor (SIF), respectively. Based on the numerical results of SIF shown in Fig. 2, the magnetostrictive or electrostrictive interfacial fracture behavior is explained through the following physical mechanisms of “initiative/passive deformation”, “magneto/electro-mechanical coupling” and “piezomagnetic/piezoelectric stiffening”.

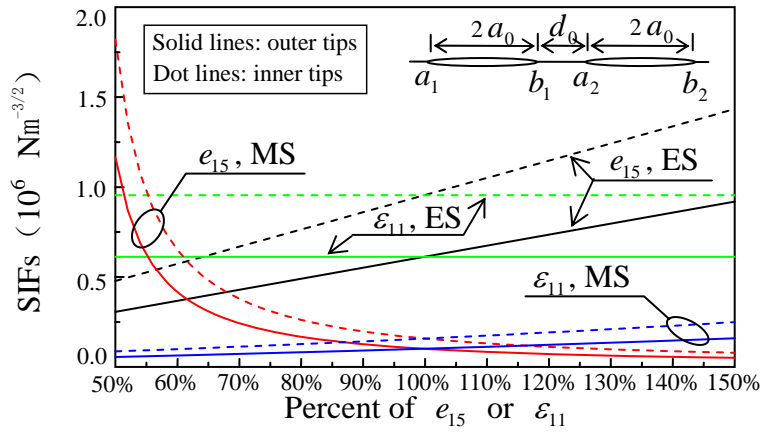
(a) The shear modulus (i.e. material stiffness) affects the SIF via the mechanism of initiative and passive deformation.

(b) The piezoelectric and dielectric coefficients affect the SIF in the ES case via the mechanism of electromechanical coupling, but they affect the SIF in the MS case through the mechanism of piezoelectric stiffening.

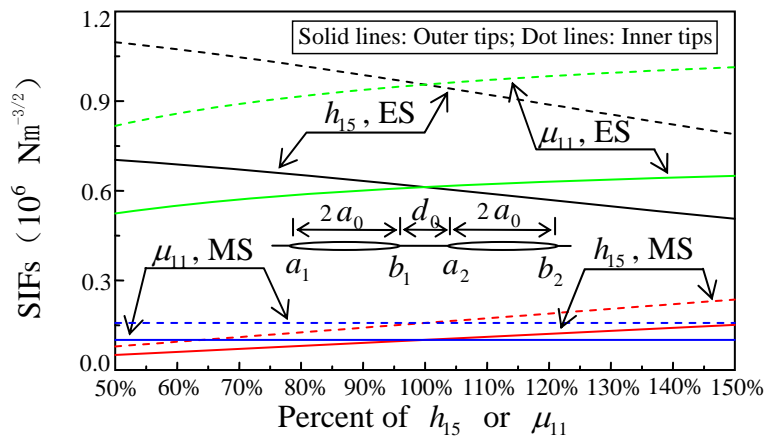
(c) The piezomagnetic coefficient and magnetic permeability affect the SIF in the MS case through the mechanism of magnetomechanical coupling, but they take effect in the ES case through the mechanism of piezomagnetic stiffening.



(a) Perturbation of shear modulus



(b) Perturbation of piezoelectric coefficient and dielectric coefficient



(c) Perturbation of piezomagnetic coefficient or magnetic permeability

**Fig. 2** Effects of material parameters on the SIF

$$(h_1 = h_2 = 10\text{mm}; d_0 = 0.1\text{mm}; a_0 = 1\text{mm})$$

### References

- [1] W. Eerenstein, N.D. Mathur, J.F. Scott, Multiferroic and magnetoelectric materials. *Nature* 2006, 442(7104): 759–765.
- [2] Y.D. Li, K.Y. Lee, Effects of magneto-electric loadings and piezomagnetic/piezoelectric stiffening on multiferroic interface fracture. *Engineering Fracture Mechanics* 2010, 77(5): 856–866.
- [3] Y.D. Li, K.Y. Lee, Magnetostrictive fracture of a cylindrical multiferroic composite. *International Journal of Engineering Science* 2010, 48(2): 199–208.
- [4] Y.D. Li, F.X. Feng, H. Zhao, Multiple interfacial cracks in a bi-layered multiferroic composite under magnetostriction or electrostriction. *Engineering Fracture Mechanics* 2012, 90(1): 65–75.