Revisit on the fractal dimension in damage and fracture

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Abstract The catastrophic failure of materials is a complicated process, involving the nucleation, growth and coalescence of numerous cracks with a wide range of time and length scales. In this paper, the variation of fractal dimension and entropy during a damage evolution process, especially in the vicinity of a critical failure, is revisited. The results show that, as damage evolves, both the fractal dimension and entropy of the spatial distribution of microcracks decrease. A sudden drop of fractal dimension or entropy can be viewed as a likely precursor of fracture and its implications for the prediction of natural disasters such as mining-induced rock bursts and earthquakes are discussed.

Keywords Damage evolution, Fracture, Fractal dimension, Entropy

1. Introduction

The catastrophic failure of materials is a complicated natural process, which is usually induced by the nucleation, growth and coalescence of numerous cracks and/or voids with a wide range of time and length scales from micro- to macro-levels [1,2]. Although great efforts have been made on the study of the damage evolution and fracture process, it is still a big challenge of how to identify a physical parameter that can be used as a potential precursor of an impending failure. Fractal, known as the geometry of nature, has been widely applied to describe a variety of irregular, rough and fragmented structures that bear a special scaling relationship [3−9]. The study on fractal fracture has been attracting much interest from scientists in materials science and solid mechanics, and about half a hundred papers have been published each year over the last decade [10].

Experimental and simulation results have indicated that the fractal character of fracture surfaces of materials, either natural or artificial, is ubiquitous [4−10]. This finding provides both a novel method for fractography and a useful theory for building a linkage between the micro and macro-mechanics. The concept of fractal can provide insights into understanding several important issues in a damage evolution and fracture process, such as the fracture precursor and the influence of disorder on macroscopic mechanical properties.

2. Fractal dimension and entropy

Fractal refers to a geometric object or a natural phenomenon, possessing the properties of self-similarity, which can be described by a power law with the exponent defined as fractal dimension [3,5]. Let us assume that a damaged solid is divided into a number of small elements and the number of damaged elements n yields

$$n = r^{-d_f}, \qquad (1)$$

where r is a dimensionless scale (e.g., the ratio of the characteristic size of microstructures and the length of a specimen) and d_f is the fractal dimension.

In continuum damage mechanics, a simple damage variable can be defined as, $D = n/N$, where N is the total number of (damaged and undamaged) elements, and $N = r^{-d}$ (d is the Euclidean dimension and here, d = 3). If the distribution of microcracks in a material is fractal, it is necessary to introduce fractal dimension in the definition of a damage variable [11]. According to Eq. (1), the damage variable D can be expressed as

$$D = D_0 r^{d-d_f}, \tag{2}$$

where D_0 is the traditional damage parameter in the case of $d_f = d$. It is obvious that, therefore, $D_0 = 0$ and $D_0 = 1$ indicate the undamaged and fracture states, respectively.

Another simple method to measure the degree of disorder is to calculate the entropy of the spatial distribution of microcracks [12]. Entropy quantifies the diversity, uncertainty, or randomness of a system. Here, let us still assume that a solid consists of a total of N elements, and the number of microcracks in each element is evaluated just as in a box-counting method for calculating fractal dimension [3,5]. Then, the entropy S can be calculated by

$$S = -\sum_{i=1}^{N} \frac{n_i}{N} \ln \frac{n_i}{N}, \tag{3}$$

where n_i be the number of microcracks in the i-th element. The normalised entropy can be represented as S/S_0, where S_0 is the equipartition entropy, provided that microcracks are uniformly distributed in a sample [12]. Thus, we have $0 < S/S_0 \le 1$, and S_0 corresponds to the case with the maximum entropy. That is, as disorder of the spatial distribution of microcracks increases, entropy decreases.

3. A likely precursor of fracture

Based on the assumption of strain equivalence, the relationship between stress σ and strain ε in a damaged material can be written as, $\sigma = E(1-D)\varepsilon$, with E being the Young's modulus [13]. In consideration of the fractal distribution of microcracks, the constitutive relationship between stress and strain can be rewritten as

$$\sigma = E(1 - D_0 r^{d-d_f})\varepsilon, \tag{4}$$

where the fractal dimension d_f is a function of time t in a damage evolution and fracture process.

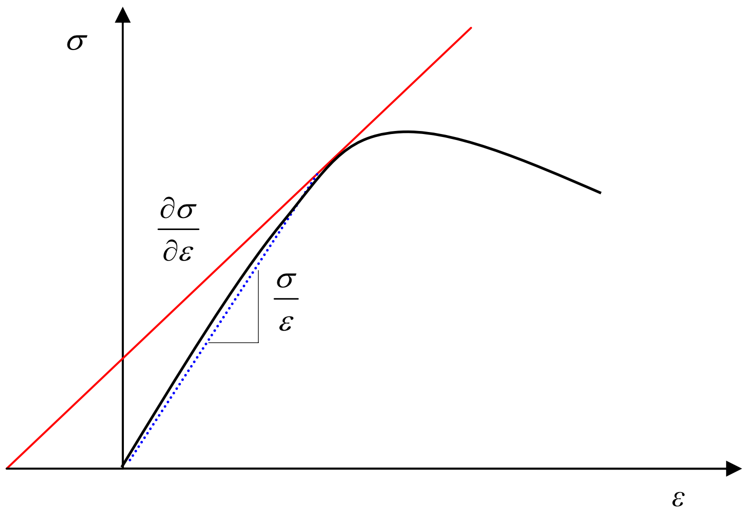

Figure 1. Illustration of a stress-strain curve during a damage evolution and fracture process, where $\partial\sigma/\partial\varepsilon$ and σ/ε are the tangent and secant slopes, respectively (adapted from [11]).

Differentiating both sides of Eq. (4), we have

$$\frac{\partial\sigma}{\partial\varepsilon} - \frac{\sigma}{\varepsilon} = -ED_0 A \frac{\varepsilon}{\dot{\varepsilon}} \frac{\partial d_f}{\partial t}, \tag{5}$$

where $\dot{\varepsilon} = \partial\varepsilon/\partial t$, $\partial\sigma/\partial\varepsilon$ and σ/ε are the slopes of tangent and secant lines of a stress-strain curve, respectively, and $A = r^{d-d_f} \ln r < 0$ due to $0 < r < 1$ [11]. As illustrated in Fig. 1, it is seen that the following condition

$$\frac{\partial d_f}{\partial t} < 0, \tag{6}$$

is valid because of $\partial\sigma / \partial\varepsilon < \sigma/\varepsilon$, which means that, as damage evolves, fractal dimension of the spatial distribution of microcracks decreases.

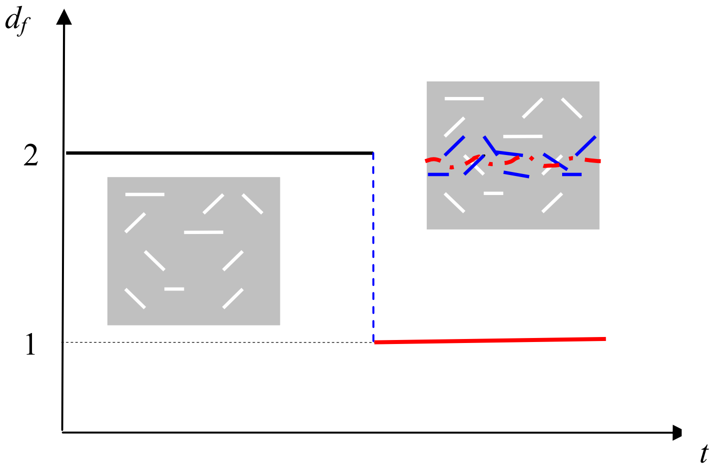

Figure 2. Schematic of fractal dimension versus time during a two-dimensional damage evolution process, where insets are two typical patterns at the initial and final damage stages (adapted from [11]).

As schematically shown in Fig. 2, in the initial damage stage, microcracks are uniformly nucleated in a specimen, but in the final stage and especially near the critical failure, cracks are localized on a surface, which will be the preferential site of fracture. As for the variation of fractal dimension, this corresponds to a roughly one-dimensional fracture profile (or a two-dimensional surface) in a two-dimensional (or three-dimensional) material. As we know, large stress or strain fluctuations in a damage evolution process indicate the divergence to failure, which is usually viewed as a critical phenomenon. As the damage localization approaches a critical point, a sudden drop of fractal dimension occurs.

4. Mining-induced rock bursts: a case study and its implications

As a case study, let us examine the data of a rock-burst-prone pillar in a galena mine, monitored by acoustic emission techniques [14]. The distribution of rock noise locations for 5 days prior to a major rock burst was collected. Based on Eqs. (1) and (3), fractal dimension and entropy of the spatial distribution of daily rock bursts can be calculated [6,11]. Fig. 3 shows the variation of fractal dimension and normalised entropy, and it is seen that both the fractal dimension and entropy decrease as the increase of time. The increase of the clustering degree of microcracks can be intuitively observed from the spatial distribution of rock bursts [14]. Prior to the major rock burst on 24 May, a sudden drop of fractal dimension or entropy occurs. This is in good agreement with our theoretical analysis. Thus, the reduction and sudden-drop of fractal dimension or entropy can be considered as a precursor of a catastrophic failure. However, the prediction of an exact occurrence time is still very difficult since there are small fluctuations of fractal dimension or entropy with time caused by floor de-stressing, and even there is a quiescent period before the main rock burst [11].

An earthquake is rock failure at a large scale. The well-know frequency-magnitude relationship (or the Gutenberg-Richter law) is equivalent to a fractal (power-law) distribution between the number of earthquakes and the characteristic size of faults, and the value of fractal dimension of regional or world-wide seismicity is twice the famous *b*-value in seismology, that is, $d_f = 2b$ [15−17]. Therefore, a sudden drop of fractal dimension corresponds to the reduction of *b*-value, which is also considered as a possible precursor of earthquakes [18].

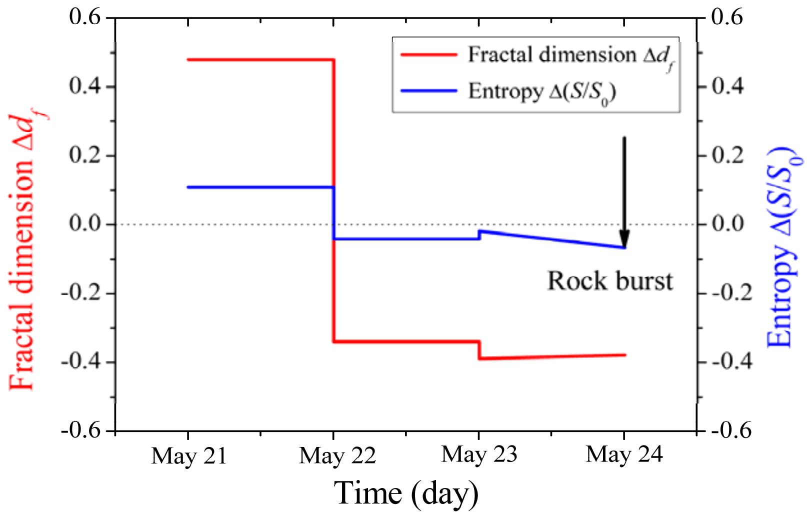

Figure 3. The variation of fractal dimension and normalized entropy versus time in a galena mine, where a major rock burst occurred on 24 May 1979 (data from [14]).

It is worth noting that, however, as the *b*-value or fractal dimension is a highly concentrated parameter, more physical details on the reduction of fractal dimension or *b*-value, especially at critical failure, are needed for reliable prediction. This can be resorted to the variation of high-order or multifractal dimensions.

5. Conclusions

In summary, the definition of a damage parameter can be extended to consider the fractal character of the spatial distribution of microcracks. It is shown that there is a universal character in a damage evolution and fracture process: the decrease of fractal dimension or entropy and the ordering of the spatial distribution of microcracks. A sudden drop of fractal dimension or entropy provides a quantitative indicator of the damage localization, which can be viewed as a likely precursor prior to catastrophic failure. Similarly, the decrease of *b*-value in seismology can be considered as a precursor for a large earthquake. These results imply that the strong fluctuations of physical parameters near the critical point such as the concentration of microcracks (mining-induced rock bursts or seismicity) could provide useful clues for the prediction of fracture and earthquakes.

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