Fracture of rubbers under biaxial loading: A criterion based upon the intrinsic defect concept

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Abstract Since the use of rubbers has been widespread in many industrial applications these last decades, the structures integrity including rubber parts requires the mechanical capabilities of such materials to be known and mastered. To prevent failure in the designing process, it is necessary to provide strong criteria taking into account not only their highly extensible capability but also the complex multiaxial loadings to which they could be subjected. In this work, the intrinsic defect concept is introduced and coupled in the fracture mechanics framework with the *J* integral in order to derive a multiaxial fracture criterion. Mechanical tests up to failure on rubber specimens subjected to monotonic biaxial loading paths were achieved on two materials. The fracture criterion requires as an input the critical value of the *J* integral which was also experimentally measured on CCT specimen. A generalized expression of the *J* integral under biaxial loading is proposed on the basis of finite element calculations on a representative volume element containing a small circular defect. The estimated failure elongations were found in very nice agreement with experimental data on the two kinds of rubber materials. Moreover, we have also outlined the predicting capability of this approach when applied to thermoplastic elastomers.

Keywords Rubber, *J* integral, fracture criterion, intrinsic defect.

1. Introduction

The fracture mechanics approach [1, 2], and its extension to rubbers [3-11], has shown its capability when dealing with crack problems. Nevertheless, if the material contains no visible crack or defect, this approach can be adapted assuming that all materials generally contain defects. These defects originated from material process can grow when subjected to mechanical loading and provoke failure of the component. When dealing with rubbers, these defects may be due to the reinforcing particles added to the neat matrix. Even the filler size is generally very small (less than 100 nm) but it can form aggregates, the size of which can have strong effects on the strength [12, 13] or the fatigue life properties [14-16]. In this work, an attempt to predict the fracture of specimen containing no cracks, but assumed to contain intrinsic flaws, by using the fracture mechanics approach is explored. The main assumption which is made is that these intrinsic flaws, as classical cracks, act as potential stress concentrators which are responsible of the fracture. This analysis uses experimental data obtained from mechanical tests under different biaxial loading paths [17, 18].

2. Experimental study

2.1. Materials and experiments

Two materials are used in this study: a Natural Rubber (NR) and a Styrene Butadiene rubber (SBR).

Different loading paths are used: uniaxial tension, pure shear, equal biaxial tension and biaxial tension with three biaxiality ratios. The biaxial loading was obtained by inflating a membrane (elliptical for biaxial loading and circular for equal biaxial loading) up to failure [17, 18]. For each test, the stretches λ_i (defined as the ratio of the actual length over the initial length in the *i* principal direction) in the two loading directions are measured up to complete failure giving the critical values. In the thickness direction λ_3 is estimated by using the incompressibility assumption:

$$\det \mathbf{F} = \lambda_1 \lambda_2 \lambda_3 = 1, \tag{1}$$

where **F** is the deformation gradient tensor. The critical *J* integral J_c was measured on center-cracked tension (CCT) specimens [11] for the two hyperelastic materials. The J_c average values were 21 kJ/m² and 13 kJ/m² for SBR and NR, respectively.

2.2. Constitutive laws

Th above mentioned tests were used to identify the parameters of the constitutive laws [17, 18]. For the NR material, it was found the best fitting of the experimental data is given by the Yeoh strain energy density (SED) function [19] while for the NR material a second order Ogden function was preferred [20]. The two SED functions are given by equations (2) and (3) respectively:

$$W = C_{10}\left(I_1 - 3\right) + C_{20}\left(I_1 - 3\right)^2 + C_{30}\left(I_1 - 3\right)^3, \tag{2}$$

$$W = \frac{\mu_1}{\alpha_1}\left(\lambda_1^{\alpha_1} + \lambda_2^{\alpha_1} + \lambda_3^{\alpha_1} - 3\right) + \frac{\mu_2}{\alpha_2}\left(\lambda_1^{\alpha_2} + \lambda_2^{\alpha_2} + \lambda_3^{\alpha_2} - 3\right), \tag{3}$$

where *W* is the SED, I_1 is the first invariant of the right Cauchy-Green strain tensor, λ_i are the principal stretches and C_{ij}, μ_i and α_i are material constants to be determined using a least square method. Table 1 reminds the values of all the material constants.

Table 1. Material constants for the NR and SBR materials.

	C_{10} (MPa)	C_{20} (MPa)	C_{30} (MPa)	
NR	0.298	0.014	0.00016	
	μ_1 (MPa)	α_1	μ_2 (MPa)	α_2
SBR	0.638	3.03	-0.025	-2.35

2.3. Intrinsic flaw size

To determine the intrinsic defect size, the formulation given by Rivlin and Thomas [3] can be used. For plates containing a crack of length *a* and submitted to tensile loading, these authors expressed the tearing energy *T*, equivalent to the *J* integral, as follows:

$$J = T = 2k\left(\lambda\right)Wa\,, \tag{4}$$

where k is a proportionality factor depending on the stretch ratio, which can be expressed for single edge cracked specimen (SENT) specimens, according to Lindley [21] in the following form:

$$k = \frac{2.95 - 0.08(1-\lambda)}{\sqrt{\lambda}}, \tag{5}$$

For CCT specimens, the expression proposed by Lake [22] is slightly different:

$$k = \frac{\pi}{\sqrt{\lambda}}, \tag{6}$$

Therefore, the intrinsic defect size can be estimated using equation (4). It comes:

$$a_{th} = \frac{J_c}{2k(\lambda_c)W_c}, \tag{7}$$

in which J_c is the critical value of J mentioned earlier in section 2.1, W_c corresponds to the SED at break of a smooth specimen loaded in uniaxial tension which can be computed using equations (2) and (3) and λ_c is the stretch at break under uniaxial tension. Since the intrinsic flaw is supposed embedded in the bulk, the flaw is taken centered in this investigation and equation (6) is used to calculate the k factor. Using equation (7) the size was found equal to 120 μm for NR while for SBR the size is 160 μm.

3. Finite element analyses

3.1. Representative volume element

Assuming a circular defect in order to avoid preferential propagation direction under biaxial loading, a representative volume element (RVE) was introduced to achieve finite element calculations. Since the defect size is estimated from uniaxial tension tests, the size of the square shape RVE was fixed equal to the width of the tensile specimen which is 10 mm as schematically shown in figure 1.

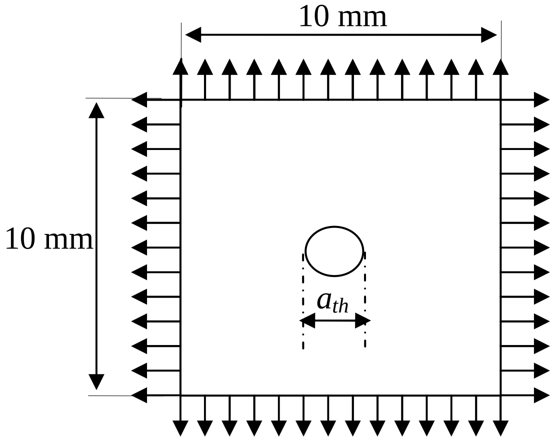

Figure 1. Reference volume element containing a circular defect.

Due to the symmetries of the RVE, only a quarter of the specimen (the shaded part) is modeled in the finite element program. Such a modeling allows getting uniaxial tension, pure shear and different biaxial loadings by varying the ratio in the assigned displacements respectively in the horizontal and vertical directions.

3.2. *J* integral determination

The numerical analysis is performed under plane stress and finite strain conditions using a total Lagrange framework, the constitutive laws being an input given by SED functions previously introduced in equations (2) and (3). The FE program was Marc MS software.
To calculate *J*, the Begley-Landes method [23] based upon the energy interpretation of *J* was used. which allows to express *J* in the following form:

$$J = -\frac{\partial U}{\partial A}, \tag{8}$$

where *U* is the stored elastic energy and *A* is the crack (defect) area. This method requires to estimate the potential energy for varying crack lengths and to plot the energy per unit thickness function of the crack area for a given displacement. The evolution is fitted by straight lines the slope of which gives the *J* value. In this work, for each material, seven defect sizes close to the reference value (i.e. that calculated by using equation (7)) were used to estimate *J*. In order to verify the validity of the Begley-Landes method, preliminary calculations on a RVE containing a sharp crack were achieved. The values obtained were compared to the direct calculation of the *J* integral available on the software. The results, shown in figure 2 for uniaxial tension, highlight the validity of this method, especially for high level of strain.

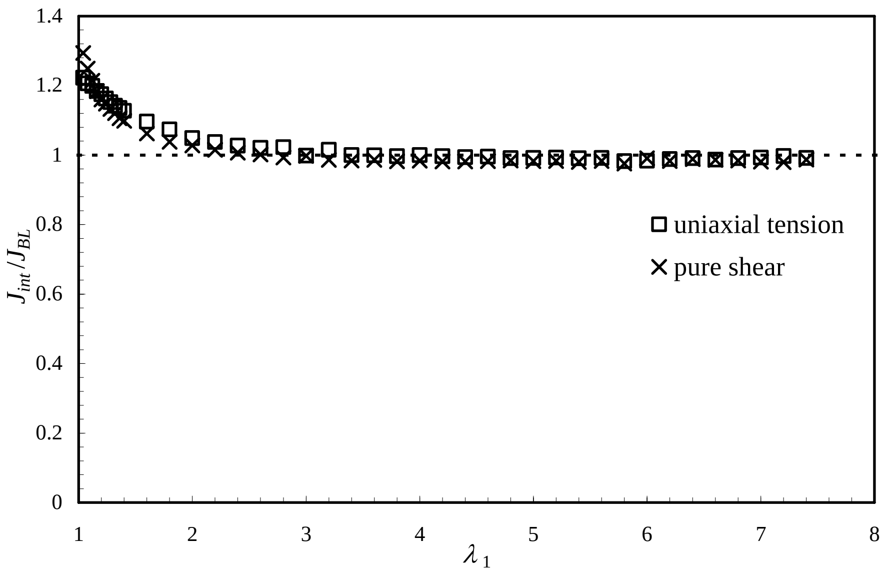

Figure 2. Evolution of the ratio J_{int}/J_{BL} as a function of the principal stretch λ_1 (SBR material).

3.3. *k* factor evolution

Finite element simulations were achieved for uniaxial tension, pure shear and four biaxiality ratios *b* defined as:

$$b = \frac{\lambda_2}{\lambda_1}. \tag{9}$$

The k factor was then computed using equation (4). Figure 3 shows the evolution of this factor as function of the equivalent stretch expressed as follows:

$$\lambda_{eq} = \sqrt{\frac{I_1}{3}}, \tag{10}$$

where I_1 is the first invariant of the right Cauchy-Green strain tensor.
The influence of the loading path is clearly highlighted on this figure. Moreover, we have not found any effect of the kind of material.

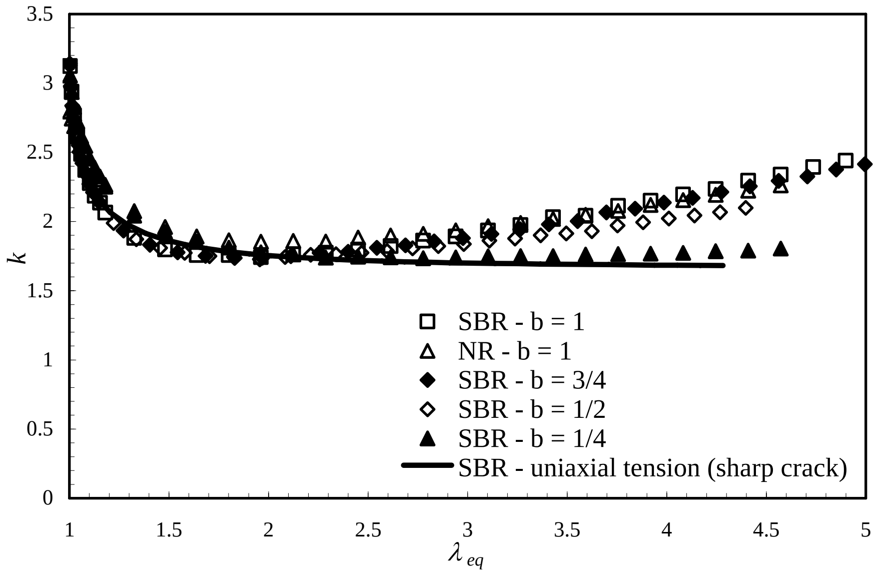

Figure 3. *k* factor as a function of the equivalent stretch λ_{eq} (circular defect, a = 0.06 mm).

4. Results

4.1. A new unified expression of *J*

Since a strong dependence of *k* on the biaxiality ratio is pointed out, the calculation of *J* using equation (4) requires to analytically express the *k* factor for every value of *b*. Another idea is to try to unify these results leading to rewrite *J* in the following form:

$$J = 2f\left(\lambda_{eq}\right)Wa, \tag{11}$$

where *f* is a function of the equivalent stretch which can be expressed in a multiplicative form as follows [24]:

$$f\left(\lambda_{eq}\right) = \left(\lambda_{eq}\right)^{\gamma} g\left(\lambda_{eq}\right), \tag{12}$$

where *g* is another function of the equivalent stretch and γ is an exponent depending on the biaxiality ratio [24] and varying from 1.05 for uniaxial tension to 1.35 for equibiaxial tension. Equation (11) therefore becomes:

$$J = 2\lambda_{eq}^{\gamma} g\left(\lambda_{eq}\right)Wa. \tag{13}$$

In this case, as shown in figure 4, all the data can be shifted to get a master curve and the evolution of g can be written in a polynomial form as follows:

$$g\left(\lambda_{eq}\right)=0.255+\frac{2.837}{\lambda_{eq}^{2}}-\frac{2.888}{\lambda_{eq}^{4}}+\frac{2.507}{\lambda_{eq}^{6}}, \tag{14}$$

As shown on figure 4 all the data (dots) are well fitted by equation (14) represented by the continuous line.

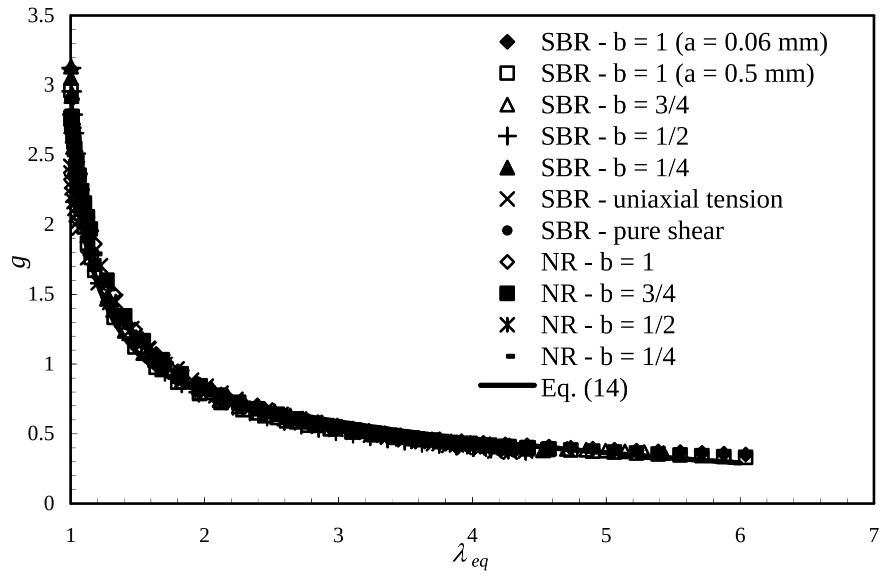

Figure 4. g as a function of the equivalent stretch λ_{eq} (a = 0.06 mm if not reported).

Figure 5 shows the validity of equation (11) to estimate the J integral. Indeed, the values obtained from equation (11) are compared to the data issued from the Begley-Landes method in the case of biaxial loadings.

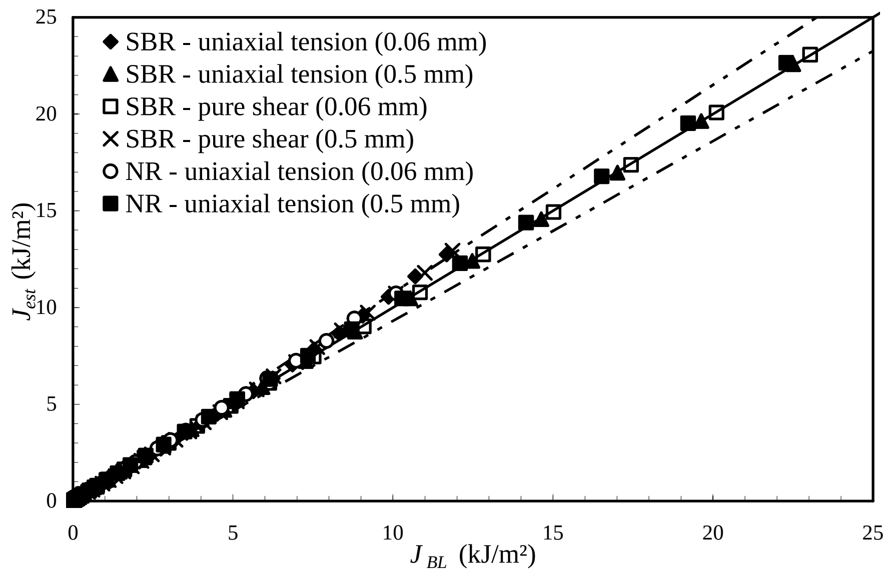

Figure 5. Comparison between J calculated from equation (11) and J_{BL} issued from the Begley-Landes method for biaxial loadings.

The maximum deviation observed is less than 5% (dashed lines). Note, even not reported on the figure, that the accuracy for uniaxial tension and pure shear is improved.

4.2. Fracture criterion

At crack initiation J takes its critical value expressed as follows:

$$J_c = 2\lambda_{eq_c}^{\gamma}\left(0.255+\frac{2.837}{\lambda_{eq}^2}-\frac{2.888}{\lambda_{eq}^4}+\frac{2.507}{\lambda_{eq}^6}\right)W_{0_c}a_c\,, \tag{15}$$

where the subscript c denotes the critical value reached at crack initiation. From equation (15) the critical stretches can be therefore derived for any loading path.

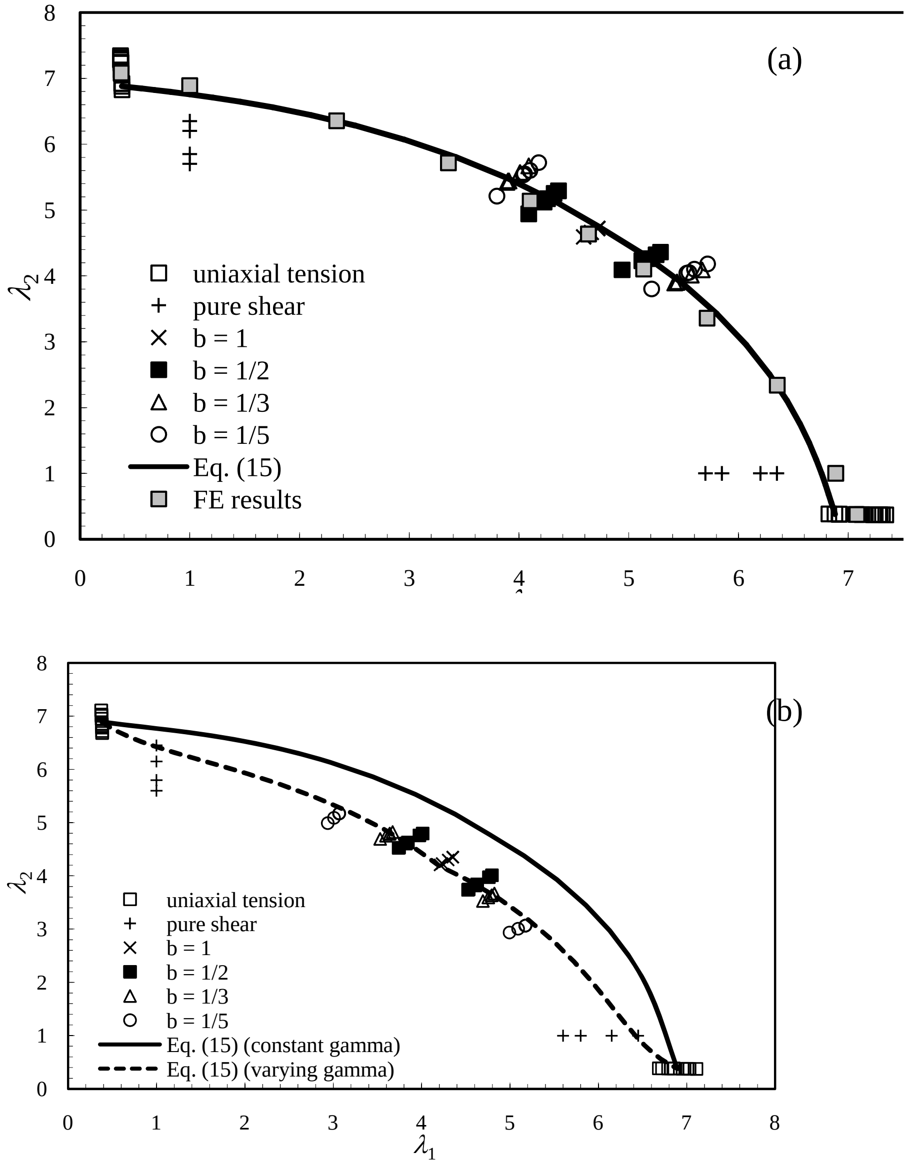

Figure 6. Failure envelopes for (a) NR and (b) SBR.

Figure 6 shows the predictive capability of the proposed approach. The failure envelope in terms of principal stretches is well predicted when using the intrinsic defect concept coupled with fracture mechanics. For NR, the exponent γ can be taken constant while for SBR it is necessary to introduce a slight variation with the biaxiality ratio.

5. Conclusion

A fracture criterion based upon the intrinsic defect concept is an original way to predict the critical stretches at failure when dealing with rubbers. The methodology we have proposed is quite easy to use and only requires two data: the stretch at break under uniaxial tension and the fracture toughness in terms of J_c. The constitutive law of the material is also an input which is necessary. We have also developed a general formulation of J which can be used, whatever the loading path in 2D conditions.

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