# A Damage Model for Sandwich Plate with Viscoelastic Core in three-Points Bending Fatigue Experiments

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**Abstract** In three-points bending fatigue experiments of sandwich plate with visoelastic core and steel facplates, recovering effects of fatigue life are found for its viscoelastic properties of the core. If the fatigue experiment is interrupted and rest for some time before reloading, there will appear a phenomenon that the deflection doesn't reach the former value immediately but after a specific time or cycles. From defining the damage variable *D* as the ratio between the differences of maximum allowable bending deflection and processing deflection in experiments, the fatigue damage equation are established. To considering the recovering effects, a formula which is similar with deflection of isotropic viscoelatic beam under concentrated load in midspan is introduced in damage equation. The experiments result shows that this damage model considering recovering effects can demonstrate the damage very well for sandwich plate with viscoelastic core. And the existing of viscoelastic core can increase the fatigue life of sandwich beam specimen.
**Keywords** Sandwich, Viscoelastic Core, Fatigue, Damage Model

## 1. Introduction

Sandwich plate with viscoelastic core is widely used in civil engineering and shipbuilding fields [1]-[3] for weight reduction, rapid reparation, vibration isolation, noise reduction and resisting impact. It also has better fatigue resistance characteristic than ordinary steel structures. From the fatigue experiment, it is found that the existing of recovering effects by intermittent loading have influences on the fatigue life. Based on damage theory, a fatigue damage model is proposed to elaborate the damage of specimen considering recovering effect. From intermittent test, the recovering effect is affirmed and damage model is verified that it have a certain accuracy.

## 2.Fatigue experiment phenomenon

In the fatigue experiment process of practical engineering, structural fatigue load applied on structure load is often not continuous, but intermittent. For the exiting of viscoelastic core in sandwich plate, it makes the responses of sandwich beam specimen under three-points bending fatigue load different from ordinary sandwich plate's responses. From the three-points bending fatigue experiment process, the following phenomenon are found:

(1) Under pressure-pressure fatigue load to three-points bending specimen, along with increasing of cycle number, vertical deflection is also increased, for specimens with smaller interlayer cohesive force, the first crack appears in one end and gradually propagates as cycles increasing. When the vertical displacement reaches a certain level, there doesn't appear fracture or crushing phenomenon. But there will be propagation of the crack or tearing of core.

(2) When fatigue loading is intermittent, that is, interrupt fatigue load and unload, after a long time (such as a few hours), fatigue loading is applied once again. Then it is found that the vertical

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displacement cannot directly reach the previous loading position under the same load, but after a certain time or cycles before reaching the former position.

In the entire loading-stop-reloading process, overall fatigue life will increase than under continuous fatigue load. As shown in Figure.1, the curve of ‘a’ is the first load curve. When the loading process reaches a certain positions (at‘d’ or ‘e’), stop and unload for a while, then reload. The cure will follow the curve ‘b’ or curve ‘c’ before reaches the intersection point superimposed with the original curve ‘a’, rather than a direct continuation of load curve ‘a’. In this process, the number of cycles corresponding to the portion shown in the dashed line means the additional growth of fatigue life. The following attempts to establish a fatigue damage model to describe this phenomenon.

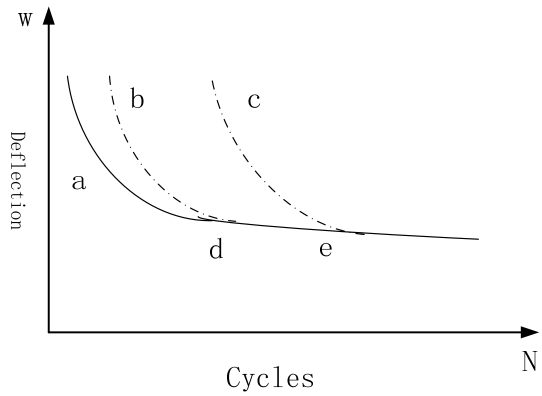



Fig.1 Loading-stop-reloading process

## 3. The fatigue damage model considering intermittent effects

Substantially the first few cycles of all fatigue tests exist nonlinear characteristic. For the metal material, these cycles does not exceed 100 before reach stable stage, but for the polyurethane elastomer sandwich panel, the presence of viscoelastic core make the initial cyclic loading can last a long time. When the load level is low, the number of cycles can even reach thousands of times. For the whole fatigue life of material, it cannot be ignored. Here according to the classical linear fatigue damage theory [], fatigue damage model will be established to consider the intermittent effect.

In a one-dimensional fatigue damage theory, the damage of the material can be represented by the damage variable *D*, it is usually a function of load cycles:

$$D=f\left(\Delta\sigma,\overline{\sigma},N,\cdots\right) \tag{1}$$

In which $\Delta\sigma$ is the stress amplitude, $\overline{\sigma}$ is stress mean, *N* is the cycles. Then the material damage can be defined by the damage variable:

$$\begin{aligned} D&=0, \qquad N=0 \\ D&=1, \qquad N=N_{\max} \end{aligned} \tag{2}$$

In order to describe the influences of the recovering effect, here introduce the damage variable $D_{eff}$, which is defined as the effective damage. When the variable is equal to 1, the material is destructive. It can be obtained by uninterrupted fatigue loading process. It also can be gotten by analysis the curve shown in Fig.1, the solid line corresponds to the effective fatigue life. The dashed parts of the curve can be considered to be invalid damage, but it need to be included in the whole fatigue life.

Suppose it is piecewise loading throughout the cycling loading process, and the loading frequencies are the same. Define the degree of recovery function $N_b\left(t,\Delta\sigma,\overline{\sigma}\right)$, if the stop time is $t_{n-1}$ before the *n-th* loading, after reloading *N* cycles, the damage of reloading can be written as

$$D_{eff,n}=\Delta D_n=\begin{cases} 0 & \mathrm{N}_n \le \mathrm{N}_b \\ f((\mathrm{N}\text{-}\mathrm{N}_b),\Delta\sigma_n,\overline{\sigma_n}) \end{cases} \tag{3}$$

In which $D_{eff,n}$ is the damage of *nth* loading, and $\mathrm{N}_n$ represents the cycles of *n-th* loading. $f((N-N_b),\Delta\sigma_n,\overline{\sigma_n})$ is the function to calculate cumulated damage. Then whole damage of specimen can be written as:

$$D_{eff,n}=\sum_{n=1}^{n}\Delta D_n \tag{4}$$

Whole loading cycles is:

$$N_{total}=\sum_{n=1}^{n}\mathrm{N}_n \tag{5}$$

Cycles corresponding effective damage is

$$N_{eff}=\sum_{m=1}^{m}N_m\text{-}mN_b \tag{6}$$

In which *m* are times of stop-reload. To identify fatigue damage model considering the effect of recovering, most important thing is to define recover degree of function, it depend on theoretical analysis or experiments.

In order to simplify the analysis, the damage function uses linear cumulative damage model, based on the analysis of the Miner [4], damage parameters is related with the number of cycles, and has nothing to do with the load order. The analysis here supposes the damage is only related with the previous loading:

$$D\left(n\right)=\frac{n-n_{if}}{N_f-n_{if}}\quad n\ge n_{if} \tag{7}$$

$n_{if}$ is the threshold of recover degree of function, and *n* is the loading cycles. For constant amplitude fatigue tests, the fatigue damage after load *N* times should be modified as follows:

$$D\left(N\right)=\frac{\sum_{m=1}^{m}N_m\text{-}mN_b}{N_{eff}\text{-}N_b}\quad N\ge N_b \tag{8}$$

## 4. Recover degree function $N_b$

The Recover degree function is generally determined by the experiment, for the sandwich beam with viscoelastic core it can be roughly determined the by theoretical analysis. For no obvious or small plastic deformation case, the loading loop is stable, and the average displacement can be calculated by followings:

$$w_{2n}^{middle}=\frac{1}{2}(w_{(2n)t_1^+}^{n\to\infty}+w_{(2n)t_1^-}^{n\to\infty})=\frac{Q_1+Q_2}{2}\frac{l^3}{48E_fI_f}+\frac{1}{E_0}\frac{l(1+\mu_c)}{2(A_c)}\frac{1}{1+e^{\frac{E_0}{\eta_0}t_1}}(Q_1+e^{\frac{E_0}{\eta_0}t_1}Q_2)$$
$$w_{2n+1}^{middle}=\frac{1}{2}(w_{(2n)t_1^+}^{n\to\infty}+w_{(2n)t_1^-}^{n\to\infty})=\frac{Q_1+Q_2}{2}\frac{l^3}{48E_fI_f}+\frac{1}{E_0}\frac{l(1+\mu_c)}{2(A_c)}\frac{1}{1+e^{\frac{E_0}{\eta_0}t_1}}(Q_2+e^{\frac{E_0}{\eta_0}t_1}Q_1) \tag{9}$$

In which $w_{2n}^{middle}$ and $w_{2n+1}^{middle}$ are the means of displacements when cycles are equal to *2n* and *2n+1* respectively. $E_f$ is modulus of elasticity, $I_f$ is moment on inertia of specimen; *b* is the width; *l* is the gauge length; $A_c$ is section area of core. $E_0$, $\mu_c$ and $\eta_0$ are viscoelastic parameters of core; $Q_1$ and $Q_2$ are maximum pressure load and minimum pressure. When the specimen is reloaded after loading a certain cycles, it is equivalent to load a reverse static effect ($Q_0$) based on former deformation, so the displacement of unload can be obtained by subtracting the two displacements. Here take the mean of deflection of when cycles are equal to *2n*:

$$w_{2n}^{unload}=w_{2n}^{middle}-w(t)$$
$$=(\frac{(Q_1+Q_2)}{2}-Q_0)\frac{Q_0l^3}{48E_fI_f}+\frac{1}{E_0}\frac{l(1+\mu_c)}{2(A_c)}\frac{1}{1+e^{\frac{E_0}{\eta_0}t_1}}(Q_1+e^{\frac{E_0}{\eta_0}t_1}Q_2)-\frac{Q_0l(1+\mu_c)}{2(A_c)}\frac{1}{E_0}(1-e^{-\frac{E_0t}{\eta_0}})$$
$$=(\frac{(Q_1+Q_2)}{2}-Q_0)\frac{Q_0l^3}{48E_fI_f}+\frac{1}{E_0}\frac{l(1+\mu_c)}{2(A_c)}\left[\frac{1}{1+e^{\frac{E_0}{\eta_0}t_1}}(Q_1+e^{\frac{E_0}{\eta_0}t_1}Q_2)-Q_0(1-e^{-\frac{E_0t}{\eta_0}})\right] \tag{10}$$

After stop a certain time $t_0$, reload the specimen for *k* cycles, Specimen displacement $w_{2n}^{reload}$ can be written as:

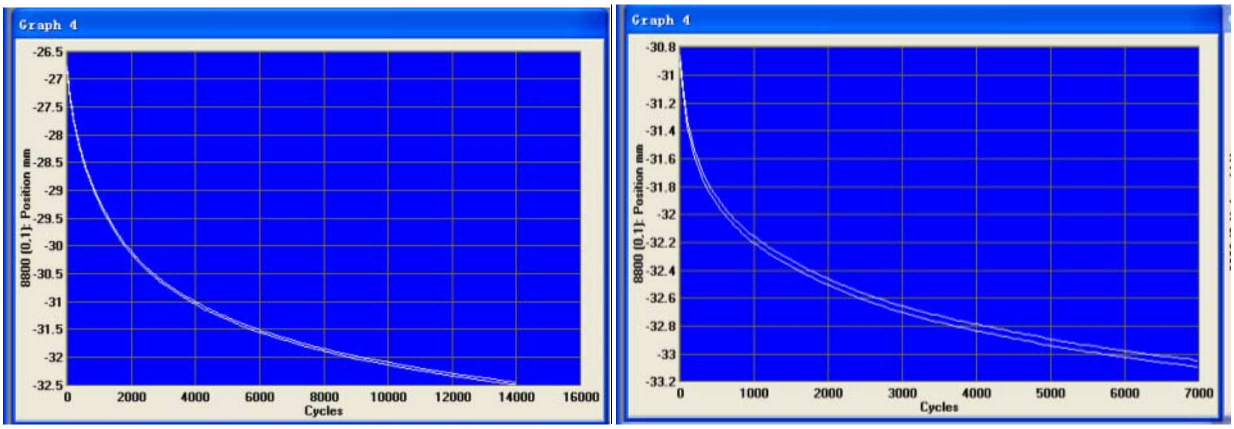
$$w_{2k}^{reload}=w_{(2k)t_1^+}+w_{2n}^{unload}\Big|_{t=t_0}$$
$$=Q_1\frac{l^3}{48E_fI_f}+\frac{1-e^{-\frac{E_0}{\eta_0}2kt_1}}{1+e^{\frac{E_0}{\eta_0}t_1}}\frac{1}{E_0}Q_1\frac{l(1+\mu_c)}{2(A_c)}+\frac{e^{\frac{E_0}{\eta_0}t_1}(1-e^{-\frac{E_0}{\eta_0}(2k+2)t_1})}{1+e^{\frac{E_0}{\eta_0}t_1}}\frac{1}{E_0}Q_2\frac{l(1+\mu_c)}{2(A_c)}$$
$$+(\frac{(Q_1+Q_2)}{2}-Q_0)\frac{Q_0l^3}{48E_fI_f}+\frac{1}{E_0}\frac{l(1+\mu_c)}{2(A_c)}\left[\frac{1}{1+e^{\frac{E_0}{\eta_0}t_1}}(Q_1+e^{\frac{E_0}{\eta_0}t_1}Q_2)-Q_0(1-e^{-\frac{E_0t_0}{\eta_0}})\right] \tag{11}$$

Make $w_{2k}^{reload}=w_{2n}^{middle}$, loading times *k* can be approximated solved, which is the extra fatigue life from the reload. When recovering function is determined, we can calculate the extra fatigue life of intermittent fatigue loadings.

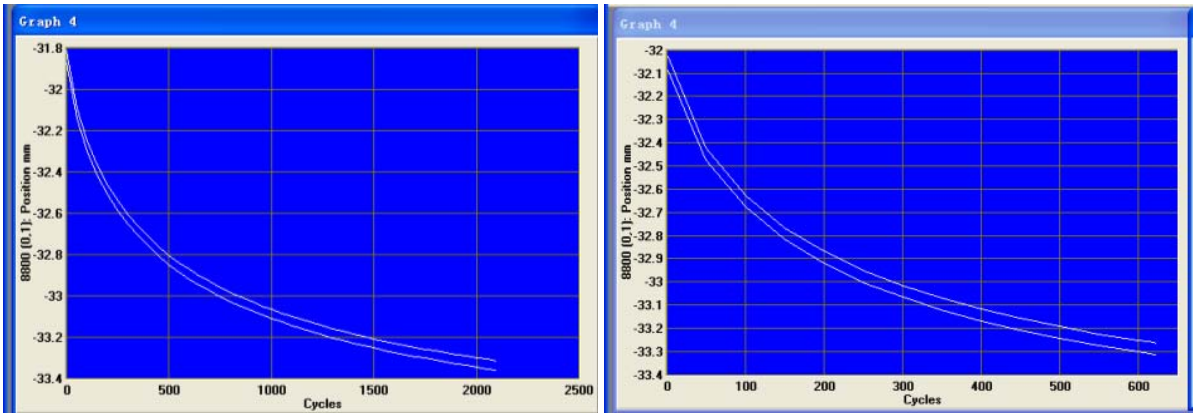
## 5 Intermittent fatigue experiment

In order to verify the proposed fatigue damage model considering intermittent loading, intermittent fatigue experiments are carried out and the theoretical result are also calculated by using of equation (11). Specimens are tested which is shown in Figure. 2 and the loading level is equal to 0.7; ratio of stress is equal to 0.9. The loading frequency is 2Hz, after loading 14000 times, the response of specimen is stable, then unloading and resting for 1 hour, the cycles-deflection curve are shown in fig.2

In the first time loading process, the deflection of specimen is about 10.31mm. Reloading after 1hour, when the deflection reached 10.31mm again, the corresponding cycles is about 2100. If taking deflection as damage value, it means the extra fatigue life from the rest. The results calculate by using of equation(11) are shown in Fig3, the rising curve of the graph presents the data of $w_{2k}^{reload}$ - $w_{2n}^{middle}$ specimen after resting for 1hour. The straight line presents the coordinate value which is equal to 0. From the Fig3a, the rising curve is infinitely close the straight line, which means in ideal state, the reloading curve is approaching to the straight line of steady state. But in actual experimental conditions, the steady line is an oblique line which declines with cycles increase. So we consider it intersected when the gap between two curves is 0.01mm. In this case, the corresponding cycles of Fig.3a is about 1580. Comparing Fig.2 and Fig.3, a certain error exists which is about 24.7%. One reason is viscoelastic model of polyurethane core is simplified to Kelvin model, another reason is theoretical calculation doesn't consider the plastic deformation of specimen in cyclic loads. According to theoretical analysis, if the rest time is long enough, each rest has same recovering fatigue life. But from the experimental result, the recover fatigue lives are different for multiple reloading. From Fig.2c and Fig.2d, the recover life of third reloading is 1300 cycles, and at the ninth reloading, recover life is 650 cycles. This means in multiple reloading processes, there are damages in the specimen, which cause the decrease of recover fatigue life.

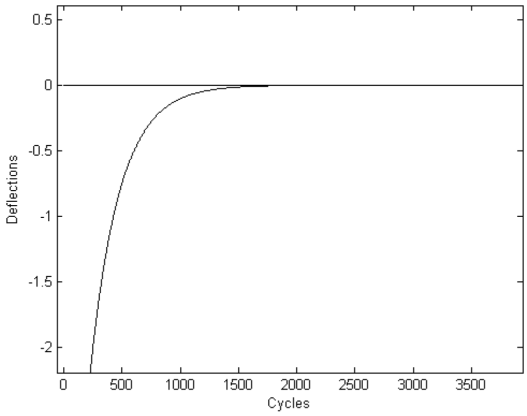


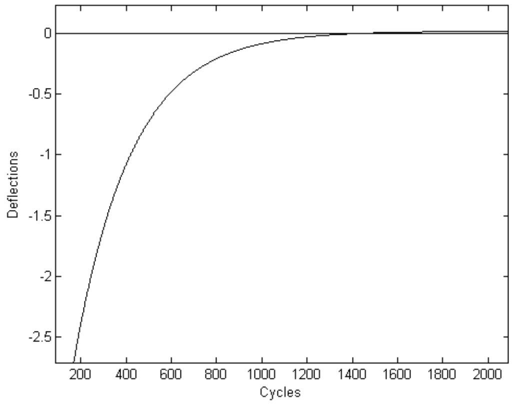

a 1st loading 14000 cycles　　b 2nd loading after resting 1hour




c 3rd loading curve　　d 9th loading curve

Fig.2 Deflection curve for load-rest-reload process




a Rest for 1 hour（60min） b Rest for 6min

Fig.3 Recovering fatigue life curve

Fig.4 is the recover fatigue life calculation results obtained in different rest time by using of formula (11) in ideal state. As can be seen from the graph, the rest time of specimen need is very short. After rest for 750s the specimen can reach a stable state. And the corresponding recover fatigue life is approximately 1580 times, it is same to Fig.2a.

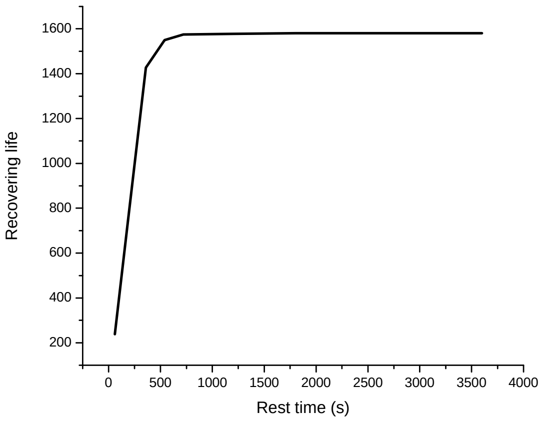



Fig.4 Recovering fatigue life for different rest time

Table.1 Experimental results for single specimen

| Rload No. | Rest time（hour） | vertical deflection (mm) | Recover fatigue life | Cycles before rest | Damage |
|---|---|---|---|---|---|
| 1 | 1 | 10.31 | 2100 | 14000 | 0.080 |
| 2 | 1 | 10.84 | 2060 | 7000 | 0.040 |
| 3 | 2 | 10.84 | 1790 | 2060 | 0 |
| 4 | 1 | 10.88 | 1680 | 2100 | 0 |
| 5 | 1 | 10.88 | 1360 | 1680 | 0 |
| 6 | 16 | 10.88 | 1140 | 1360 | 0 |
| 7 | 1 | 10.88 | 890 | 1040 | 0 |
| 8 | 1 | 10.88 | 710 | 890 | 0 |
| 9 | 1 | 10.88 | 600 | 710 | 0 |
| 10 | 1 | 15.03 | 2230 | 32700 | 0.188 |

| | | | | | |
|---|---|---|---|---|---|
| 11 | 14 | 20.1 | 2140 | 37300 | 0.214 |
| 12 | 15 | 25.3 | 2210 | 40700 | 0.234 |
| 13 | 14 | 30 | | 41900 | 0.241 |
| Total | | | | 187440 | 1 |

Table.1 is intermittent experimental results for single specimen, and it is also found that after multiple times reloading, if the deflection of last reloading is surpass the former loading, the recover fatigue life will increase. This is can be explained by damage viewpoint, that is because the new plastic deflection make the damage-resistance refreshed.

Equivalent damage of each reloading is also calculated by former method and listed in Table.1. Fatigue life result of each experiment is listed too. From Table, the total fatigue life is 187440, in which effective damage cycles is 173600. The recover fatigue life increases about 5.36% than specimen without rest and stop.

## 6 Conclusion

Recovering effects for sandwich beam with viscoelastic core are found in three-pionts bending fatigue test. A fatigue damage model considering recovering effects and recovering degree function of sandwich beam are proposed here. The intermittent fatigue experiment is carried out and the results show that the recovering fatigue life of specimen should not be ignored in loading-rest-reloading test and the damage model proposed here can describe the fatigue damage considering intermittent effects in certain accuracy.

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