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API 579 G-factors for K Calculations and Improvements For Assessment of Crack-like Flaws in Pipelines

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Abstract

API 579-1/ASME FFS-1 2007 gives a complete guideline of Fitness-For-Service (FFS) assessment of crack-like flaws in structural components based on the Failure Assessment Diagram (FAD) method. The assessment is carried out in FAD using a non-linear relationship between the load ratio (L_r) and the toughness ratio (K_r) where the toughness ratio (K_r) is defined as the ratio of the Mode I stress intensity factor (K_I) to the toughness of the material (K_{mat}). One of the methods for Mode I stress intensity factor (K_I) calculation in API 579-1/ASME FFS-1 2007 is FEA based, which correlates five influence coefficients, namely G_o to G_4, for K_I calculation. These influence coefficients are dimensionless and are tabulated for a range of two ratios; the wall thickness to the internal radius (t/R_i) and the flaw depth to the wall thickness (a/t) for infinite cracks and three ratios (crack depth to half crack length (a/c) besides a/t and t/R_i) for finite cracks. API 579-1/ASME FFS-1 2007 allows interpolation of the influence coefficients, Gs for intermediate values. Even though it is not explicitly mentioned about the method of interpolation, a linear interpolation is generally considered. Another possible method is explored here with the polynomial function fitted by the FEA data provided in the table given in API 579. A detail discussion on obtaining the influence coefficients for an intermediate value is outlined here. Case studies are provided to illustrate the advantages and disadvantages of the linear interpolation and the fitted polynomial function.

Keywords: Stress Intensity Factor, Influence Coefficients G_i, G Interpolation, FAD, Level 2 Limiting Pressure

1 Introduction

The Mode I Stress Intensity Factor (K_I) in API 579 is a fourth order polynomial function of (a/R_i) or (a/R_o) with applied internal pressure and pipe dimension. The solution is obtained from FEA work completed by T. L. Anderson Group [1]. According to API 579, the Mode I Stress Intensity Factors for the infinite length internal and external surface cracks in axial (longitudinal) direction are given by the following equations [2] respectively.

Internal axial infinite surface crack:

$$K_I = \frac{pR_o^2}{R_o^2 - R_i^2}\left[2G_o - 2G_1\left(\frac{a}{R_i}\right) + 3G_2\left(\frac{a}{R_i}\right)^2 - 4G_3\left(\frac{a}{R_i}\right)^3 + 5G_4\left(\frac{a}{R_i}\right)^4\right]\sqrt{\pi a}$$

External axial infinite surface crack:

$$K_I = \frac{pR_i^2}{R_o^2 - R_i^2}\left[2G_o + 2G_1\left(\frac{a}{R_o}\right) + 3G_2\left(\frac{a}{R_o}\right)^2 + 4G_3\left(\frac{a}{R_o}\right)^3 + 5G_4\left(\frac{a}{R_o}\right)^4\right]\sqrt{\pi a}$$

Where, p is the internal pressure, R_i and R_o are the internal and external radii respectively, a is the flaw depth and G_o to G_4 are the influence coefficients. This equation is valid for $0.0 \leq a/t \leq 0.8$ and $0.001 \leq t/R_i \leq 1.0$. The influence coefficients are given in Table C.10 and Table C.11 in API 579 document for infinite axial and circumferential cracks respectively. The table gives values of influence coefficients for various t/R_i and a/t. t/R_i is converted to R_i/t for the convenience in calculation performed in this work. R_i/t can also be readily converted to OD/t $[= 2(R_i/t + 1)]$ that is more common in tubular performance calculation in Oil and Gas Industry. For finite surface crack $\sqrt{\pi a}$ in the above two equations are replaced by $\sqrt{\pi a/Q}$. The influence coefficients for the finite crack are function of a/c in addition to a/t and t/R_i ratios. Therefore, for infinite crack, two interpolations and for finite crack, three interpolations are required to obtain the desired influence coefficients. It is mentioned in API 579 that "interpolation of the influence coefficients, G_i, may be used for intermediate values of t/R_i and a/t" but does not state explicitely how the interpolation should be performed, linearly or otherwise. The work described in this paper takes an attempt to explain if the linear interpolation is adequate or accurate enough for this purpose. The cubic polynomial interpolation is used for the comparative study.

2 Interpolation methods

2.1 Infinite Surface Crack

Table C.10 and Table C.11 in API 579 give the influence coefficients of infinite surface crack for axial and circumferential direction respectively. Figure 1 shows part of the Table C.10 and Table C.11 from API 579. The influence coefficients shown in these tables are function of t/R_i and a/t. Therefore, two interpolations are required to obtain the desired influence coefficients for a given pipe and crack dimensions.

Table C.10

Influence Coefficients For a Longitudinal Infinte Length Surface Crack in a Cylindrical Shell

t/Ri	a/t	Internal Crack					External Crack				
		G0	G1	G2	G3	G4	G0	G1	G2	G3	G4
0.001	0	1.12	0.682	0.5245	0.4404	0.379075	1.12	0.682	0.5245	0.4404	0.379075
	0.2	1.362669	0.775768	0.577169	0.475763	0.405555	1.362492	0.77543	0.577078	0.475707	0.40532
0.00333	0	1.12	0.682	0.5245	0.4404	0.379075	1.12	0.682	0.5245	0.4404	0.379075
	0.2	1.357654	0.772719	0.575074	0.47405	0.404119	1.357654	0.772836	0.57512	0.47405	0.404212
0.01	0	1.12	0.682	0.5245	0.4404	0.379075	1.12	0.682	0.5245	0.4404	0.379075
	0.2	1.355721	0.772039	0.574618	0.473607	0.403962	1.355914	0.772205	0.574709	0.473663	0.404056

Table C.11

Influence Coefficients For a Circumferential 360° Surface Crack in a Cylindrical Shell

t/Ri	a/t	Internal Crack					External Crack				
		G0	G1	G2	G3	G4	G0	G1	G2	G3	G4
0.001	0	1.12	0.682	0.5245	0.4404	0.379075	1.12	0.682	0.5245	0.4404	0.379075
	0.2	1.322217	0.749276	0.557422	0.459379	0.392453	1.322614	0.749276	0.557516	0.459436	0.392373
0.00333	0	1.12	0.682	0.5245	0.4404	0.379075	1.12	0.682	0.5245	0.4404	0.379075
	0.2	1.326772	0.756593	0.563926	0.464829	0.397395	1.327958	0.75694	0.564112	0.464942	0.397435
0.01	0	1.12	0.682	0.5245	0.4404	0.379075	1.12	0.682	0.5245	0.4404	0.379075
	0.2	1.328921	0.764758	0.572095	0.471607	0.403497	1.332125	0.754093	0.552893	0.446495	0.394324

Figure 1 – Partial Table C.10 and Table C.11 from API 579

Interpolation is performed in two steps using the following methodology:

- For a given pipe and crack geometry, a/t and t/R_i are calculated.
- The a/t and t/R_i are then used to identify the neighboring a/t and t/R_i in the table. There are two sets of influence coefficients for each t/R_i obtained from two neighboring a/t.
- First interpolation is performed for each t/R_i between the corresponding set of a/t for the desired a/t.

- The second and final interpolation is performed between those interpolated influence coefficients for the desired t/R_i.

The order of the interpolations (starting with a/t or t/R_i) is irrelevant. Linear interpolation requires two sets of data points surrounding the desired a/t or t/R_i. But for the cubic polynomial interpolation requires four sets of data points. In the subsequent sections a detail description is given with examples on how to perform the both interpolations and is shown the difference with respect to linear interpolation. Out of the five influence coefficients, the first one, G_o has the most influence in the stress intensity function. Therefore, the comparative analysis is performed on G_o alone.

2.1.1 Nature of nonlinearity

The influence coefficients are function of a/t and inverse of R_i/t. The nature of nonlinearity of the influence coefficients with a/t and R_i/t are discussed here starting with a/t.

Figure 2 shows the nature of non-linearity of all five influence coefficients for $R_i/t = 10$. The first influence coefficient shows the highest nonlinearity and the nonlinearity diminishes for the subsequent influence coefficients. It also shows a very good fit with cubic polynomial. For other R_i/t the cubic polynomial function will change. Figure 3a shows the first influence coefficient (G_o) for $R_i/t = 10$ and $R_i/t = 5$. It can be visually inferred from Figure 3a that linearity can be assumed between any two consecutive points without losing a considerable amount of accuracy. The effect of non-linearity on G_o can be observed from Figure 3b. Non-linearity reduces with lesser R_i/t as well as lesser a/t.

There are seven points identified in the Figure 3a. They are -

- Point A: a/t=0.6 and $R_i/t=5$
- Point B: a/t=0.6 and $R_i/t=10$
- Point C: a/t=0.8 and $R_i/t=5$
- Point D: a/t=0.8 and $R_i/t=10$
- Point E: a/t=0.7 and $R_i/t=5$. This is one of the two interpolated points obtained after the first interpolation.
- Point F: a/t=0.7 and $R_i/t=10$. This is one of the two interpolated points obtained after the first interpolation.
- Point X: a/t=0.7 and $R_i/t=7$. This is the final value after the second interpolation between points E and F.

Point A through Point D are obtained from Table C.10. The example discussed later uses these points and shows the differences in calculating G_o using cubic polynomial interpolation and linear interpolation. Next discussion is on the nonlinear nature of G_o with respect to R_i/t.

Figure 4a shows the nature of the nonlinearity of G_o with respect to R_i/t. It is observed from the earlier figures that the non-linearity reduces with decreasing a/t as well as with decreasing R_i/t. Even more interestingly, Figure 4a shows that the nonlinearity of G_o vanishes beyond a value of R_i/t. This value varies with variation of a/t. Assuming the maximum $R_i/t = 40$ after which the influence of R_i/t vanishes, Figure 4b shows a set of cubic polynomials of R_i/t for various a/t. These are not as good fit as we have seen earlier with cubic polynomial of a/t even though the maximum of R_i/t is reduced down to 40. Improvement can be made in the curve fitting is to perform the curve fitting around the desired value. In explaining this, two piecewise curve fittings are performed and shown in Figure 5. Curve fittings with R_i/t from 1 to 10 (left in the figure) and from 10 to 40 (right in the figure) are considered in this figure.

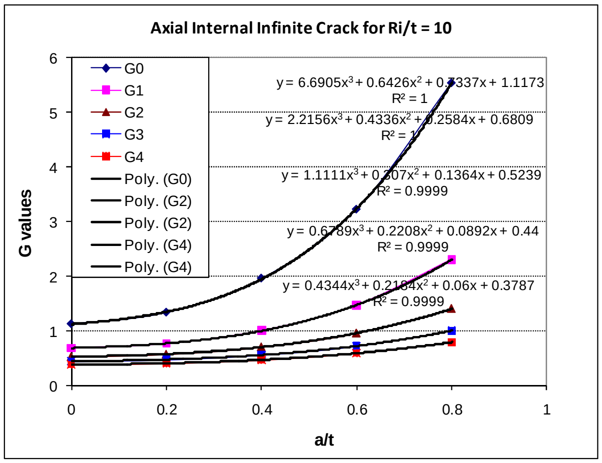
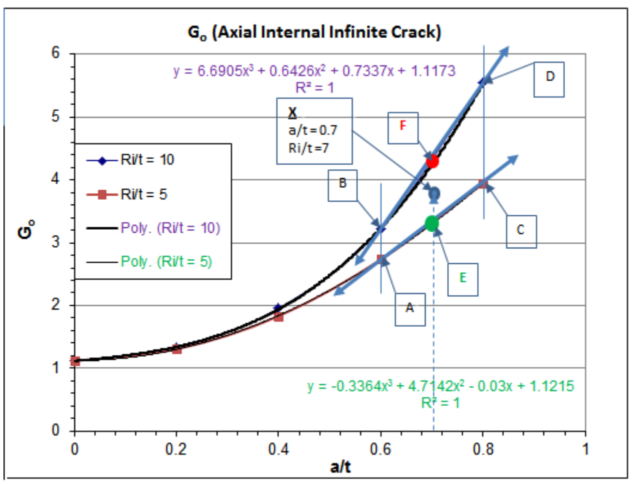
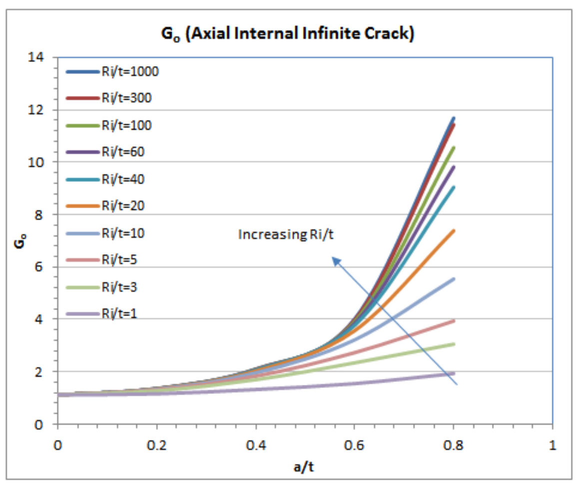


Figure 2 – Nonlinear behavior of the influence coefficients of the axial internal infinite surface crack for $t/R_i = 10$



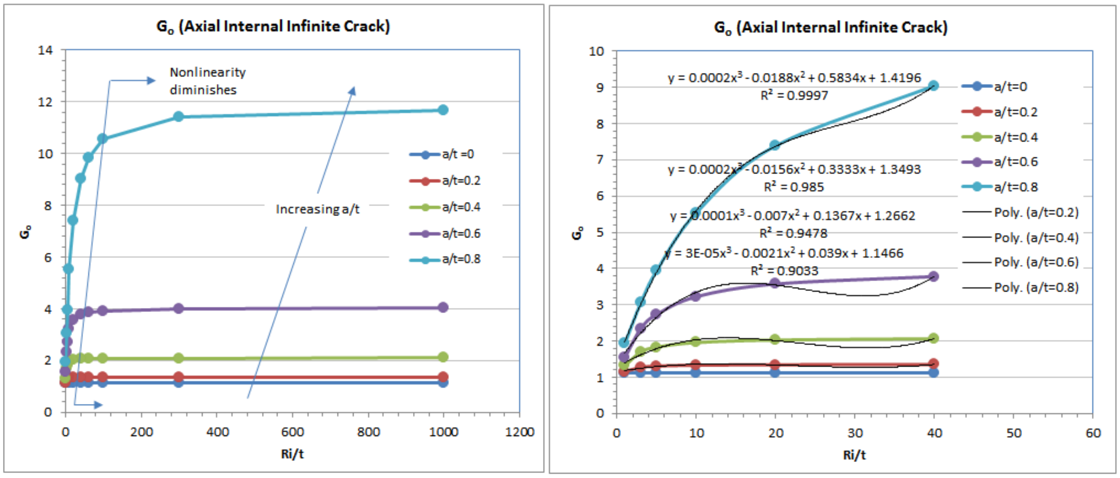
(3a)



(3b)

Figure 3 – (3a) First influence coefficient (G_o) for $t/R_i = 10$ and 5 and (3b) Effect of non-linearity in G_o with increasing R_i/t

Figure 6 shows the similar arrangement of points A through D described earlier along on the cubic curve fit between R_i/t of 1 to 10. It also shows the linear approximation lines. It can be clearly inferred from this figure that linearity can be assumed between two adjacent points as shown in the previous case.



(4a) **(4b)**

Figure 4 – (4a) The first influence coefficient (G_o) with respect to R_i/t and (4b) Nonlinear behavior of the first influence coefficient with respect to R_i/t

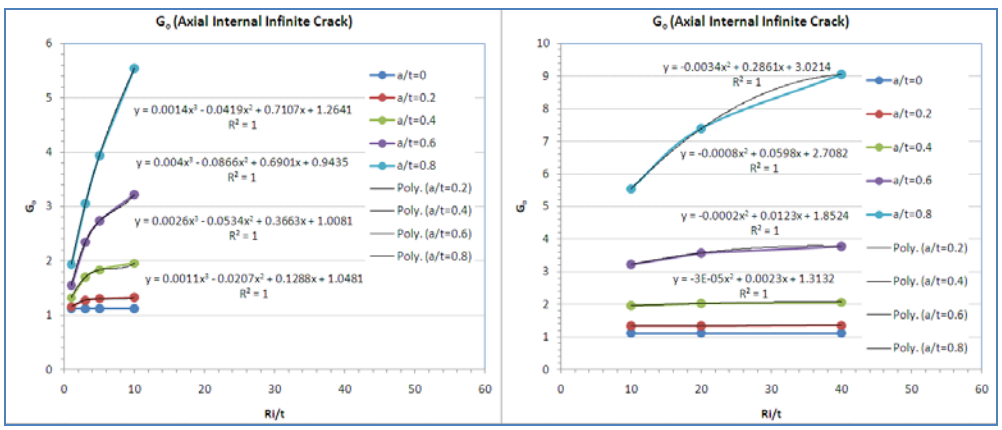


Figure 5 – Local cubic polynomial of the first influence coefficient with respect to R_i/t. (a) $R_i/t \leq 10$ (left figure) (b) $10 \leq R_i/t \leq 40$ (right figure)

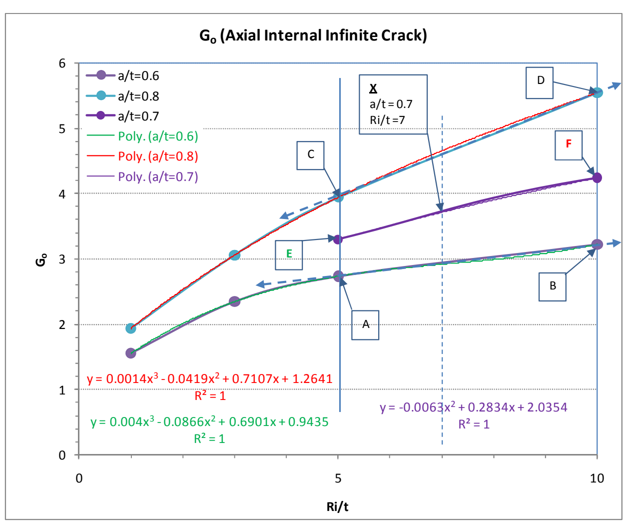


Figure 6 – First influence coefficient (G_o) for a/t = 0.6 and 0.8

Therefore, linearity between two adjacent a/t points and two R_i/t points can be assumed without losing much accuracy. Following example takes an attempt of quantifying the accuracy. The first influence coefficient (G_o) is selected because it has the most influence in stress intensity function (Equation is given at the beginning).

2.1.2 Example

Let us assume that the influence coefficients for a/t=0.7 and R_i/t=7 are needed for calculating stress intensity factor. Table 1 shows a portion of Table C.10 from API 579 for R_i/t of 10 and 5 and for a/t from 0 to 0.8. The values we are looking for are between R_i/t of 10 and 5 and a/t of 0.6 and 0.8. These values are marked in the table. As it is mentioned earlier that two interpolations are required. The first interpolation is performed for a/t = 0.7 that is in between a/t of 0.6 and 0.8. This yields two sets of influence coefficients; one for R_i/t = 10 and the other for R_i/t = 5. The next and final interpolation is performed for R_i/t = 7 that is in between R_i/t of 10 and 5. It is also important to note that the interpolations are performed between values of a/t and R_i/t that are adjacent to the desired values.

Table 1 – Influence coefficients for Axial Internal Infinite Crack

		Internal Crack				
Ri/t	a/t	G0	G1	G2	G3	G4
10	0	1.12	0.682	0.5245	0.4404	0.379075
10	0.2	1.332691	0.763153	0.569758	0.470495	0.401459
10	0.4	1.957764	1.002123	0.702473	0.556857	0.467621
10	0.6	3.223438	1.466106	0.953655	0.718048	0.585672
10	0.8	5.543784	2.300604	1.398958	1.000682	0.789201
5	0	1.12	0.682	0.5245	0.4404	0.379075
5	0.2	1.307452	0.753466	0.564298	0.466913	0.398757
5	0.4	1.8332	0.954938	0.676408	0.539874	0.454785
5	0.6	2.734052	1.28757	0.857474	0.656596	0.54072
5	0.8	3.940906	1.739955	1.10621	0.81823	0.661258

2.1.3 Interpolation between a/t

Following are couple of important notes that can be inferred from Figure 2:

- The first influence coefficient (G_o) is more nonlinear than other influence coefficients.
- For lesser a/t, the function is more linear than higher a/t.
- A linear behavior may be assumed between two adjacent a/t.

Figure 3 shows the first influence coefficients for R_i/t = 10 and R_i/t = 5. Two vertical straight lines are drawn between a/t = 0.6 and a/t = 0.8. Four points are identified on these two lines. The values of the points are given in Table 2 along with the values computed using cubic functions and the associated % difference. It is important to note that even at the known values of a/t and R_i/t the cubic functions yield difference even though the % difference are minor.

Table 2 – Values of four points used in the interpolations

Point	a/t	Ri/t	Go	Go using cubic function	% Difference
A	0.6	5	2.734052	2.727950	-0.22%
B	0.6	10	3.223438	3.234004	0.33%
C	0.8	5	3.940906	3.942351	0.04%
D	0.8	10	5.543784	5.541060	-0.05%

Next, we calculate G_o for a/t = 0.7 by using cubic polynomial function and by using linear interpolation method. Linear interpolation method uses the tabular values of G_o for a/t = 0.6 and a/t = 0.8. The points E and F in Figure 3 are the interpolated points for a/t = 0.7 for $R_i/t = 5$ and $R_i/t = 10$ respectively. Table 3 shows the values of points E and F by using cubic and linear interpolations. The % difference shown in the table indicate that the linear interpolation yields 1.27% and 2.36% higher G_o in reference to the cubic interpolated values. It is important to remember that the values using cubic functions associate with some difference.

Table 3 –Interpolation for a/t=0.7 between 0.6 and 0.8

Point	a/t	Ri/t	Go using cubic function	Go using linear function	% Difference
E	0.7	5	3.295073	3.337479	1.29%
F	0.7	10	4.240606	4.383611	3.37%

2.1.4 Interpolation between Ri/t

Figure 6 shows influence coefficient points A through F on R_i/t axis. A cubic curve fit is obtained for a/t=0.7. Since cubic polynomial interpolation requires four points to perform the calculation we calculated G_o for a/t=0.7 and $R_i/t=20$. Using the cubic polynomial function for a/t=0.7, G_o is calculated for $R_i/t=7$. Finally, linear interpolation is performed between Ri/t = 5 and 10 for Ri/t = 7 on a/t = 0.7. The final interpolated values for point X using cubic and linear interpolation are shown in Table 4. The overall % difference is 1.22% for linear interpolated value in reference to the cubic interpolated value.

Table 4 – Interpolation for Ri/t=7 between 5 and 10 at a/t=0.7

Point	a/t	Ri/t	Go using cubic function	Go using linear function	% Difference
E	0.7	5	3.295073	3.337479	1.29%
F	0.7	10	4.240606	4.383611	3.37%
	0.7	20	5.189129		
X	0.7	7	3.710500	3.755932	1.22%

2.1.5 Findings of the example

- Cubic interpolation introduces complexity in the calculation without improving much accuracy in the results.
- Besides computational complexity following are some comments and observations on cubic polynomial and curve fitting:
    - Cubic polynomial interpolation requires four consecutive points if local curve fitting is intended otherwise global curve fitting may be employed.
    - Global curve fitting introduces more difference in the interpolation.
    - Cubic polynomial function needs to be obtained for each set of Ri/t and a/t that will introduce complexity in the calculation and increase computation time.
- The % difference in reference to the cubic interpolation is reasonable if linear interpolation is performed.
- Similar arguments for linear interpolation on other influence coefficients can be made.

2.2 Finite Surface Crack

Figure 7 shows partial tables for influence coefficients of finite axial inside and outside surface cracks. Unlike the tables for infinite crack, these tables contain one more level of variable that is the ratio of crack depth to the half crack length (a/c). Hence, to obtain the desired influence coefficients, three interpolations are required.

Table C.12

Influence Coefficients For A Longitudinal Semi-Elliptical Surface Crack in A Cylinder - Inside Surface

t/Ri	a/c	a/t	Gi	A0	A1	A2	A3	A4	A5	A6
0	0.03125	0	G0	0.1965046	2.9373464	-5.2582823	7.4889153	-6.9282667	3.3673349	-0.6677966
		0	G1	0.005178	0.175028	2.771868	-4.6457154	4.6780502	-3.276809	0.9840994
		0.2	G0	0.208076	3.0112422	-5.1048701	7.6348715	-6.8347547	2.7940766	-0.3882688
		0.2	G1	0.0084834	0.2406767	2.4574292	-3.6452421	3.6142837	-2.8451814	0.9270638
0	0.0625	0	G0	0.2695332	2.1626001	-1.6551569	-1.2970208	4.5604304	-4.3163876	1.4010655
		0	G1	0.0138667	0.1827458	2.5749608	-3.9044679	3.3556301	-2.1772209	0.6420134
		0.2	G0	0.2845892	2.2264055	-1.454619	-1.5760719	5.1131083	-4.9485443	1.6207574
		0.2	G1	0.0199077	0.2210874	2.4642047	-3.5898625	3.1624039	-2.240378	0.6965751

Table C.13

Influence Coefficients For A Longitudinal Semi-Elliptical Surface Crack in A Cylinder - Outside Surface

t/Ri	a/c	a/t	Gi	A0	A1	A2	A3	A4	A5	A6
0	0.03125	0	G0	0.1965046	2.9373464	-5.2582823	7.4889153	-6.9282667	3.3673349	-0.6677966
0	0.03125	0	G1	0.005178	0.175028	2.771868	-4.6457154	4.6780502	-3.276809	0.9840994
0	0.03125	0.2	G0	0.208076	3.0112422	-5.1048701	7.6348715	-6.8347547	2.7940766	-0.3882688
0	0.03125	0.2	G1	0.0084834	0.2406767	2.4574292	-3.6452421	3.6142837	-2.8451814	0.9270638
0	0.0625	0	G0	0.2695332	2.1626001	-1.6551569	-1.2970208	4.5604304	-4.3163876	1.4010655
0	0.0625	0	G1	0.0138667	0.1827458	2.5749608	-3.9044679	3.3556301	-2.1772209	0.6420134
0	0.0625	0.2	G0	0.2845892	2.2264055	-1.454619	-1.5760719	5.1131083	-4.9485443	1.6207574
0	0.0625	0.2	G1	0.0199077	0.2210874	2.4642047	-3.5898625	3.1624039	-2.240378	0.6965751

Figure 7 – Partial Table C.12 and Table C.13 from API 579

Interpolation is performed using the following methodology:

- For a given pipe and crack geometry, t/R_i, a/c and a/t are calculated.
- The a/t a/c and t/R_i are then used to identify the neighboring a/t, a/c and t/R_i in the table. There are two sets of influence coefficients with the neighboring a/t for each t/R_i and a/c.
- First interpolation is performed for each t/R_i and a/c between the corresponding set of a/t for the desired a/t.
- The second interpolation is performed between those interpolated influence coefficients for the desired a/c for each t/R_i.
- Finally the third and final interpolation is performed for the desired t/R_i.

2.3 Effect of interpolation on limiting pressure prediction

In the earlier section, we discussed and showed in the example the differences in the influence coefficients when using the cubic polynomial interpolation instead of the linear interpolation. With the finite crack we will calculate Level 2 limiting pressure using both interpolation methods. Level 2 limiting pressure for a known crack is defined as the pressure at which the toughness ratio (K_r) equals to the toughness ratio calculated from the Level 2 FAD equation for the calculated load ratio (L_r). This is the maximum operation pressure beyond which the crack will be unacceptable for fitness for service according to Level 2 FAD. FAD is a relation between K_r and L_r, which is described as follows:

$$K_r = [1 - 0.14(L_r)^2]\left\{0.3 - 0.7e^{\left[-0.65(L_r)^6\right]}\right\} \quad for\ L_r \leq L_{r\,max}$$

Where, $L_{r\,max} = 1.25$ for Carbon Steels, $K_r = K_I/K_{mat}$ and $L_r = \sigma_{ref}/\sigma_{YS}$, K_I is the mode I stress intensity factor, K_{mat} is the materials fracture toughness, σ_{ref} is the reference stress, based on yield load and limit load solutions for the configuration of interest, and σ_{YS} is the yield strength of the material. Figure 8 shows the FAD using the above equation. If the assessment point lies inside the FAD, the structure is considered safe and the crack is acceptable for fit. The unacceptable crack is predicted when the assessment point falls outside the FAD.

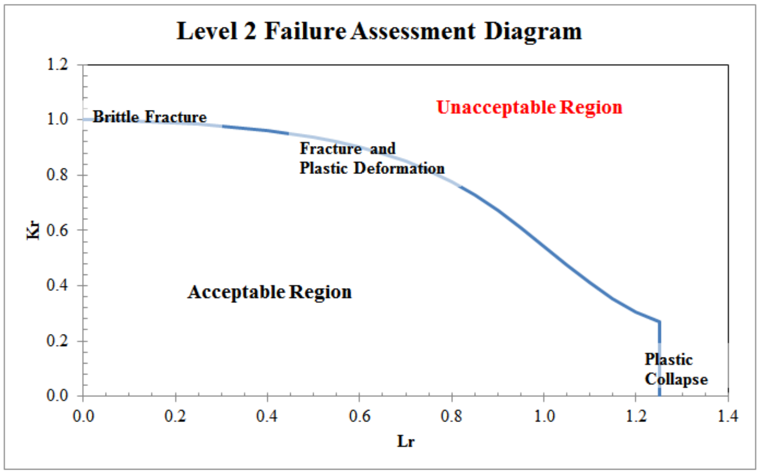


Figure 8 – FAD for Level 2 Assessment

Both K_I and σ_{ref} calculated for a given crack are function of operating pressure. The limiting pressure is that operating pressure at which the calculated K_r for the given crack equals to the K_r calculated from the above FAD equation. Therefore, a difference in K_I due to the use of linear instead of cubic interpolation method for estimating influence coefficients is carried in K_r calculation and there on to the limiting pressure prediction.

In this example, we choose a 914.4 mm (36 in.) X65 pipe with the wall thickness of 6.35 mm (0.25 in.) and 12.7 mm (0.50 in.). The internal pressure for this example is assumed as 4,356 kPa (70% SMYS). The cracks used in this example are defined as a/t varying from 0.1 to 0.8 and as a/c from 2 to 0.03125. Three material toughness values (20, 100 and 200 MPa√mm), K_{mat} are used in this example.

Figure 9 shows the percent difference in Level 2 limiting pressure versus the percent difference in K_r. The variation in K_r due to the use of interpolation method introduces a variation in limiting pressure for various crack dimensions. We can infer the following from the figure –

- The difference in limiting pressure is inversely varying with the difference in K_r.
- Difference in predicted limiting pressure diminishes with higher material toughness value. For a higher material toughness, the difference in limiting pressure is considerably lower than a lower material toughness value. It is also observed that with a higher toughness (i.e. K_{mat} = 200 MPa√mm) and shallower crack (i.e. a/t <0.4) the difference in limiting pressure is zero or negligible.

- The maximum difference in limiting pressure observed is less than 6%. However, it can be critical for a particular application. Therefore for a deep surface crack with low material toughness it is important to be cautious in predicting Level 2 limiting pressure.

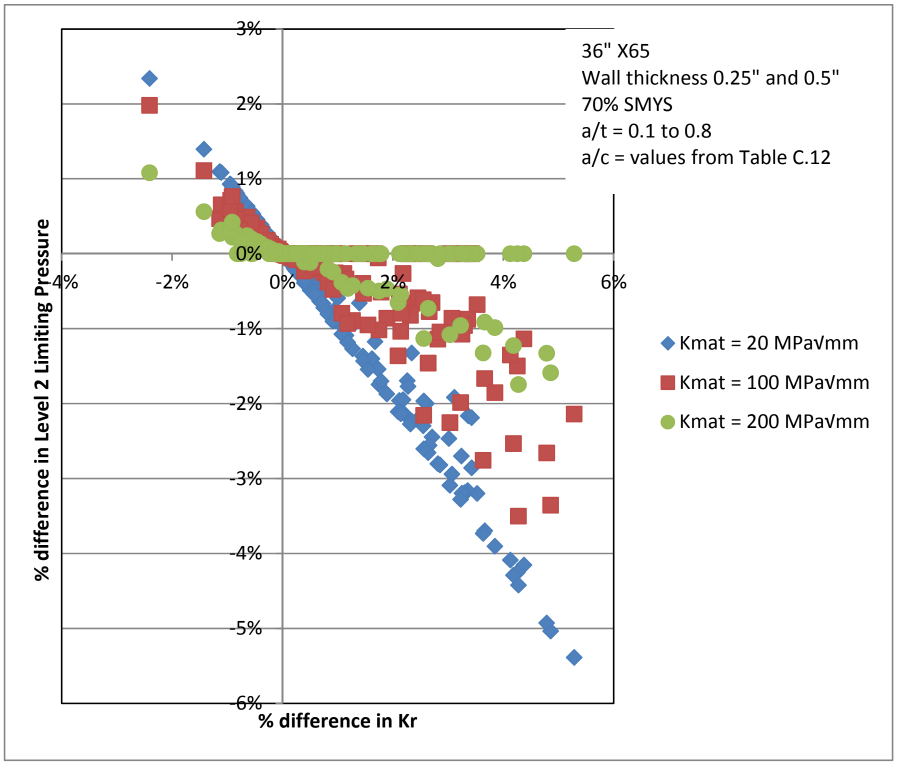


Figure 9 – Percent difference in the Level 2 limiting pressures

3 Conclusions

Even though the values of the influence coefficients given in the tables in API 579 fit with some cubic polynomial functions for a/t and also for R_i/t, the nonlinearity is found more in the first influence coefficient (G_o). For a/t equal or less than 0.4, the influence coefficients can be assumed linear. It is also concluded that shallower cracks (a/t <0.4) with a relatively high material toughness, the linear interpolation does not produce significant difference in predicting limiting pressure with respect to nonlinear interpolation. While for the deep surface cracks with low material toughness may be critical for a particular application and should require caution in predicting Level 2 limiting pressure. However the overall difference in K_I as well as in Level 2 limiting pressure by using the linear interpolation rather than the cubic polynomial interpolation is less than 6% which is reasonably within the acceptable error margin.

4 Reference

[1] Ted L. Anderson, et. al, Development of Stress Intensity Factor Solutions for Surface and Embedded Cracks in API 579, Bulletin 471, Welding Research Council, Inc. New York, NY, May 2002

[2] API 579-1/ASME FFS-1 2007 Fitness-For-Service, Equation C.175 and C.176.