Fracture problem of the thin superconducting strip with transverse crack

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Abstract This paper presents the fracture behaviors in thin type-II superconducting strips with a transverse crack induced by the electromagnetic body force under a perpendicular applied field. Different from the longitudinal crack, the effect of the transverse crack on the current in the thin strip is considered. The Bean and Kim model of critical state are adopted to calculate the electromagnetic force. Based on the finite element method, the stress intensity factors of the transverse crack are obtained during one full cycle applied field (the field increasing process and decreasing process). Furthermore, the influences of the crack length on the stress intensity factors are also studied in this paper. Generally speaking, all the results are useful for understanding the critical state model and the fracture mechanism of high-temperature superconductors, which could provide a theoretical guide for the ITER CICC designer.

Keywords Thin superconducting strip, Bean model, Kim model, Transverse crack

1. Introduction

High-temperature superconductor (HTS) tapes have provided a wide range of engineering applications, such as ac power transmission cables [1, 2] and filters for wireless communication [3]. With the development of engineering applications, improvements on mechanical performances of HTS bulks in the presence of high magnetic fields have attracted considerable interest. The stress and the magnetostriction in bulk superconductors subjected to high magnetic fields were analyzed for various geometrical cases, critical state models, and magnetization conditions [4-7]. Cracks in samples can be explained by tensile stresses that occur during the magnetization process due to the stored flux density gradient and may exceed the tensile strength of the material [8]. In recent years, a multitude of theoretical analyses for the crack problem of long rectangular slabs and long cylinders under electromagnetic force have been achieved [9-11]. Owing to the strong demagnetization effects in the perpendicular geometry, the central crack in a thin superconducting strip was studied according to the Bean critical state model [12].

However, it is observed experimentally that the critical current density usually depends on the flux density in real materials. The body forces induced by flux-pinning and magneto-elastic behavior are significantly different in magnitude between the Kim model and Bean model in thin superconducting strip [13]. It is found that the magnetostrictive behavior for the Kim model is in much better agreement with experiment than that for the Bean model at low temperatures. The Kim model is chosen because it has been shown to fit well the $J_c\left(H_a\right)$ measured in thin YBCO films

[14]. Therefore, it is of great practical importance to investigate crack problem of HTS thin films for the Kim model. It is to be known that stress intensity factor is a good fracture criterion and can provide successful prediction for fracture behavior. Although the longitudinal crack problem was extensively studied, the influence of transverse crack on the transport current has not been considered for thin films in the literature. Our main aim of this paper is to study the fracture behaviors of transverse crack in a HTS thin film based on the framework of field-dependent critical current model [15]. Applying finite element method, the stress intensity factors of the transverse crack are obtained during one full cycle applied field (the field increasing process and decreasing process) for the Bean model and Kim model. The comparisons are made between the results from the transverse crack and the longitudinal crack. The influence of the crack length is also studied in this paper.

2. Basic equations

We consider a thin type-II superconducting strip with a transverse crack of length $2a$ in an applied perpendicular field. As shown in Figure 1 (a), the thin strip is located in the Cartesian coordinate system. The width and the thickness of the film is $2w$ and d, respectively. The film is assumed to be isotropic and infinite along the x axis. Different from the longitudinal crack, the effect of the transverse crack on the current distribution in the thin strip should be considered (see Figure 1 (b)).

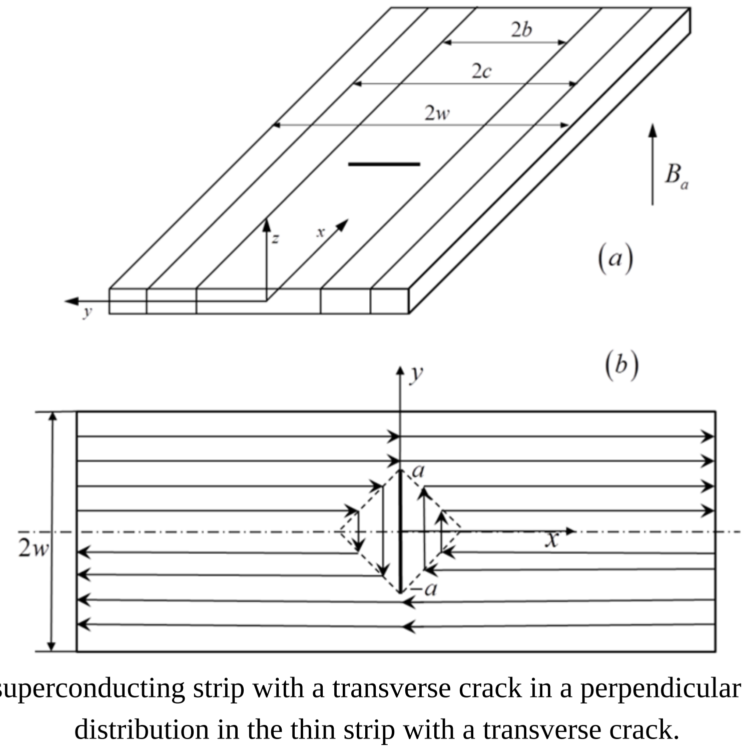

Figure 1. (a) A thin superconducting strip with a transverse crack in a perpendicular field. (b) The current distribution in the thin strip with a transverse crack.

When the superconducting strip is in a perpendicular field which exceeds the lower critical field B_{c1}, the flux and current starts to penetrate at the edge of the strip. Based on the critical state model, the shielding current density J is less than the critical current density J_c. Two classical critical state models, Bean model and Kim model, are adopted to analyze this problem. For the Bean model, the critical current is constant, i.e. $J_c = J_{c0}$. The current density is dependent on the flux density in the Kim model, i.e.

$$\frac{J_c\left(B_z\left(y\right)\right)}{J_{c0}} = \frac{B_0}{B_0 + \left|B_z\left(y\right)\right|}, \tag{1}$$

where B_0 is a constant field that characterizes the degree of field dependence.

After ignoring the effect of the lower critical field, the current profile can be obtained by conformal mapping analytically for the Bean model [16]. When applied field increases from zero to a maximum B_{am}, the shielding current and penetrated flux are

$$J(y) = \begin{cases} \frac{2J_c}{\pi} \arctan \frac{y}{w} \sqrt{\frac{w^2 - b^2}{b^2 - y^2}}, & |y| < b \\ \frac{y}{|y|} J_c, & b < |y| < w \end{cases}, \tag{2}$$

$$B(y) = \begin{cases} 0, & |y| < b \\ B_f \operatorname{arctan h} \frac{w}{|y|} \sqrt{\frac{y^2 - b^2}{w^2 - a^2}}, & b < |y| < w \end{cases}, \tag{3}$$

$$\tanh\left(\frac{B_a}{B_f}\right) = \frac{\left(a^2 - b^2\right)^{1/2}}{a}, \tag{4}$$

and the scaling field B_f is defined by

$$B_f = \frac{\mu_0 J_{c0} d}{\pi}, \tag{5}$$

where J_{c0} and μ_0 is the zero-field critical current and the permeability of free space, respectively.

When applied field decrease from the maximum B_{am} to $-B_{am}$, the current and penetrated flux can be obtained as

$$J\left(y, B_a, J_c\right) = J\left(y, B_{am}, J_c\right) - J\left(y, B_{am} - B_a, 2J_c\right), \tag{6}$$

$$B\left(y, B_a, J_c\right) = B\left(y, B_{am}, J_c\right) - B\left(y, B_{am} - B_a, 2J_c\right). \tag{7}$$

However, it can be found from the experimental results that the critical current density j_c is related to local flux density B. The results of magnetostriction in our preceding paper [13] also that the Kim model is in better agreement with experiments than Bean model at low temperatures. Unfortunately, one cannot obtain the analytical solutions based on Kim model. However, for the applied magnetic field increased from zero to B_{am}, the profiles of current and flux density can be calculated numerically from the equations below [15]

$$J_x(y)=\begin{cases}-\dfrac{2}{\pi}y\sqrt{b^2-y^2}\displaystyle\int_b^w \dfrac{J_c\left(B_z\left(y'\right)\right)dy'}{\left(y'^2-y^2\right)\sqrt{y'^2-b^2}}, & |y|<b\\ -\dfrac{y}{|y|}J_c\left(B_z\left(y\right)\right), & \mathrm{b}<|y|<w\end{cases}, \tag{8}$$

$$B_z(x)=B_f\,|x|\sqrt{x^2-b^2}\int_b^w \frac{J_c\left(B_z\left(x'\right)\right)}{J_{c0}}\frac{dx'}{\left(x^2-x'^2\right)\sqrt{x'^2-b^2}},\ b<|x|\neq w, \tag{9}$$

$$B_a=B_f\int_b^w \frac{J_c\left(B_z\left(y'\right)\right)}{J_{c0}}\frac{dy'}{\sqrt{y'^2-b^2}}. \tag{10}$$

When the applied magnetic field is increased from initial value B_{am} to B_a, the flux density changes in the outer regions ($c<|x|<w$) but is unchanged in the inner regions ($b<|x|<c$), and the central region ($|x|<b$) remains free of flux. Therefore, the distributions of current and flux density can be obtained by solving the following equations

$$J_x(y)=\begin{cases}\dfrac{2}{\pi}y\sqrt{c^2-y^2}\left[\begin{array}{l}\displaystyle\int_c^w \dfrac{J_c\left(B_z\left(y'\right)\right)}{\left(y'^2-y^2\right)\sqrt{y'^2-c^2}}dy'\\ \displaystyle-\dfrac{2}{\pi}J_{c0}\int_b^c \dfrac{B_z\left(y'\right)}{B_f}\dfrac{dy'}{\left(y'^2-y^2\right)\sqrt{c^2-y'^2}}\end{array}\right], & |y|<c\\ \dfrac{y}{|y|}J_c\left(B_z\left(y\right)\right), & \mathrm{c}<|y|<w\end{cases}, \tag{11}$$

$$B_z(y)=|y|\sqrt{y^2-c^2}\left[\begin{array}{l}\displaystyle-B_f\int_c^w \frac{J_c\left(B_z\left(y'\right)\right)}{J_{c0}}\frac{dy'}{\left(y^2-y'^2\right)\sqrt{y'^2-c^2}}\\ \displaystyle+\frac{2}{\pi}\int_b^c \frac{B_z\left(y'\right)}{\left(y^2-y'^2\right)\sqrt{c^2-y'^2}}dy'\end{array}\right],\ \mathrm{c}<|y|\neq w, \tag{12}$$

$$B_a=-B_f\int_c^w \frac{J_c\left(B_z\left(y'\right)\right)}{J_{c0}}\frac{dy'}{\sqrt{y'^2-c^2}}+\frac{2}{\pi}\int_b^c \frac{B_z\left(y'\right)}{\sqrt{c^2-y'^2}}dy'. \tag{13}$$

The body force (Lorentz force density) in the film is

$$\boldsymbol{f} = \boldsymbol{J} \times \boldsymbol{B} . \tag{14}$$

Because the thickness is very small compared with other two dimensions, this is a plane stress problem. For the symmetric model, it is convenient to analyze one quarter part of the film by means of finite element method. The famous virtual crack closure technique [17] is adopted to calculate the stress intensity factor, which can save our computational time and has high precision with fewer meshes. In our calculations, the Young's modulus and the Poisson's ratio equal to 125 GPa and 0.3, respectively.

3. Results and discussions

In this section, we will discuss the development of the stress intensity factor for applied field increasing to different maximum (B_f, $2B_f$, $3B_f$, $4B_f$) based on Bean model and Kim model. The crack length is the half of strip width. In addition, the crack length effect on the stress intensity factor is also discussed for applied field decreasing from $4B_f$ to B_f. The stress intensity factor is normalized by $K_0 = 1.2153 \times 10^6 \,\mathrm{Pa} \cdot \mathrm{m}^{0.5}$ for applied field increasing to B_f with Kim model.

Figure 2 shows the development of the stress intensity factor in a full cycle applied field with maximum B_f. It can be found that the stress intensity factor is positive as applied field increases from zero to B_f. The depth of the penetrated flux has not reached the crack in this case. Therefore, all the body force is compressive and along y direction. The tensile stress along x direction induces positive stress intensity factor. The stress intensity factor for the Kim model is large than for the Bean model.

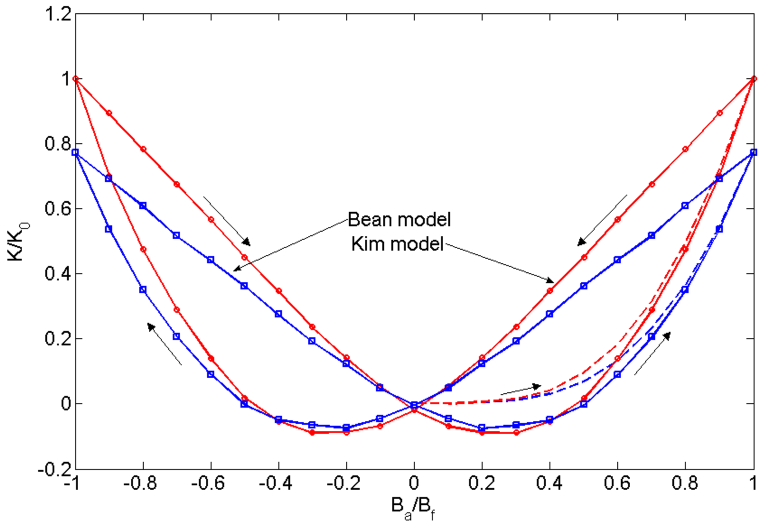

Figure 2. The development of the stress intensity factor in a full cycle applied field with maximum B_f.

If the applied field increases from zero to $2B_f$, the depth of the penetrated flux will reach the crack. The body force near the crack is along x direction in this case. Therefore, the stress intensity factor will decrease when applied field exceeds a certain value (see Figure 3). The value for the Kim model is less than for the Bean model since the filed dependence of J_c leads to a larger amount of penetration in a given applied field. The stress intensity factor increases firstly, and then decreases during the field decent. Moreover, it cannot recover zero when applied field decreases to zero because of remnant field.

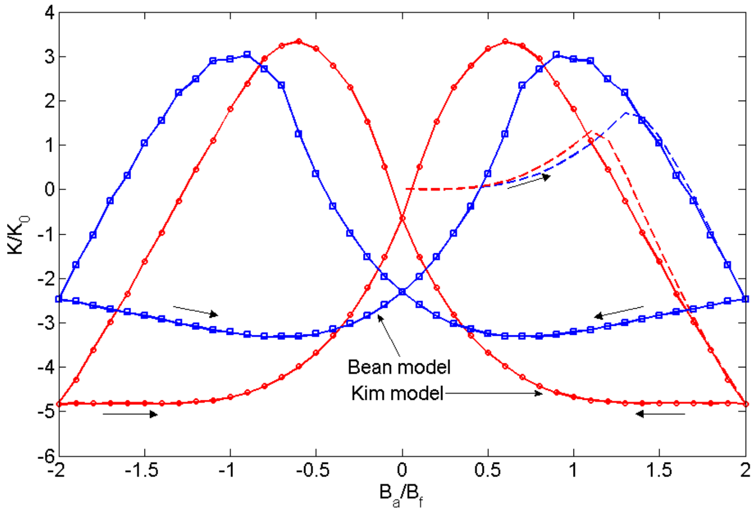

Figure 3. The development of the stress intensity factor in a full cycle applied field with maximum $2B_f$.

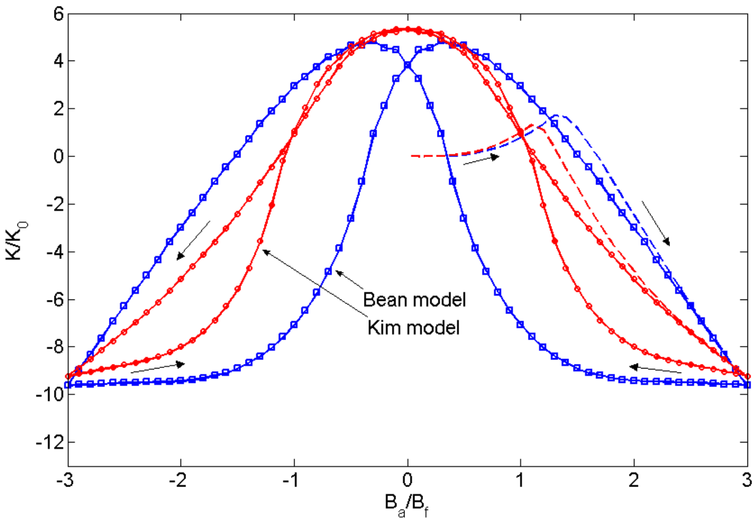

Figure 4. The development of the stress intensity factor in a full cycle applied field with maximum $3B_f$.

When applied field continue to increase, it can be found from Figure 4 that the stress intensity factor profiles is becoming complex. The development of the stress intensity factor during field decent is similar to Figure 3. However, different from Figure 3, the remnant stress intensity factor is positive when applied field decreases to zero. It is interesting that the stress intensity factor is maximal in the absence of applied field for the Kim model during field decent, and it is larger than for the Bean model.

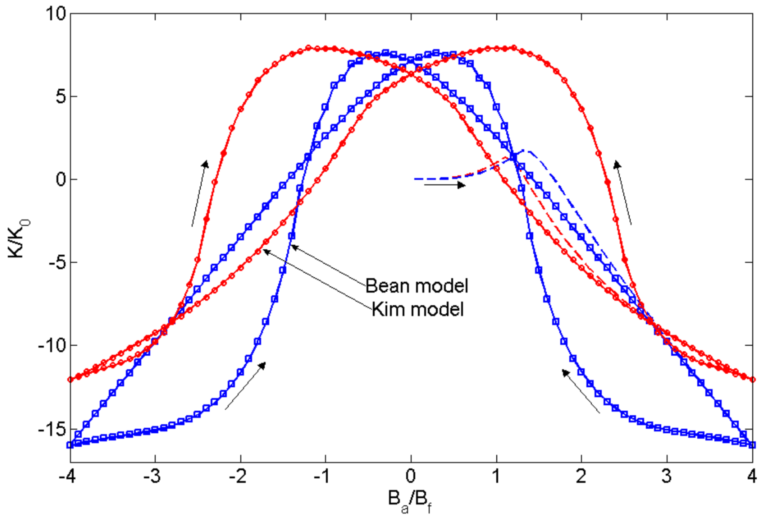

Figure 5. The development of the stress intensity factor in a full cycle applied field with maximum $4B_f$.

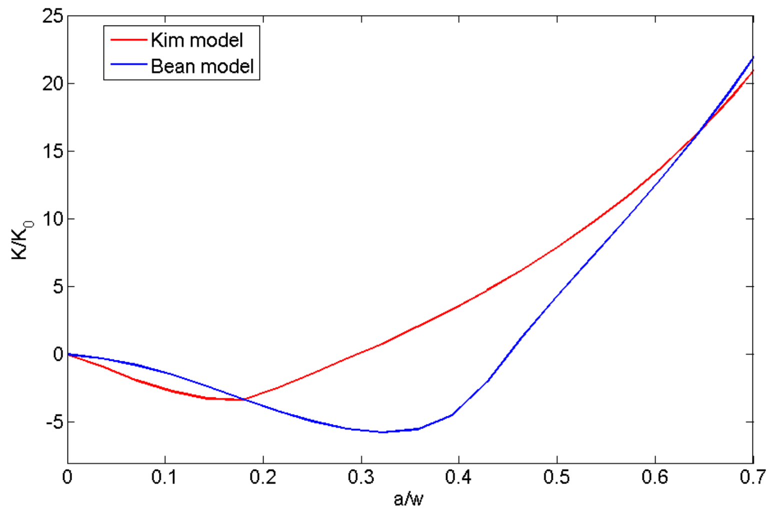

Figure 6. The stress intensity factor for different crack lengths in a perpendicular field which decreases from $4B_f$ to B_f.

Shown in Figure 5 is the development of the stress intensity as applied field increases to $4B_f$. In this case, one can see that the stress intensity factor profiles become even more complex. The maximum of the stress intensity during field decent for both Bean model and Kim model is larger than that in Figure 4. Therefore, it is more dangerous in this case. Moreover, the decreasing field corresponding to the maximal stress intensity factor for the Kim model is greater than for the Bean model.

Finally we discuss the effect of crack length on the stress intensity factor for applied field decreasing from $4B_f$ to B_f. One can see from Figure 6 that the stress intensity factor firstly decreases as the crack length increases, and then increases when the crack length exceeds a certain value. The value for the Kim model is less than for the Bean model. The reason is that the depth of current change for the Kim model is larger than for Bean model. Moreover, Figure 6 demonstrates that it is more dangerous for large crack than for small crack.

4. Conclusions

The problem of thin superconducting strip with transverse crack in a perpendicular field is investigated in this paper. The Bean model and Kim model are adopted to analyze this problem. Different from the longitudinal crack, the transverse crack is more complex because of the effect of the transverse crack on the current. The results are useful for understanding the fracture mechanism of high-temperature superconductors, which could provide a theoretical guide for the ITER CICC designer.

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