Analysis of resistance cracking process on the experiment of meso and macroscopic of quasi-brittle materials

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Abstract: Engineering materials such as cast iron and concrete are quasi-brittle materials, and there is a fracture process zone in front of opening crack and it exist micro and macro defect and energy dissipation. The damage development area is simplified into fictitious crack, and its action stress distribution and opening displacement or deformation are along the cohesive crack. In order to investigate the constitutive relation of cohesive crack opening displacement and its stress distribution in quasi brittle materials, cast iron and concrete were used to test their fracture process by electronic equipment and micro view experiment facility. Through the analytical method of applied mathematics and mechanics, the material deformation stress constitutive relations were gotten for the cohesive crack. The theoretical calculation and test results are basically in agreement with each other. From the fracture criterion of Double K and the mechanics model of cohesive crack, the bearing capacity of structure with crack was estimated in this present paper.

Key Words: Quasi brittle materials; constitutive relation of cohesive crack; mesoscopic damage; fracture process testing; inverse problem

Engineering materials such as cast iron or concrete, not same as carbon steel and other reinforcement materials, its stress and deformation or constitutive behavior shows softening characteristics when stress or strain reaches a certain value. So them are classified to the quasi brittle materials. Those materials fracture phenomenon has attracted many scholars research. Hillerborg etc presented virtual crack model for concrete fracture based on the Dugdal-Barenblatt elastic-plastic fracture model, also other researchers have double parameters, equivalent crack, double K fracture model and size effect model ad so on[1-4]. In fact, this kind of material, its fracture process zone or a fuzzy damage crack area has action force each other when external load exerting among the area. If the strip area was named fictitious crack of being not completely separate, and it has bridging role on resistance crack opening. The cohesive force must be put along the fictitious crack segment. The mechanics behavior of those quasi-brittle materials will be analyzed in the present paper by combining cohesive crack model with macro and mesoscopic experiment investigation.

1. The cohesive crack model

As shown in Fig.1，a model of cohesive crack has been formed with both segments. One is completely opening crack segment without any force, and the other is the cohesive crack segment with stress distribution showing the fracture process damage area. The damage region was regarded as part of crack, the cohesive stress distribution express the interaction of material media. The medium outside of damage or cohesive crack is regarded as a linear elastic medium. The cohesive crack tip point is named as the origin of rectangular coordinate, and the direction of level axis is for right. As the length of cohesive crack segment is indicated to '**b**', and the coordinate '**x**' to be the position of some point, then the follow polynomial express the opening displacement distribution of cohesive crack.

$$U(x)=\frac{4b}{E\pi}\sum_{n=1}^{N}c_n\left(\frac{x}{b}\right)^{n-1/2}\qquad(0\le x\le b)\tag{1}$$

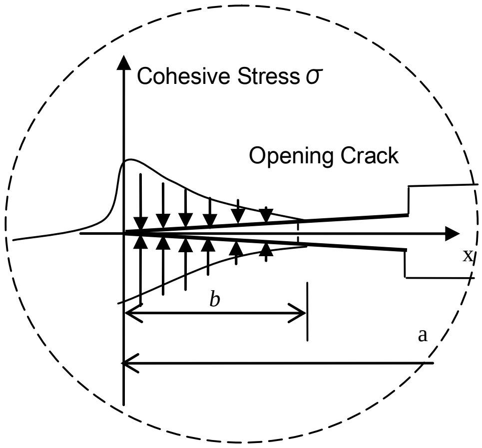


Figure 1. The cohesive crack model with cohesive stress and opening displacement in front of smooth crack

The Crack Opening Displacement or $\tilde{U}$ can be regarded as the superposition of both an opening displacement or V_K coursed by load P far from crack tip and a closing displacement done by the cohesive stress σ along crack together. Then, an integral equation was been formed with the unknown function σ being of cohesive stress distribution, based on the superposition principle of stress and deformation of the solid structure.

$$\int_0^b \tilde{G}(\xi,x)\sigma(\xi)d\xi=V_K(P,x)+\tilde{U}(x,u)\qquad(0\le x\le b)\tag{2}$$

Here, $\tilde{G}(\xi,x)$ is a Green function about the displacement of point x by unit force at ξ point. The solution of equation was been worked out, and the formula of cohesion stress distribution is as follows [5].

$$\sigma(x)=\frac{K_I}{\sqrt{2\pi(b-x)}}+\frac{-1}{2\pi\sqrt{b}}\sum_{n=1}^{N}c_n\sqrt{\frac{1}{b-x}}\sum_{r=0}^{n-1}\left(\frac{x}{b}\right)^{n-r-1}\frac{2^{-2r}(2r)!}{(r!)^2}+\frac{1}{\pi\sqrt{b}}\sum_{n=2}^{N}c_n\sqrt{b-x}\sum_{r=0}^{n-2}\frac{2^{-2r}(n-r-1)(2r)!}{(r!)^2}\left(\frac{x}{b}\right)^{n-r-2}\qquad(0\le x\le b)\tag{3}$$

Here, E is Young's elastic modulus of matrix materials for plane stress state. For the plane strain state the $E/(1-\nu^2)$ will be instead of the E, and ν is Poisson's ratio. The $c_i\,(i=1,2,...,N)$ is undetermined parameter with stress dimension. The K_I is the crack stress intensity factor by far field load P.

2 Experiment of micro detection and electronic measurement

2.1 The micro detection of iron and concrete

For the fracture process zone of quasi brittle material, its resistance part of crack opening is an area reflecting the characteristics of material nonlinear mechanical behavior. For the strip area with damage or plastic deformation, its microscopic characteristics are also complicated with load changing [6,7]. As shown in Figure 2, it is the microstructure morphology for cast iron or concrete. The status pictures of graphite piece in cast iron were shown in Fig2 (a, b) before and after loaded on the specimen. The former are not stress action in the pictures, and the latter shows the graphite flake being open or taking shape of crack in ferrite. As the area still has connection force among, this just be the mesoscopic experimental evidence of existing cohesive stress along virtual crack. Of course, the crack deformation of material with graphite flake will increase along with the increasing of load. The micro appearance of concrete were shown in Fig 2 (c, d), the interface crack between cement and stone being in picture 'c' and cracks in cement base doing in picture 'd'. The cracks existing in media is the increasing cause of deformation or strain.

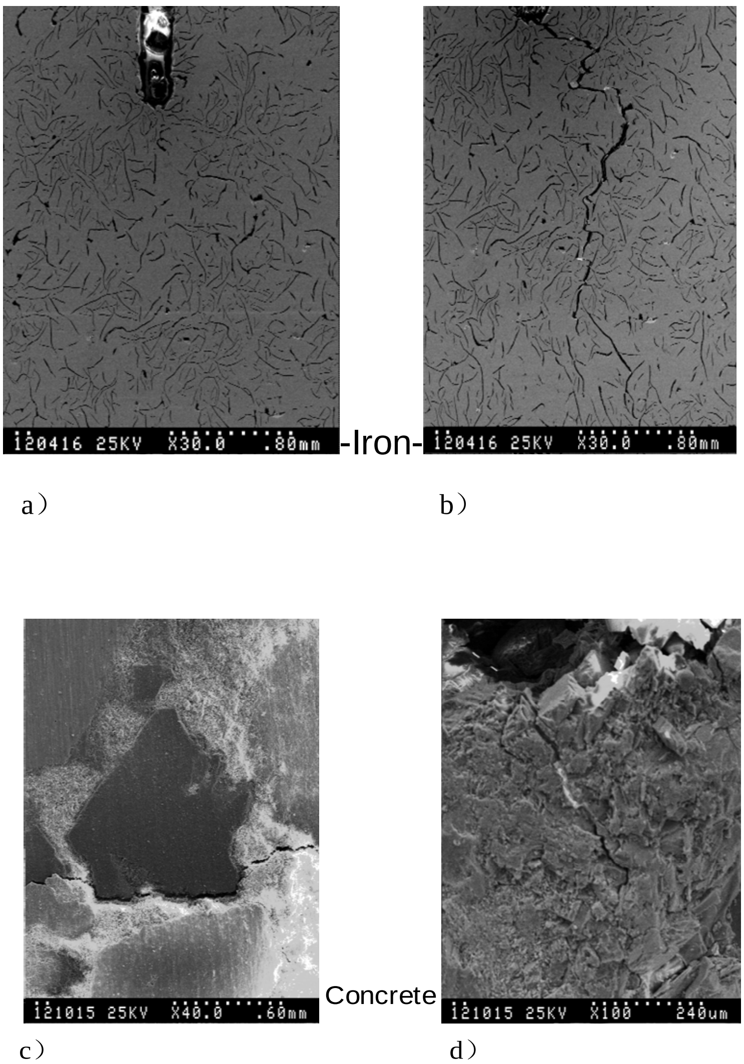


Figure 2. Electron microscopy (sem) surface images of Iron (a,b) and Concrete (c,d).

2.2 The fracture process test of materials

The specimen of iron and concrete with prefabricated crack were tested by three point bending or wedge splitting loading way, as shown in Fig 3. In order to record the process of crack ligament deformation, a few pieces of strain gauges were been pasted on front of prefabricated crack. Only, were the both sides of a strain gauge just posted at the materials surface in case of preventing the gauge film early to be pulled apart [8].

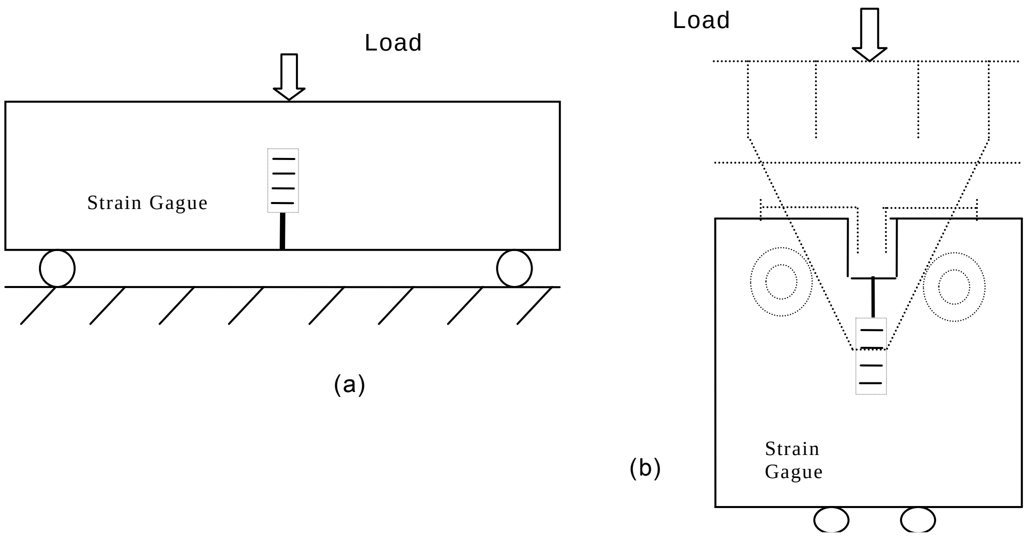


Figure 3. The experiment device of fracture process: a) Iron loaded by three point bending; b) Concrete loaded by wedge splitting pull

As shown in Fig 4, it is the deformation distribution of specimen ligament of iron or concrete under different load. The deformation is the relative displacement between its two sides of a gauge posted on materials ligament. In Fig 4 (i), the experimental curves are for cast iron specimen by the loading of three-point bending. The appearance size are respectively 'Length * Thick * High = 300 * 50 * 65 (mm*mm* mm), and loaded span is 260 mm, prefabricated crack length 5 mm. In Fig 4 (ii), the experiment curves are for concrete by loading of wedge splitting pull. The appearance size is that ‘Wide*High*Thick=500*520*150(mm*mm*mm)’ and the prefabricated crack length is about 120 mm.

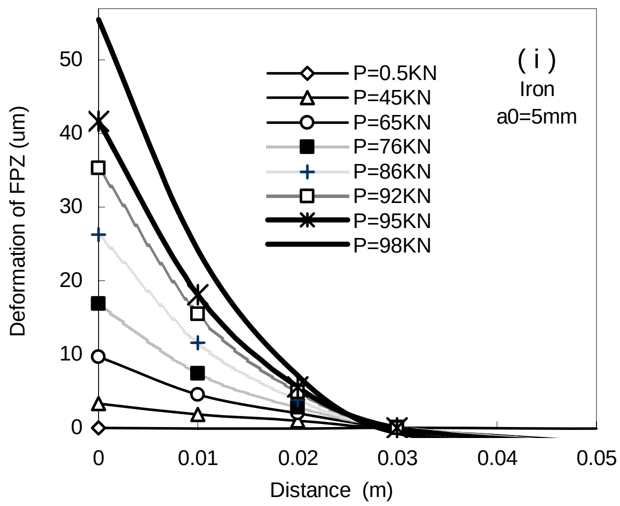
Of course, the cohesive crack deformation distribution can be seen from the graphs of test on fracture process zone. Obviously, no matter what kind of material, it is the deformation changing with the load increasing.

3 The constitutive relation of cohesive stress and opening displacement of fictitious crack

From above formula (1) and (3) for the cohesive crack model, the simple calculation equations can be worked out. As the ‘N’is selected to be 2 in Eq(1,3), the distribution of cohesive stress or opening displacement of virtual crack can be expressed as following.

$$u(x) = \frac{8K_I}{3E}\left(\frac{x}{b}\right)^{3/2}\sqrt{\frac{2b}{\pi}} \qquad \text{(4a)}$$

$$\sigma(x) = \frac{K_I\sqrt{2}}{b}\sqrt{\frac{b-x}{\pi}} \tag{4b}$$



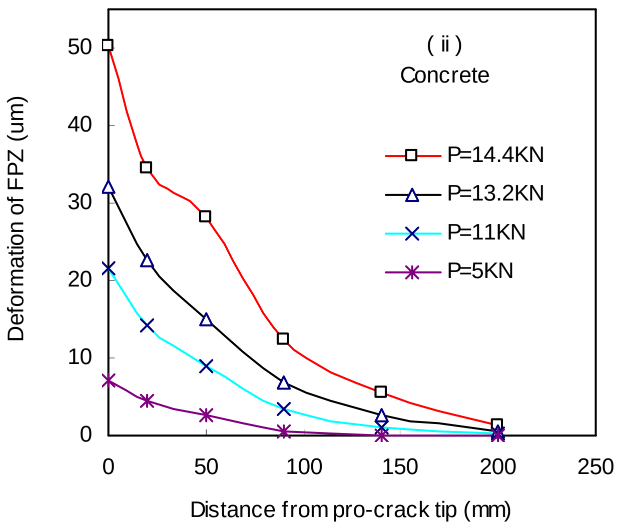


Figure 4. The deformation distribution along the crack ligament of （i） iron （ii） concrete under different load

If the coordinate x is taken away from Eq 4 (a,b), the constitutive equation of fictitious crack can be worked out between cohesive stress and opening displacement of cohesive crack. Refering to Fig 3 and 4, the displacement distribution can be calculated from the cast iron test data in Ref [8] through above Eq 4 (a). In Fig 5, the theoretical curve is compared with the experimental results of iron specimen, and the calculation curve is basely consistent with experiment data. In Fig 6, it is the constitutive relation curves between the cohesion stress and crack opening displacement for concrete specimens under different load. There is a little difference about concrete constitutive relation between different loading condition, but it also be consistent with reference early results [9,10].

4 The bearing capacity estimate of structure with crack

As the fracture process zone length changing with variable load in experiment specimen, the quantitative relationship of its loading and damage zone length can be worked out according to the fracture criterion of Double-K and the theoretical analysis results of cohesive crack [9]. For fracture specimen configuration such as three-point bending beam with crack, the equation can be written as followings.

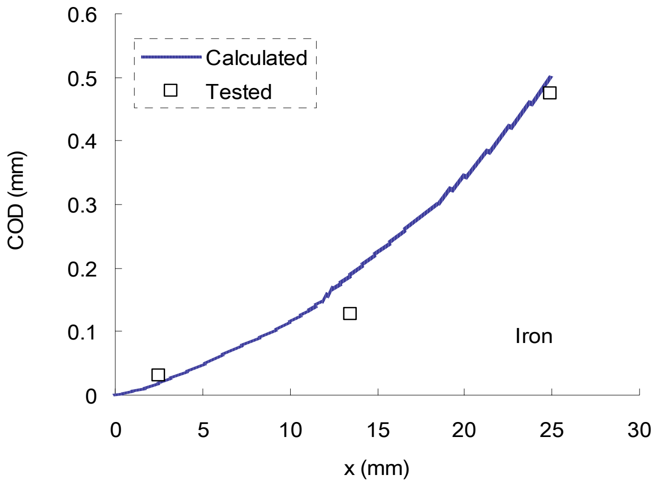


Figure 5. The compare of theory curve of cast iron cohesive crack opening displacement with experimental data

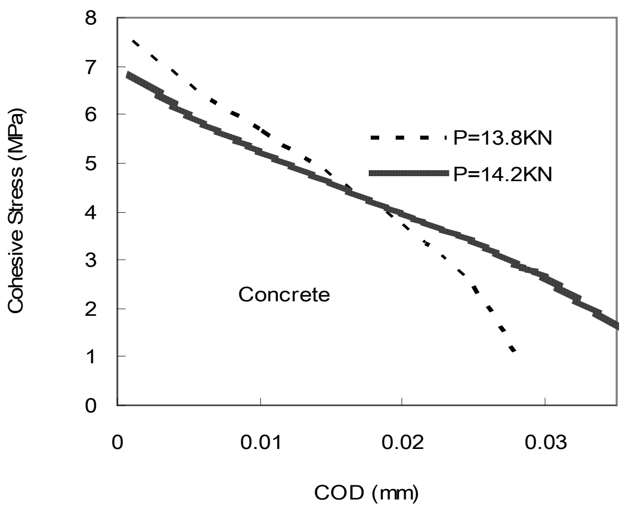


Figure 6. The constitutive relation between cohesive stress and opening displacement of cohesive crack for concrete being under different load

$$P = \frac{2Bh^2\left(k_0 + \eta\sqrt{\frac{b}{2\pi}}\right)\left(1 + 2\frac{a_0 + b}{h}\right)\left(1 - \frac{a_0 + b}{h}\right)^{3/2}}{3s\sqrt{a_0 + b}\left[1.99 - \frac{a_0 + b}{h}\left(1 - \frac{a_0 + b}{h}\right)\left(2.15 - 3.93\frac{a_0 + b}{h} + 2.7\left(\frac{a_0 + b}{h}\right)^2\right)\right]} \quad (4)$$

Where, k_0 is the stress intensity factor of initiation crack; and a_0 is for the prefabricated crack length. Refer to Fig 3, the symbol *s* and *h* are span and high respectively. Regarding to the specimens with pre-crack length 5mm and 20 mm, the changing length of damage zone respectively with loading was shown in Fig 7 for iron materials [8]. Continuous curve is the theoretical calculation value, and scatter icons indicate experimental test results. From the figure it can be seen that theories values are in good agreement with experiment, and the maximum bearing capacity value of structure with cracked can be estimated from the curve peak.

5 Conclusion

(1) Through nonlinear crack model analysis, both the cohesive stress distribution and opening displacement of fictitious crack can be calculated by analytical equation.

(2) Regarding the cohesive crack model, specimen of cast iron or concrete was observed or tested in order to investigate the mechanics behavior of quasi-brittle materials. And its theoretical calculation is consistent with experimental observation.

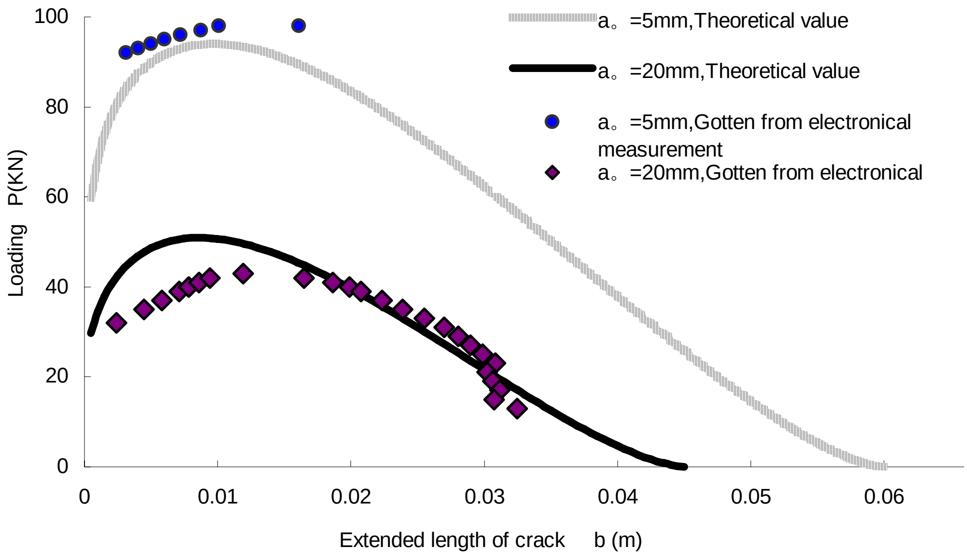


Figure 7. The relation between the loading of three point bending beam with crack and the length of extended crack for iron material

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