Experiments on cracks under spatial loading

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Abstract

The basic loading cases (Mode I, Mode II and Mode III) of cracks are generally defined by the near-field solutions for the stress distribution at the crack front. Cracks, whose stress near fields are symmetric due to geometry and/or loading of the structure, are called Mode I-cracks. In case of spatial loaded cracks the stress fields near the crack tip are unsymmetrical. The fracture mechanical treatment of such Mixed-Mode-loaded cracks is consequently more complicated as of pure Mode I-loaded cracks.

For experimental investigations of 3D-Mixed-Mode-loaded cracks the CTSR-specimen (Compact-Tension-

Shear-Rotation-specimen) will be proposed. The CTSR-specimen with the corresponding loading device enables the generation of pure Mode I, pure Mode II, pure Mode III and several combinations of the three basic fracture modes.

In this paper the CTSR-specimen, the loading device and various experimental results for fracture and fatigue loading situations are illustrated. Furthermore these results will be compared with existing fracture criteria for 3D-Mixed-Mode-problems.

Keywords

3D-Mixed-Mode, fracture, fatigue, CTSR-specimen

1. Introduction

In real structures fracture processes are in many cases of a three dimensional character. Different defects, e.g. pre-cracks, which often exist in materials and structures, may experience complex loading conditions. Cracks under complex loading are subjected to a superposition of the three basic fracture modes I, II and III. In this case cracks tend to kink and/or twist, Fig. 1.



Figure 1. Crack screwing under combined loading

For static loading the stress field near the crack front (Eq. 1) here is not only defined by the stress intensity factor $K_{\rm I}$, but also by $K_{\rm II}$ and $K_{\rm III}$. In this case the calculation of a comparative stress intensity factor $K_{\rm V}$ is important (Eq. 2) [1].

$$\sigma_{ij} = \frac{1}{\sqrt{2\pi \cdot r}} \cdot \left[K_{I} \cdot f_{ij}^{I}(\varphi) + K_{II} \cdot f_{ij}^{II}(\varphi) + K_{III} \cdot f_{ij}^{III}(\varphi) \right]$$
(1)

$$K_{\rm v} = \frac{K_{\rm I}}{2} + \frac{1}{2} \cdot \sqrt{K_{\rm I}^2 + 5,336 \cdot K_{\rm II}^2 + 4 \cdot K_{\rm III}^2}$$
(2)

In this regard the fatigue crack growth then is governed by the cyclic stress intensity factors ΔK_{II} , ΔK_{II} and ΔK_{III} respectively the cyclic comparative stress intensity factor ΔK_{V} , which can be derived from Eq. 2:

$$\Delta K_{\rm V} = \frac{\Delta K_{\rm I}}{2} + \frac{1}{2} \cdot \sqrt{\Delta K_{\rm I}^2 + 5,336 \cdot \Delta K_{\rm II}^2 + 4 \cdot \Delta K_{\rm III}^2}$$
(3)

The *K*-concept for spatial Mixed-Mode-loading is based on the fact that unstable crack growth occurs, if the comparative stress intensity factor K_V reaches the fracture toughness value K_{IC} for Mode I. In case of fatigue crack growth the crack is propagable, if the cyclic comparative stress intensity factor ΔK_V for spatial Mixed-Mode-loading reaches or exceeds the threshold value ΔK_{th} . Both contexts can be illustrated clearly in a K_I - K_{II} - K_{II} -diagram, Fig. 2.

Unstable crack growth will occur, if a local loading condition along the crack front reaches a point on the fracture limit surface. Fatigue crack growth or stable crack growth develops, if points characterizing the local crack front loading conditions are lying between the threshold and the fracture limit surfaces.



Figure 2. Fatigue crack growth limits at spatial Mixed-Mode-loading

Precisely because the fracture mechanical treatment of such three dimensional Mixed-Mode-loaded cracks is very complicated compared to pure Mode I-loaded cracks, the prediction of the above mentioned 3D-fracture-process is not yet well understood.

In addition there is a shortage of experimental investigations and findings regarding general spatial Mixed-Mode-fracture in order to compare the correlation between the experimental results and

existing fracture criteria for 3D-Mixed-Mode-loading conditions. Experimental investigations on spatial Mixed-Mode-loading are also necessary, in order to get a solid basis, on which, on the one hand, the existing fracture criteria for 3D-Mixed-Mode could be enhanced, on the other hand, new fracture criteria or hypotheses including the desired understanding could be established.

2. Mixed-Mode-criteria for 3D-loading conditions

For a complete prediction of crack growth behavior under combined loading not only the crack growth direction is required, but also the determination of comparative stress intensity factors, like K_V or ΔK_V , see Eq. 2 and Eq. 3.

Compared to characteristic fracture mechanical values, e.g. threshold value or fracture toughness value, conclusions could be drawn on crack growth behavior.

In this purpose some criteria for characterizing the crack growth under spatial Mixed-Mode-loading were established:

- Crack growth criterion by POOK [2-4]
- σ_1 '-criterion by SCHÖLLMANN et al. [5, 6]
- Criterion by DHONDT [7]
- 3D-criterion by RICHARD [1]

A comparison of these criteria is given in e.g. [1].

3. Experiments on cracks under general loading

In order to understand the 3D-fracture-process completely not only further theoretical, but also experimental investigations have to be performed. Currently several types of specimens are available for experimental investigations of fatigue crack growth and fracture under various Mixed-Mode-loading conditions [8-17]. None of these specimens enables investigating the full range of all basic fracture modes or any combinations thereof.

But the AFM-specimen with the corresponding loading device and the so-called CTSR-specimen (Compact-Tension-Shear-Rotation-specimen) in combination with the special loading device fulfill these high requirements. Some of the experimental results of both specimen types are shown below.

3.1. Experiments on AFM-specimen

As already mentioned the AFM-specimen, developed by RICHARD, allows the investigation of crack problems under general loading conditions by using a simple uniaxial testing machine [18]. In the past experiments under static load were performed in order to determine the fracture limit surface and the crack deflection angles φ_0 and ψ_0 [19], see Fig. 3.

For fatigue tests this loading device is less suitable, due to its high weight and deformation. Only low test frequency, which leads to high test duration, can be realised for fatigue experiments. Making use of this background the CTSR-specimen (Fig. 4a) with the corresponding loading device (Fig. 4b) was developed.



Figure 3. Crack growth direction under different loading conditions

3.2. Experiments on CTSR-specimen

This concept was developed especially for investigating Mixed-Mode-loaded cracks under cyclic loading. The loading device is designed in a way, with which any ratio of Mode I to Mode II/Mode III can be generated by the adjustment of the loading angle α (see Fig. 5). Furthermore, the ratio of Mode II- or Mode III-load can be set by rotating the so-called turret inside the loading device (see Fig. 6).



Figure 4. a) CTSR-specimen b) Loading device for CTSR-specimen



Figure 5. Setting of the ratio of Mode I to Mode II/Mode III

The superposition of Mode I, Mode II and Mode III takes place by the adjustment of both angles α and γ . Thereby the load line of action always passes through the center of the specimen.



Figure 6. Setting of the ratio of Mode II- or Mode III-load

In this paper some experimental results on CTSR-specimen will be presented. On the one hand experiments on PMMA have been performed, in order to determine the fracture limit surface (cf. Fig. 2) for this material. On the other hand fatigue tests were performed on an EN AW-7075-T651 aluminium alloy, in order to identify the threshold value surface. The results of both experiment types are illustrated and discussed in the next chapter.

4. Results of experimental investigations

Figure 7 illustrates the measured fracture toughness values for PMMA material under several Mixed-Mode-loading combinations.

The fracture toughness values K_{IIIC} for pure Mode III-loading, measured on CTSR-specimen, exhibits a significant variation in comparison with the 3D-criterion by RICHARD. The resulting values for Mode III are around factor 2.7 above the hypothesis. Similar significant variations have already been observed on other specimen, e.g. [15]. Furthermore it is noticeable, that the difference between measured $K_{\rm C}$ values and the hypothesis decreases with decreasing Mode III-ratio. The

values are very close to the criterion as soon as there is no Mode III-loading. The same trend can be observed by the comparison with other criteria, mentioned in this paper. All these criteria are conservative.



Figure 7. Comparison of experimental results with 3D-criterion by Richard

Due to these results and the significant difference to the hypothesis, an approximation of the 3D-criterion by Richard on the fracture toughness values for PMMA material is proposed. In Figure 8 the approximation of the criterion by RICHARD is shown. Here the α_2 parameter was changed to 0.36. This approximation is still conservative, but agrees very well with the experimental results.

In addition, fatigue tests on an ENAW-7075-T651 aluminium alloy were performed. The experimentally measured threshold values for different Mixed-Mode-loading conditions are pictured in Figure 9. Compared to the threshold value surface by RICHARD the experimentally determined threshold values depict also a significant variation under pure Mode III-loading. Here the $\Delta K_{\text{III,th}}$ values are around factor 2.2 above the hypothesis.



Figure 8. Approximation of the 3D-criterion by Richard on experimental results



Figure 9. Threshold values for several Mixed-Mode-loading conditions

Moreover, for a complete description of crack growth behavior under general Mixed-Mode-loading the crack kinking and twisting angles were established by using an optical 3D-scanner. In order to prove the reliability of the criteria the determined crack deflection angles φ_0 and ψ_0 were compared with the predictions of the criteria. The comparison of the crack kinking angle φ_0 is shown in barycentric coordinates in Figure 10.



Figure 10. a) Comparison of kinking angle with criterion by RICHARDb) Comparison of kinking angle with criterion by SCHÖLLMANN et al.c) Comparison of kinking angle with criterion by DHONDT

d) Comparison of kinking angle with criterion by POOK

The measured crack kinking angle φ_0 coincides very well with the predictions of the hypotheses by RICHARD as well as by SCHÖLLMANN et al. and by DHONDT (see Fig. 10a-c). The maximal deviation here one finds at pure Mode III-loading. In Comparison with the predictions of the criterion by POOK the real crack kinking angle differs considerably from its predictions as soon as Mode III-loading part occurs (Fig. 10d).

The measurement results of the crack twisting angle ψ_0 exhibit also a very well agreement with the criterion by RICHARD as well as by SCHÖLLMANN et al. (Fig. 11a, 11b). The predictions of the criteria by DHONDT and by POOK show the greatest deviations from the real crack twisting angle (Fig. 11c, 11d).





5. Conclusion

In this paper the suitability for experimental investigations under 3D-Mixed-Mode-loading conditions of the CTSR-specimen in combination with its loading device was presented. The experimental results show obviously higher threshold and fracture toughness values for high Mode III-loading ratios than by criteria predicted. The most exactly predictions indicate the criteria by RICHARD and by SCHÖLLMANN et al.

These criteria give the best predictions regarding the crack kinking and twisting angle. Due to their exactness of predictions, they should be implemented in numerical calculation programs.

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