The effect of thickness on components of the non-singular T-stress under mixed mode loading

Yury G. Matvienko[1,*], Alexander S. Chernyatin[1], Igor A. Razumovsky[1], Hui-Ji Shi[2], Zhao-Xi Wang[2,3]

[1] Mechanical Engineering Research Institute of the Russian Academy of Sciences, 101990, Russia
[2] Department of Engineering Mechanics, Tsinghua University, 100084, China
[3] Suzhou Nuclear Power Research Institute, 215004, China
* Corresponding author: ygmatvienko@gmail.com

Abstract. Distributions of the singular (K_I, K_{II}) and the non-singular (T_{xx}, T_{zz}) terms along the 3D crack front are analyzed for the case of mixed (mode I and II) loading. Relations between the singular, non-singular terms and specimen thickness are calculated in the specimen middle plane for different values of the mixity parameter. The values of the mixity parameter are varied through 1 (pure mode I), 0.75, 0.5, 0.25–0 for pure mode II. It is shown that the increase of thickness in 8 times influences on values of K_I , K_{II} and T_{zz} . The strong effect of thickness and mixed mode loading conditions on the T_{zz}-stress is observed. At the same time, there is not the effect of thickness on the non-singular T_{xx} at the same loading conditions.

Keywords T-stress, Mixed mode, In-plane and out-of-plane constraint, Specimen thickness

Introduction

The development of modern fracture mechanics is closely related to the formation of two-parameter approaches and criteria, which introduce and nonsingular components of the stress field at the crack tip, such as the T-stress. Primarily it is due to the fact that the fracture toughness of structural materials shows a significant effect of the specimen geometry and thickness, load conditions, etc. (e.g. [1, 2]). The results of analytical and numerical calculations show that stress fields in the vicinity of the crack tip, in many cases, are strongly dependent on the non-singular terms of the stress field [3, 4]. Introduction of an additional parameter, namely, the nonsingular T_{xx}-stress in the criteria allows to eliminate the above-mentioned effects [5, 6]. It should be noted that the T_{xx}-stress describes in-plane constraint. In contrast to the T_{xx}-stress, out-of-constraint is connected with the T_{zz}-stress. However, investigation of joint influence of the nonsingular T-stress (T_{zz} and T_{xx}) terms on the fracture mechanics parameters is not carried out yet.

Present paper deals with numerical analysis of three-dimensional stress fields in the vicinity of through-thickness crack tip under mixed mode (I + II) loading. The combined effect of thickness on the effective stress intensity factor and T-stress terms are discussed.

Statement of the problem

An analysis of 3D stress field in the vicinity of the crack front is performed on the CCCD-specimen with the thorough-thickness crack of arbitrary space orientation. The specimen is loaded by 2 compressive forces acting in the vertical direction (Fig. 1). This configuration of the specimen is very suitable to create different conditions of mixed loading [7].

Finite element method is employed to estimate the 3D stress field in the vicinity of crack front. The orientation of the crack plane with respect to the disk can be arbitrary, since it is determined by both the angle α (Fig. 1) and the other two spatial angles. Changing the combination of angles can provide almost any relationship between the magnitudes of the K_I, K_{II}, K_{III}. The geometrical parameters of the model are given in dimensionless form, namely, $b=B/R$, $l=a/R$ (Fig. 1). Note that the developed procedure for the construction of the model allows taking into account the spatial symmetry of the problem and getting the best mesh in terms of accuracy and speed of calculation.

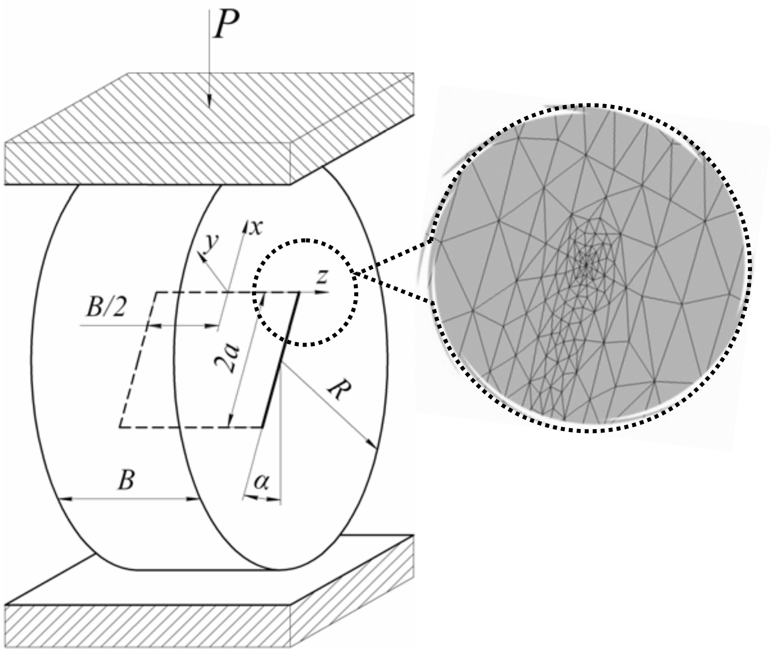

Figure 1. Model of the CCCD-specimen

The stress field in the vicinity of the crack tip of mixed type (I+II) can be represented in the form [8]:

$$\begin{aligned}
\sigma_{xx} &= \frac{1}{\sqrt{2\pi r}}\left[K_I \cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) - K_{II}\sin\frac{\theta}{2}\left(2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)\right] + T_{xx}, \\
\sigma_{yy} &= \frac{1}{\sqrt{2\pi r}}\left[K_I \cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) + K_{II}\sin\frac{\theta}{2}\left(\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)\right], \\
\sigma_{xy} &= \frac{1}{\sqrt{2\pi r}}\left[K_I \sin\frac{\theta}{2}\left(\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right) + K_{II}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)\right], \qquad (1) \\
\sigma_{zz} &= \frac{2\nu}{\sqrt{2\pi r}}\left[K_I \cos\frac{\theta}{2} - K_{II}\sin\frac{\theta}{2}\right] + T_{zz}, \\
T_{zz} &= E\varepsilon_{zz} + \upsilon T_{xx}.
\end{aligned}$$

where x is the direction formed by the intersection of the plane normal to the crack front and the crack plane, y is the direction orthogonal to the crack plane, z is the direction orthogonal to x and y directions (tangent to crack front); r and θ are the in-plane polar coordinates in plane $x0y$. Here, E and v are the elastic modulus and Poisson's ratio, respectively.

The value of the angle α and the corresponding parameters M_e, which characterize loading mode mixity, for the specimen under consideration are shown in Table 1. Note that in the case of mode I crack (K_{II}=0) M_e=1, in the case of mode II crack (K_I=0) M_e=0.

$$M_e = \frac{2}{\pi} arctg(\frac{K_I}{K_{II}}) \qquad (2)$$

Table 1. The mixity parameter and crack orientation

α, °	0	5	10,5	18	27
M_e	1	0,75	0,5	0,25	0

Determination of K_I, K_{II}, T_{xx}, T_{zz}

The evaluation of the stress intensity factor (SIF) is based on the well-known approach that includes the calculating of the SIF in a number of points (at varied *r*) using relations obtained from Eq. (1):

$$K_I = \sqrt{\frac{\pi r}{2}}\left(2\sigma_{xx}|_{\theta=0} - \sigma_{xx}|_{\theta=-\pi} - \sigma_{xx}|_{\theta=+\pi}\right),$$
$$K_{II} = \sqrt{\frac{\pi r}{8}}\left(\sigma_{xx}|_{\theta=-\pi} - \sigma_{xx}|_{\theta=+\pi}\right) \quad (3)$$

and extrapolation of the obtained values of the SIFs to the point r=0.

The computational procedure considers the nodes of the finite element mesh as the calculation points, but the nodes are located at some small distance from the crack front. To obtain the distribution of the SIF along the crack front, this procedure is used for a number of planes (x0y) orthogonal to the crack front. Their location is characterized by local coordinate *s* along the front and starts from the center of the crack front.

To assess the accuracy of the values of SIF, the comparison of magnitudes of the *J*-integral, calculated as contour integral (Eq. 4a) and based on the known relationship (Eq. 4b), has been done.

$$J = \int_{\Gamma}\left(W(x,y)dx - \sigma_{ij}n_j\frac{du_i}{dy}ds\right) \quad (4a)$$

$$J = \frac{1-\upsilon}{E}\left[K_I + K_{II} + (1+\upsilon)K_{III}\right] \quad (4b)$$

The calculation of the T_{xx}- and T_{zz}-stress is performed using the stresses in the points on the crack surface:

$$T_{xx} = \frac{1}{2}\left[\sigma_{xx}\big|_{\theta=-\pi} + \sigma_{xx}\big|_{\theta=+\pi}\right]$$
$$T_{zz} = \frac{1}{2}\left[\sigma_{zz}\big|_{\theta=-\pi} + \sigma_{zz}\big|_{\theta=+\pi}\right] \quad (5)$$

The determination of T_{xx} T_{zz} is similar to the procedure for SIF including extrapolation to the point r=0.

The special macros that carry out the automatic computation of the SIF, *J*-integral and T-stresses (on referred above relations) along the crack front are created in ANSYS environment.

Estimation of the accuracy of the numerical model and computational procedures is based on comparison of the calculated values of the SIF and the T-stresses with one presented in [7] for the CCCB-specimen in case of R=50 mm, b=0,8, l=1/3. Comparison of the results shows that there is good coincidence and only in some cases the relative difference is close to 10%.

Discussion of the results

The results of distribution of the SIF and the T-stress along the crack front for different mixed mode loading conditions are summarized in Fig. 2 and 3. To study the effect of specimen thickness on T_{zz}-stress, the compressive load was adjusted so that the T_{xx}-stress was remained approximately constant at variation of the specimen thickness (b= 0,2; 0,4; 0,8 and 1,6) at constant l=1/3 (for the corresponding values of mixity parameter M_e).

The increase of the specimen thickness is 8 times greatly reduces (in absolute value) the T_{zz}-stress (up to 75% at M_e=0,25), whereas the value of T_{xx} is virtually unchanged (Table 2). The predicted results are consistent with the data of numerical calculations [6]. Thus, magnitudes of the T_{zz}- stress have significant correlation with thickness of the specimen and allows taking into account the level of the out-of-constraint which should be included into assessment of the fracture toughness of full-scale structures.

Table 2. The effect of thickness on the SIF and T-stresses for M_e=0,25

b	K_I, MPa·m^{0,5}	K_{II}, MPa·m^{0,5}	T_{xx}, MPa	T_{zz}, MPa
0,2	0,105	0,252	-2,691	-0,309
0,4	0,104	0,255	-2,681	-0,206
0,8	0,099	0,254	-2,647	-0,121
1,6	0,087	0,246	-2,581	-0,079

To reflect mixed mode loading conditions, an effective SIF (K_{eff}) is introduced into consideration as follows

$$K_{eff} = \sqrt{K_I^2 + K_{II}^2} \tag{6}$$

There are dependence of K_{eff} on T_{xx} and T_{zz} in the middle plane of the crack front (s=0) for varied parameters M_e at different thickness (b=0,2; 0,4; 0,8; 1,6) of the specimen and constant value of l=1/3 (Fig. 2 and 3).

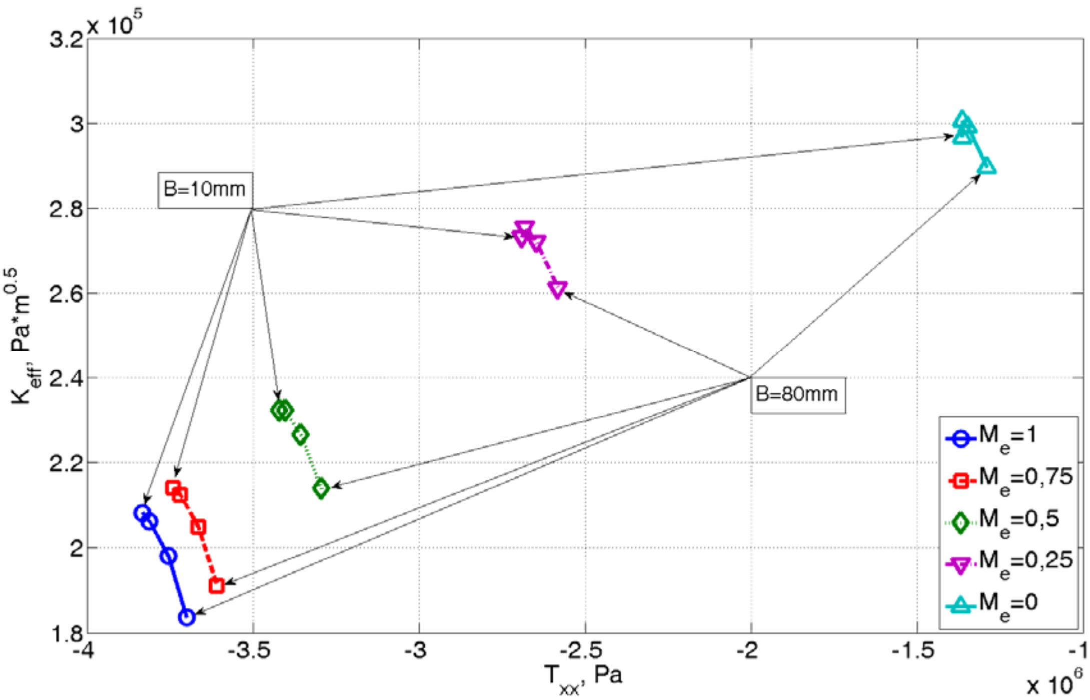

Figure 2. The effect of T_{xx}, M_e and thickness on K_{eff} at s=0

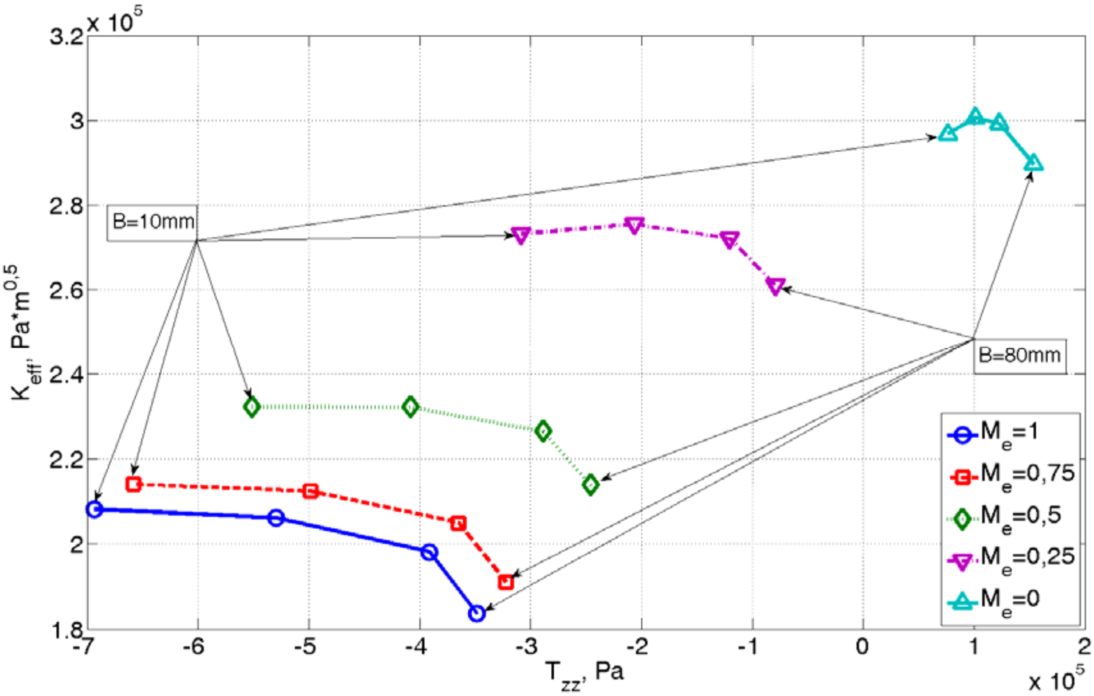

Figure 3. The effect of T_{zz}, M_e and thickness on K_{eff} at s=0 at s=0

Conclusions

The three-dimensional stress field ahead of the through-thickness crack under mixed mode (I+II) loading conditions is analyzed. The in-plane and out-of-plane constraint effect are discussed by means of the non-singular terms, namely, T_{xx}-stress and T_{zz}-stress, respectively. The distribution of the singular and non-singular terms along the crack front has been predicted. The significant effect of specimen thickness on the T_{zz}-stress in the middle plane of the crack front has been observed.

Acknowledgements

The authors acknowledge the support of the Russian Foundation of Basic Research (Grant N 12-08-91158-NSFC_a).

References

[1] S. Liu, Y.J. Chao, Variation of fracture toughness with constraint, International Journal of Fracture, 124 (2003) 113–117.
[2] X.K. Xhu, Y.J. Chao. Specimen size requirements for two-parameter fracture toughness testing, International Journal of Fracture, 135 (2005) 117–136.
[3] V.F. González-Albuixech, E. Giner, J. Fernández-Sáez, A. Fernández-Canteli, Influence of the t_{33}-stress on the 3-D stress state around corner cracks in an elastic plate, Engineering Fracture Mechanics, 78 (2011) 412–427.
[4] Yu.G. Matvienko, V.N. Shlyannikov, N.V. Boychenko, In-plane and out-of-plane constraint parameters along a three-dimensional crack-front stress field under creep loading, Fatigue and Fracture of Engineering Materials and Structures, 36 (2013) 14–24.
[5] H.M. Meliani, Yu.G. Matvienko, G. Pluvinage, Two-parameter fracture criterion ($K_{\rho,c}$-$T_{ef,c}$)

based on notch fracture mechanics, International Journal of Fracture, 167 (2011) 173-182.
[6] T. Meshii, T. Tanaka, Experimental T_{33}-stress formulation of test specimen thickness effect on fracture toughness in the transition temperature region, Engineering Fracture Mechanics, 77 (2010) 867–877.
[7] M.R.M. Aliha, M.R. Ayatollahi, D.J. Smith, M.J. Pavier, Geometry and size effects on fracture trajectory in a limestone rock under mixed mode loading, Engineering Fracture Mechanics, 7 (2010) 2200–2212.
[8] T. Nakamura, D.M. Parks, Determination of elastic T-stress along three-dimensional crack fronts using an interaction integral, International Journal of Solids and Structures, 29 (1992) 1597–1611.