Cracks under Mixed Mode loading: Questions and solutions for isotropic and graded materials

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Abstract In reality cracks often initiate and grow due to mixed mode loading. This paper deals with questions about stable and unstable crack growth and the crack growth direction under multi-axial loading conditions. For homogeneous materials many criteria and experimental confirmations exist. This paper shows a selection of these solutions and some experimental investigations. In practice innovative manufacturing and application-oriented products considering lightweight construction gain increasingly in importance. In this context structures with graded material properties are produced. However there does not exist many concepts for these graded materials. Within this paper fracture and fatigue criteria and experimental findings are presented. In particular the TSSR^B-concept is used for the determination of crack growth in fracture mechanical graded materials.

Keywords Mixed Mode, fracture criteria, isotropic materials, graded materials, TSSR-concept

1. Introduction

If the two basic fracture modes (Mode I and Mode II) temporarily or permanently occur in combination, as indicated in Fig. 1, local plane Mixed Mode loading conditions at cracks can be observed.

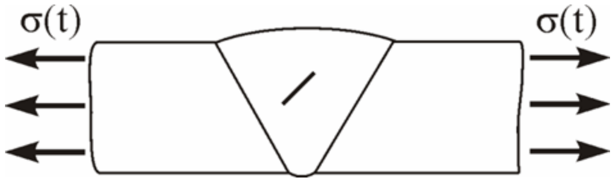


Figure 1. Mixed Mode loaded crack in a welded structure

For a fail-safe dimensioning it is important to know among other things if such a crack is able to grow, how fast and whereto does the crack grow and when unstable crack growth occurs. Answers can be found using crack propagation concepts which will be briefly described in the following chapter. An extensive description can be found in [1, 2].

2. Crack growth in isotropic and homogeneous materials

An isotropic and homogeneous structure is defined by fracture mechanical and elastic material properties which are independent of place and direction, i.e. they are the same for the whole structure. With regard to the subsequent consideration of fracture criteria for graded materials and their complexity only 2D-concepts will be regarded within this paper.

2.1. Concepts for two-dimensional Mixed Mode crack growth

In the following two theoretical concepts are presented which enable the determination of unstable crack growth. Information about further crack propagation concepts for 2D and 3D Mixed Mode situations can be found in [1-5].

2.1.1. Concept according to Erdogan and Sih

The concept of the maximum tangential stress (MTS) by Erdogan and Sih [4, 5] enables the determination of the crack growth direction as well as the start of unstable crack propagation on the basis of the tangential stress σ_φ, presented in Eq. (1).

$$\sigma_\varphi = \frac{K_I}{\sqrt{2\pi r}}\cos^3\frac{\varphi}{2} - \frac{K_{II}}{\sqrt{2\pi r}}\frac{3}{2}\sin\varphi\cdot\cos\frac{\varphi}{2} \tag{1}$$

The concept assumes that the crack propagates in the direction $\varphi_{0,MTS}$ which is perpendicular to the maximum tangential stress $\sigma_{\varphi max}$. Eq. (2) defines the kinking angle $\varphi_{0,MTS}$ according to the MTS-concept which depends on the loading condition.

$$\varphi_{0,MTS} = -\arccos\left(\frac{3K_{II}^2 + K_I\sqrt{K_I^2 + 8K_{II}^2}}{K_I^2 + 9K_{II}^2}\right) = -\arccos\left[\frac{3\left(\frac{K_{II}}{K_I}\right)^2 + \sqrt{1+8\left(\frac{K_{II}}{K_I}\right)^2}}{1+9\left(\frac{K_{II}}{K_I}\right)^2}\right] \tag{2}$$

The crack grows unstable if $\sigma_{\varphi max}$ reaches a material limit value $\sigma_{\varphi c}$ or if a maximum comparative stress intensity factor K_{Vmax} (Eq. 3), determined by the tangential stress σ_φ (Eq. 1), reaches the fracture toughness K_{IC}.

$$K_{Vmax} = \lim_{r\to 0}\left(\sigma_{\varphi max}\sqrt{2\pi r}\right) = \lim_{r\to 0}\left(\sigma_\varphi(\varphi_0)\sqrt{2\pi r}\right) = \cos\frac{\varphi}{2}\left(K_I\cos^2\frac{\varphi_0}{2} - \frac{3}{2}K_{II}\sin\varphi_0\right) \tag{3}$$

Eq. (4) shows the criterion for unstable crack growth.

$$K_{Vmax} = K_{IC} \tag{4}$$

Due to cyclic loading the tangential stress σ_φ (Eq. 1) is transformed to the cyclic tangential stress $\Delta\sigma_\varphi$ (Eq. 5) with the cyclic stress intensity factors ΔK_I and ΔK_{II}.

$$\Delta\sigma_\varphi = \frac{\Delta K_I}{\sqrt{2\pi r}}\cos^3\frac{\varphi}{2} - \frac{\Delta K_{II}}{\sqrt{2\pi r}}\frac{3}{2}\sin\varphi\cdot\cos\frac{\varphi}{2} \tag{5}$$

Fatigue crack growth starts if the cyclic comparative stress intensity factor $\Delta K_{Vmax}=\Delta\sigma_\varphi\sqrt{2\pi r}$ reaches the Threshold value ΔK_{th}. In case of cyclic loading unstable crack growth occurs if $K_{Vmax} = K_{IC}$ (Eq. 4) or if $\Delta K_{Vmax} = \Delta K_{IC} = K_{IC}\cdot(1-R)$, with the stress ratio $R = \sigma_{min}/\sigma_{max} = K_{min}/K_{max}$.

2.1.2. Concept according to Richard

The general fracture concept of Richard [1, 5, 6] is very practical and adaptive and can be used for different materials. The concept is based on a comparative stress intensity factor K_V. This value depends on the stress intensity factors K_I and K_{II} (Eq. 6).

$$K_V = \frac{1}{2}K_I + \frac{1}{2}\sqrt{K_I^2 + 4(\alpha_1 K_{II})^2} \tag{6}$$

The material parameter α_1 depends on the ratio of the fracture toughness of Mode I K_{IC} and the fracture toughness of Mode II K_{IIC}. If α_1 is set to 1.155 an excellent approximation of the fracture limit curve of the maximum tangential stress criterion is obtained. Unstable crack growth occurs as soon as K_V exceeds the fracture toughness K_{IC} for Mode I.

Furthermore the concept also enables the determination of the kinking angle φ_0 (Eq. 7), with

$\varphi_0<0$ for $K_{II}>0$, $\varphi_0>0$ for $K_{II}<0$ and K_I always larger than 0.

$$\varphi_0 = \mu\left[A\frac{|K_{II}|}{|K_I|+|K_{II}|}+B\left(\frac{|K_{II}|}{|K_I|+|K_{II}|}\right)^2\right] \tag{7}$$

In this empirical formula the parameters have to be set to: A = 155,5° and B = -83,4°.

2.2. Comparison of the theoretical concepts and experimental findings

Experimental determined kinking angles φ_0 for different almost isotropic materials (for example PMMA, Araldit B, PVC, AlZnMgCu) are shown in Fig. 2. The kinking angle doesn't depend on the material. However the K_{II}/K_I-ratio is important. Furthermore the kinking angles determined by different fracture criteria (for example Erdogan and Sih, Richard) are presented in dependency of the Mixed Mode ratio $K_{II}/(K_I+K_{II})$. It can be seen that these crack propagation criteria are able to predict the crack kinking angle for isotropic and nearly isotropic material sufficiently exact [7]. For more information about experimental investigations see [5, 8].

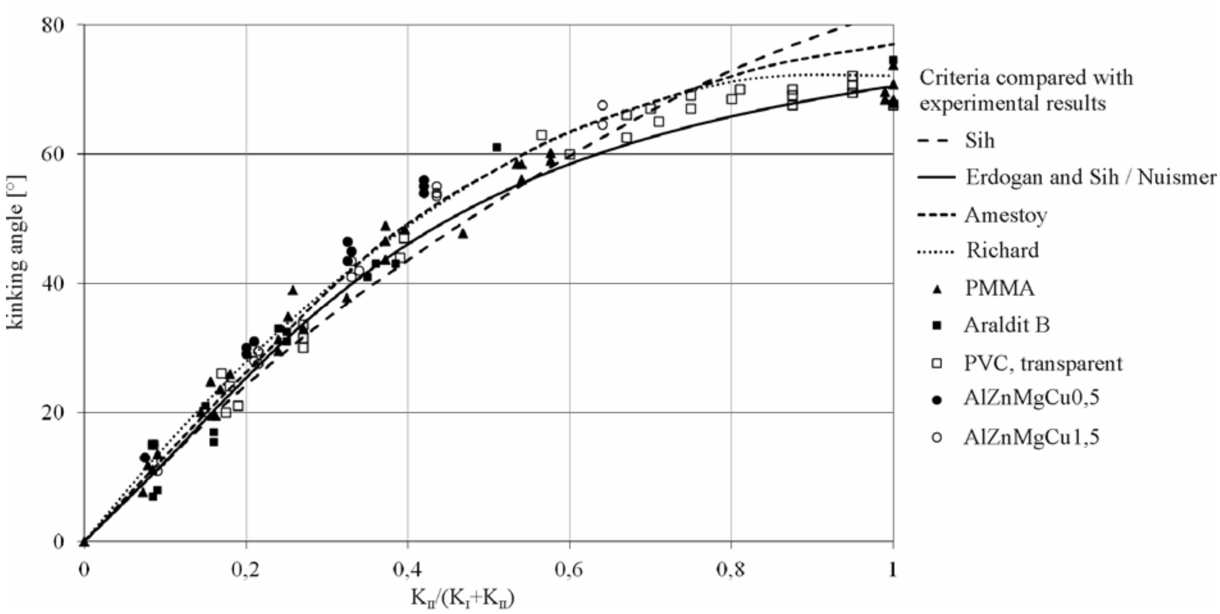
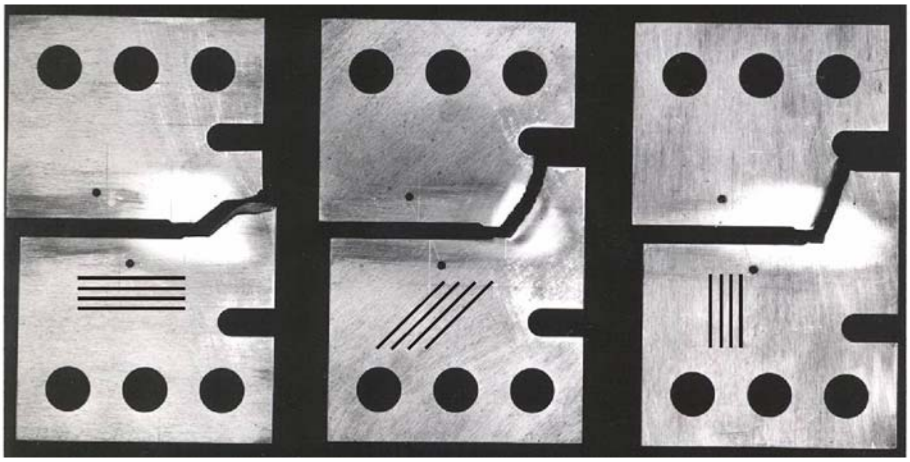


Figure 2. Kinking angle from criteria for 2D Mixed Mode cracks in comparison to experimental data

Fig. 3 shows aluminum alloy specimens which vary in their rolling direction. The specimen in Fig. 3a was rolled in crack direction, in Fig. 3b diagonal and in Fig. 3c perpendicular to the crack direction. After the production and rolling process these specimens were investigated experimentally. The resulting kinking angles φ_0 of these anisotropic structures are presented in Fig. 4. The results show that the crack propagation concepts are not able to predict the kinking angles of the rolled aluminum alloy specimens very well. The concepts are not able to consider the predominant direction due to the rolling process.



a) b) c)

Figure 3. Aluminum alloy specimens with special rolling direction: a) in crack direction, b) diagonal, c) perpendicular to the crack direction

These results lead to the knowledge that the presented concepts are able to predict the crack growth behavior very well as long as the structure is a homogeneous and isotropic one. For the description of materials with a graded property, for example in form of the shown predominant direction, other concepts have to be used considering the influence of the material gradation. Therefore hypotheses describing Mixed Mode crack growth in graded materials are presented in the following contribution.

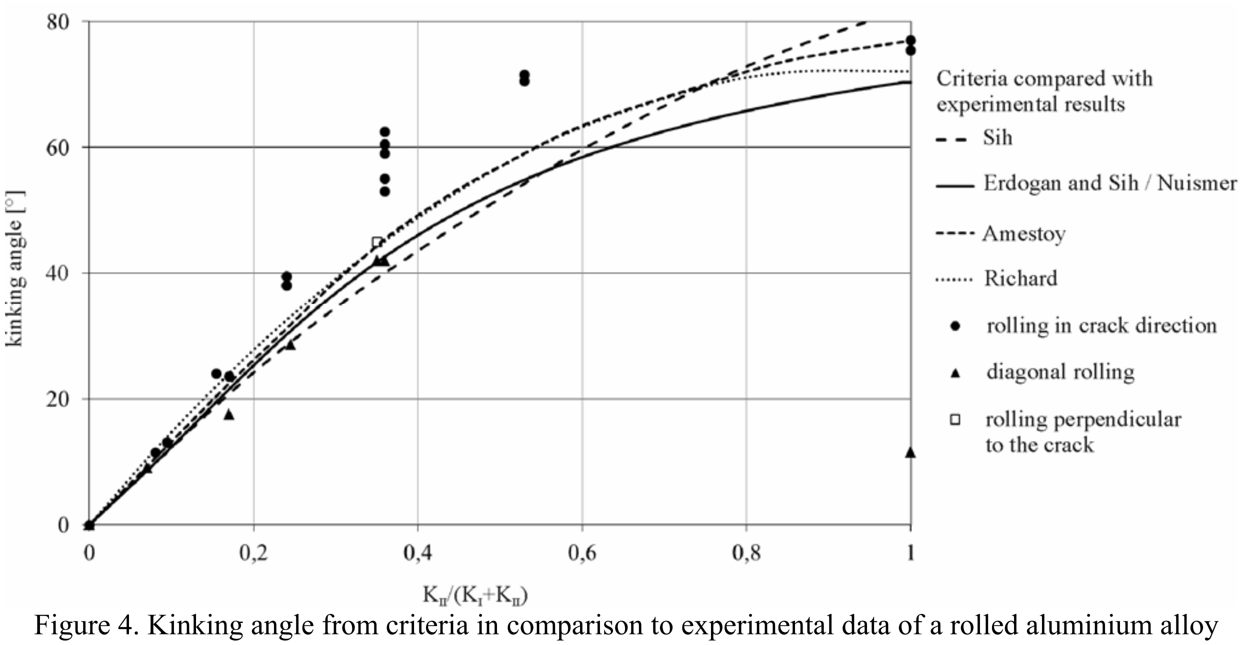


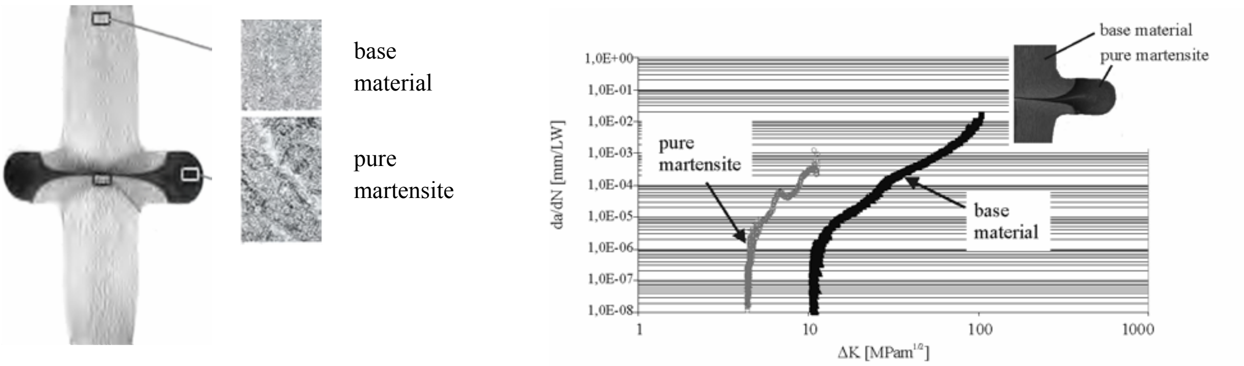
Figure 4. Kinking angle from criteria in comparison to experimental data of a rolled aluminium alloy specimens for 2D Mixed Mode cracks

3. Crack growth in functional graded materials

In technical practice innovative manufacturing and application-oriented products, considering lightweight construction, gain increasingly in importance. In this context functional graded materials and structures take a special position. Due to the fact that these materials possess an application matched functional gradation they are able to meet different local demands such as among other things absorbability, abrasion and fatigue of structures. The material gradation has a remarkable influence on the crack propagation behavior which is why the development of appropriate concepts is important. The material gradation can be realized by different properties. On the one hand the gradation can be defined by different fracture mechanical properties, on the other side by differences in the elastic parameters. Furthermore a combination of the fracture mechanical gradation and the elastic gradation is imaginable. The following considerations deal with a fracture mechanical gradation.

3.1. Influence of a fracture mechanical material gradation on crack propagation

A fracture mechanical material gradation exists for example within the flanged shaft, Fig. 5a, which is used as a demonstrator object by the collaborative research centre Transregio 30 with the title "functionally graded materials in industrial mass production". Besides the development and manufacturing of these innovative materials another important task is the characterization of the mechanical properties of these materials regarding the remarkable influence of the gradation on the fracture mechanical behavior and hence on the life time of such a graded structure [9]. Due to a thermo-mechanical production process the unformed region of the shaft shows ferritic-perlitic base material of the heat treatable steel 51CrV4, whereas the formed flange consists of a martensitic structure. Fig. 5b clarifies that the fracture mechanical properties of these microstructures (i.a. Threshold value $\Delta K_{th,}$ fracture toughness K_{IC}, crack velocity da/dN) differ extremely from each other. The elastic properties (Young's modulus E, Poisson's ratio ν) are not affected by the production process. Furthermore there is a defined distinctive transition zone between the mentioned microstructures which is neglected in this contribution.



a) b)

Figure 5. a) Metallographic micrograph of the flanged shaft [9], b) crack velocity curves for the steal 51CrV4 with different microstructures [10]

3.1.2. Influence on the limits of fatigue crack growth

Fig. 6 shows the correlation between cyclic stress $\Delta\sigma$ and crack length a in dependency of a fracture mechanical gradation for a Griffith crack (geometry factor Y = 1) [10, 11]. Below the Threshold value curves a crack is not able to grow, whereas the unstable crack growth is above the fracture limit curves. The region of stable fatigue crack growth is between both material curves.

The crack starts in base material and reaches the martensitic microstructure at the crack length a = 6mm. It can be seen that the region of stable fatigue crack growth is more distinctive for the base material than for martensite. Furthermore there is no real overlapping at the transition, hence the crack becomes unstable immediately after reaching the martensitic structure.

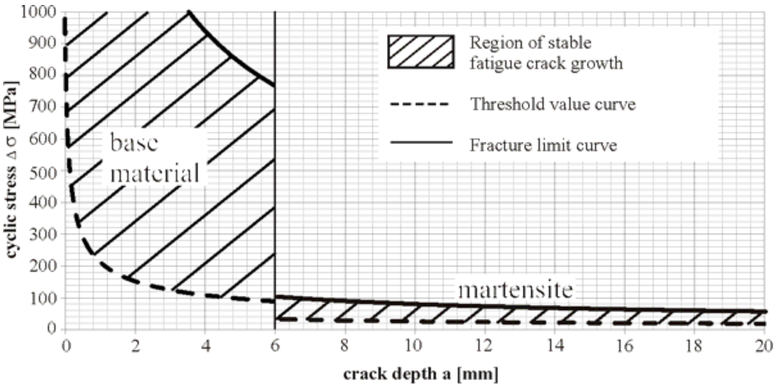


Figure 6. Δσ-a-diagram for a fracture mechanical gradation with a sharp transition at crack length a = 6mm

3.1.3. Influence on the crack growth velocity

Fig. 7a shows schematic crack velocity curves for two materials characterized by different fracture mechanical properties [10, 11]. At the beginning crack growth is starting within material 2. At reaching the other material a change to the crack velocity curve of material 1 occurs. Hence, the crack grows faster and reaches the fracture toughness much earlier as if it grows further within material 2. The worst imaginable case is that at the change to curve 1 the cyclic stress intensity factor ΔK is already larger than the cyclic fracture toughness ΔK_{C1} of material 1 leading to the immediate failure of the structure. In Fig. 7b the gradation is oriented opposite. The crack starts in the material with the worse fracture mechanical properties, material 1, and reaches material 2 afterwards. Due to the change the crack velocity slows down and more load cycles are tolerated until final fracture. At the best crack arrest occurs if the cyclic stress intensity factor ΔK is smaller than the Threshold value ΔK_{th2} of material 2 at the material change. As can be seen, a positive or a negative impact on the prospective lifetime is connected with the gradation constellation.

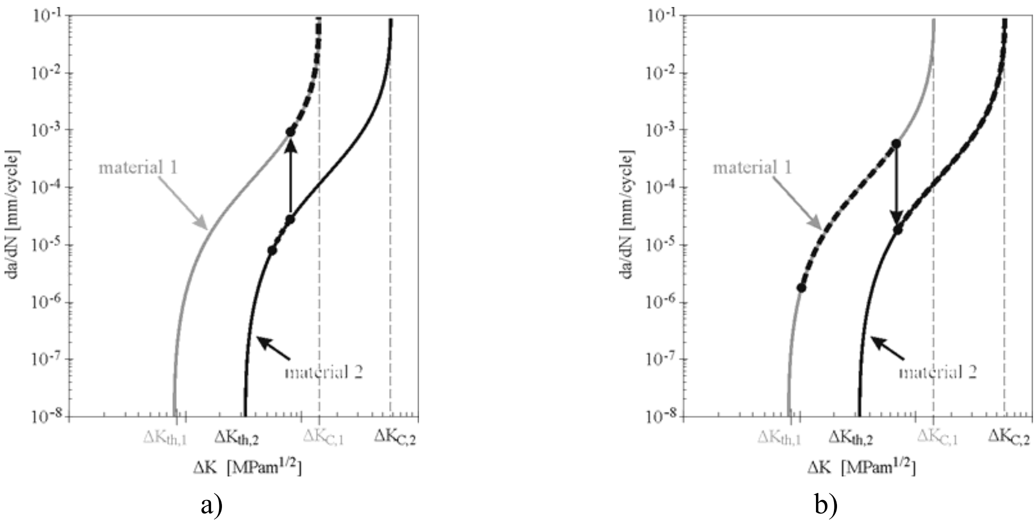


Figure 7. Schematic crack velocity curves:
a) transition from material 2 to material 1, b) transition from material 1 to material 2

3.2. TSSR^B-concept for the prediction of crack propagation in fracture mechanical graded materials

The new developed TSSR^B-concept enables the prediction of the start and the direction of the crack propagation as well as the determination of unstable crack growth in fracture mechanical graded materials [10, 11, 12]. Fig. 8 shows a crack-afflicted structure which is loaded periodically resulting in a pure Mode I loading solution. Furthermore the structure consists of two materials whose fracture mechanical properties differ from each other. The presented gradation angle $\varphi_M = 30°$ defines the position of the material transition in relation to the crack tip. The crack could kink by the kinking angle $\varphi_{0,MTS}$ which depends on the stress situation and can be determined by a crack propagation concept for homogeneous and isotropic materials (for example the MTS-concept of Erdogan and Sih). Another imaginable kinking angle is the gradation angle φ_M itself, due to the fact that the crack strives to take the way of least resistance.

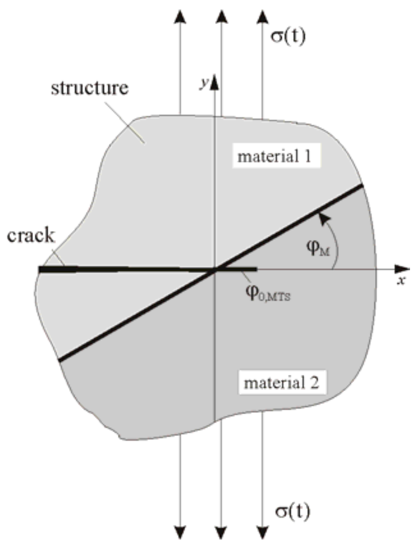


Figure 8. Fracture mechanical graded structure with gradation angle $\varphi_M = 30°$

The occurrence of fatigue crack growth as well as the entrant kinking angle φ_{TSSRB} can be determined by the TSSR^B-concept. This concept is based on the assumption that stable crack growth starts, when the cyclic tangential stress $\Delta\sigma_\varphi$ (Eq. 5) reaches a material limit value $\Delta\sigma_{\varphi,th}$ or rather when a cyclic comparative stress intensity factor ΔK_V, determined by the means of the cyclic tangential stress $\Delta\sigma_\varphi$, reaches the Threshold value curve $\Delta K_{th}(\varphi)$. This material function depends on the coordinate φ and consists of the values $\Delta K_{th,material1}$ and $\Delta K_{th,material2}$ for the different regions, see Eq. (8). Similarly the material functions $\Delta K_C(\varphi)$ and $K_C(\varphi)$ can be defined in dependency of the gradation angle φ_M.

$$\Delta K_{th}(\varphi) \begin{cases} \Delta K_{th,material1} & \text{for} \quad \varphi_M \le \varphi \le \varphi_M + 180° \\ \Delta K_{th,material2} & \quad\quad\;\; \varphi_M - 180° < \varphi < \varphi_M \end{cases} \tag{8}$$

The determination of the start and the direction of fatigue crack growth is shown in Fig. 9a. The first intersection of the stress function $\Delta\sigma_\varphi\sqrt{2\pi r}$ with the material function $\Delta K_{th}(\varphi)$ (Eq. 8) identifies the occurrence and the corresponding direction of crack growth, whereas the intersection of the stress function $\Delta\sigma_\varphi\sqrt{2\pi r}$ with the material function $\Delta K_C(\varphi)$ defines the occurrence of final failure of the structure. For static loading the occurrence of unstable crack growth can be predicted using the material function $K_C(\varphi)$.

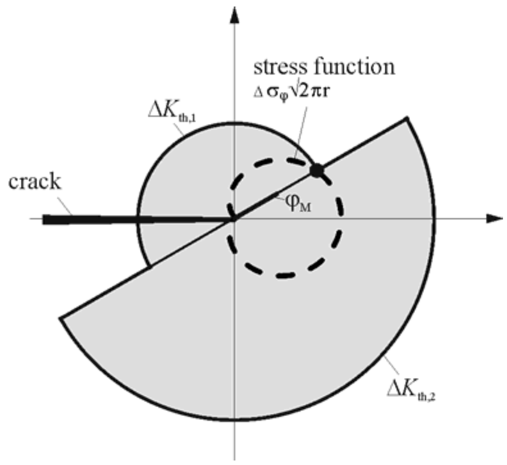
At first it is assumed that the crack is not able to grow resulting in a stress function without point of contact with the material function $\Delta K_{th}(\varphi)$. Subsequently the cyclic load will be increased until the corresponding stress function has a contact with the material function. At this point the crack is able to grow. Furthermore the point of contact defines the kinking angle φ_{TSSRB} due to the loading situation and the material gradation.

3.2.1 Mode I loading in a graded material

As a special case of a Mixed Mode loading at first a pure Mode I situation is considered. In Fig. 9a a single Mode I ($\Delta K_{II} = 0$) is considered leading to the reduced stress function $\Delta\sigma_\varphi\sqrt{(2\pi r)}$, see Eq. (9), derived from Eq. (5).

$$\Delta\sigma_{\varphi}\sqrt{2\pi r} = \Delta K_I \cos^3\frac{\varphi}{2} \qquad (9)$$

The point of contact can be found at the polar coordinate $\varphi_M = 30°$ resulting in the kinking angle $\varphi_{TSSRB} = \varphi_M = 30°$. First experimental investigations considering the same loading condition and the same gradation angle φ_M confirm this theoretical concept (Fig. 9b). In spite of pure Mode I the crack kinks due to the material gradation. For the further crack propagation the kinked crack evokes a Mixed Mode loading situation.



a)

b)

Figure 9. a) Point of contact for the determination of occurrence and direction of fatigue crack growth for a Mode I loaded crack, b) experimental confirmation of the TSSR^B-concept

3.2.2 Mixed Mode loading in a graded material

The TSSR^B-concept can also be used for a Mixed Mode loading situation. Thereby the Mixed Mode ratio V, see Eq. (10), defines the combination of the pure loading cases Mode I and Mode II. Fig. 10 shows the stress functions $\Delta\sigma_\varphi\sqrt{(2\pi r)}$ (Eq. 11) for a Mixed Mode ratio V = 0,23, see also Eq. (5).

$$V = \frac{K_{II}}{K_I + K_{II}}\,. \qquad (10)$$

$$\Delta\sigma_{\varphi}\sqrt{2\pi r} = \Delta K_{I}\cos^{3}\frac{\varphi}{2} - \Delta K_{II}\frac{3}{2}\sin\varphi\cdot\cos\frac{\varphi}{2}. \tag{11}$$

The structure in Fig. 10a and Fig. 10b possess different gradation angles φ_M. This difference in the gradation angle φ_M leads to a different kinking angle φ_{TSSRB} due to the loading condition and the material gradation. In Fig. 10a the gradation angle φ_M is 30°. The point of contact is at this coordinate leading to the kinking angle $\varphi_{TSSRB} = \varphi_M = 30°$. In Fig. 10b the structure possesses the gradation angle $\varphi_M = 60°$. Here the point of contact can be found at the coordinate with the largest stress. Hence the kinking angle $\varphi_{0,MTS} = -30°$ due to the loading condition, according to the MTS-concept of Erdogan and Sih, defines the kinking angle φ_{TSSRB} in this case.

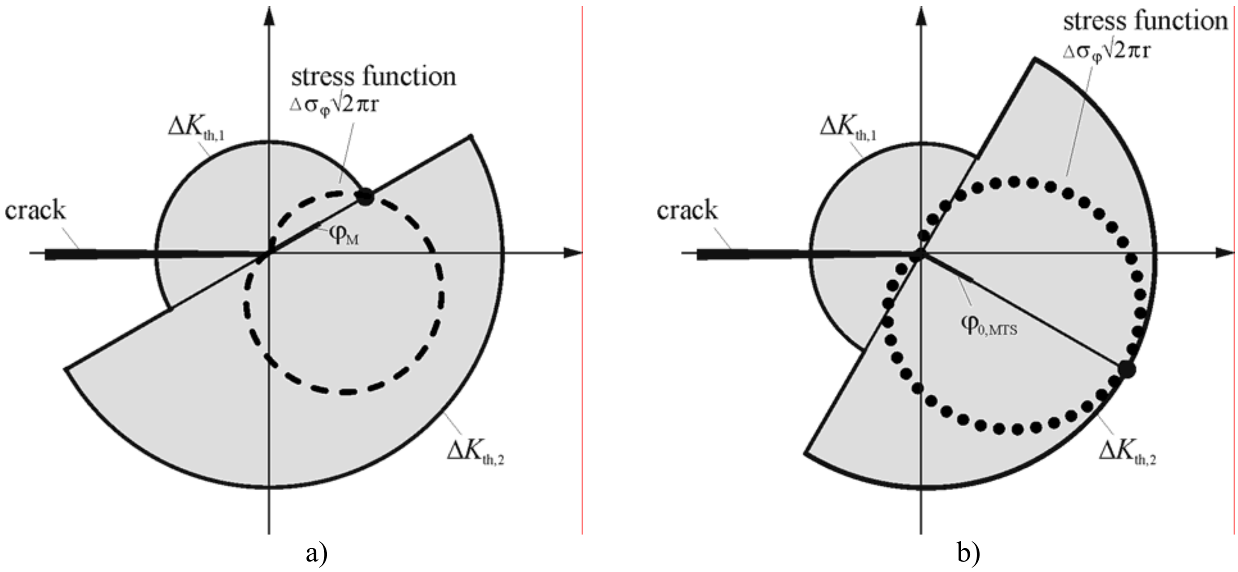


Figure 10. Occurrence and direction of fatigue crack growth for Mixed Mode loading in a fracture mechanical graded material: a) gradation angle $\varphi_M = 30°$, b) gradation angle $\varphi_M = 60°$.

In graded materials the crack propagation direction depends on the Mixed Mode loading situation and the material gradation. Which crack growth occurs can be determined with the $TSSR^B$-concept. Furthermore the concept enables the determination of the starting of fatigue crack growth and of unstable crack growth.

4. Conclusion

Using the presented $TSSR^B$-concept crack propagation as well as the direction of crack growth can be determined in fracture mechanical graded materials. By comparing the stress function $\Delta\sigma_\varphi\sqrt{2\pi r}$ with the material functions $\Delta K_{th}(\varphi)$, $\Delta K_C(\varphi)$ and $K_C(\varphi)$ the occurrence, the direction as well as the start of unstable fatigue crack growth and the start of unstable failure due to static loading can be determined. Furthermore this concept can also be modified for the application for an elastic material gradation as well as for the combination of a fracture mechanical and elastic material gradation.

Acknowledgements

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