Three-parameter approaches for three-dimensional crack-tip stress fields

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Abstract Two three-parameter descriptions for the three-dimensional (3D) crack-tip stress fields have been introduced. The three-parameter solution K-T-T_z is developed to describe the linear elastic crack-tip stress state, and the J-Q_T-T_z is to elastic-plastic crack-tip field. The conventional two-dimensional solutions such as K, K-T, HRR and the extended J-Q description which considers the in-plane constraint modification can hardly provide satisfied description for the three-dimensional crack front fields, especially for the out-of-plane stress near the crack front. It is shown that a consideration of the out-of-plane constraint and use of the three-parameter description is necessary and efficient to predict the 3D stress fields near the crack front.

Keywords Fracture mechanics, Three-dimensional stress field, Out-of-plane stress constraint

1. Introduction

The complicated three-dimensional (3D) stress fields near the crack front play a vital role in the strength of materials [1], and control the initiation and propagation of cracks [2]. The character of the stress fields near the crack front has long been extensively studied. The classical linear elastic and elastic–plastic fracture mechanics are based on the theory stemming from the one singular term of asymptotic expression and its amplitude the stress intensity factor (SIF, K) [3] and HRR solution [4, 5], respectively. Then more accurate two-parameter approaches, such as K–T [6], J–T [7], J–Q [8, 9] and J–A_2 [10, 11], have been developed to describe the crack-tip field. These approaches have been applied successfully in engineering designs though they are limited to describe the effect of the in-plane constraint on the crack-tip field and fracture toughness. In fact, fracture toughness depends on the 3D out-of-plane stress level near the crack front also [12]. It is well known that fracture toughness depends highly on the thickness of the test specimen until a threshold thickness, beyond which the toughness does not decrease further. The toughness at this thickness is called plane strain fracture toughness. It is less than the fracture toughness of thinner plates and is a material property. So the variable fracture toughness is inconvenient in the engineering applications if the 3D out-of-plane stress level is not considered accurately.

In order to describe the out-of-plane stress level, the out-of-plane stress constraint factor T_z was introduced by Guo [13-15], the factor is defined as

$$T_z = \frac{\sigma_{33}}{\sigma_{11}+\sigma_{22}}, \qquad (1,\ 2,\ 3)=(x,\ y,\ z) \text{ or } (r,\ \theta,\ z) \tag{1}$$

where r, θ, x and y are coordinates in the conventional polar and Cartesian systems with origin at the crack tip and z is the third coordinate (parallel to the crack front) in both systems. The corresponding coordinate system and a normal sheet element of a through-straight crack are shown in Fig.1. In the state of plane stress, T_z=0. In the state of plane strain, T_z changes from the Poisson's ratio v of the linear elastic material to 0.5 for elastic-perfectly plastic material.

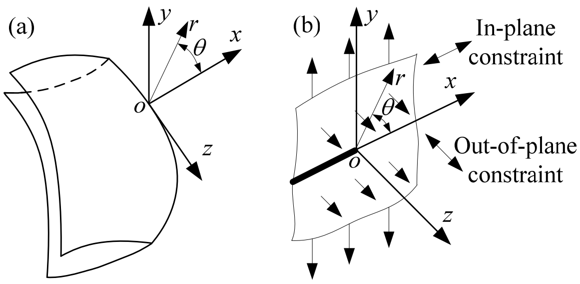


Fig. 1 The coordinate system and a normal sheet element of a through-straight crack.

The out-of-plane constraint factor T_z plays an important role in the determination of fracture toughness of a structural element. The effect of Tz on 3D crack-front fields and fracture toughness were systematically studied by Guo, then the 3D two-parameter principles of K–T_z, J–T_z have been proposed [12-16]. Combining with the in-plane constraint T or Q, the 3D three-parameter principles of K-T-T_z and J-Q_T-T_z have also been proposed [13-15, 17-22].

In this paper, the recent researches on the 3D three-parameter principles of K-T-T_z and J-Q_T-T_z are emphasisly summarized. The comparisons of the three-parameter principles with two-parameter and one-parameter principles are presented.

2. Three-parameter principle *K–T–T*$_z$ for the linear elastic material

Based on the SIF K, the more accurate two-parameter approach K–T was proposed by Williams [6] for linear elastic material. For the 3D crack case, the three-parameter principle K-T-T_z was developed by Guo [13-15, 17-19] based on the K-T approach, which can be expressed as

$$\begin{cases} \sigma_{ij} = \dfrac{K}{\sqrt{2\pi r}} f_{ij}(\theta) + T\delta_{1i}\delta_{1j} \\ \sigma_{zz} = T_z(\sigma_{ii} + \sigma_{jj}) \end{cases} \qquad (i,\ j) = (x,\ y),\ (r,\ \theta)\ \text{or}\ (1,\ 2) \tag{2}$$

where T is the T-stress,

$$f_{11}(\theta) = \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \tag{3}$$

$$f_{22}(\theta) = \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \tag{4}$$

$$\tau_{12}(\theta) = \cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2} \tag{5}$$

The three-parameter principle K-T-T_z can describe not only the in-plane constraint by T but also the out-of-plane stress constraint by T_z. Then the crack-tip stress fields described by the principle K-T-T_z for the typical cracks such as through-the-thickness crack, quarter elliptical corner crack, semi-elliptical surface crack and embedded elliptical crack are discussed. The geometry of a plate with a quarter elliptical crack under uniform tension is presented in Fig.2, the corresponding FE mesh is shown in Fig.3. The 3D singular elements with four mid-side nodes at the quarter points are used around the crack front to simulate the inverse square root singularity at the crack tip. The FE model in Fig.3 can be used to simulate the semi-elliptical surface crack and embedded elliptical crack by altering the corresponding displacement boundary conditions.

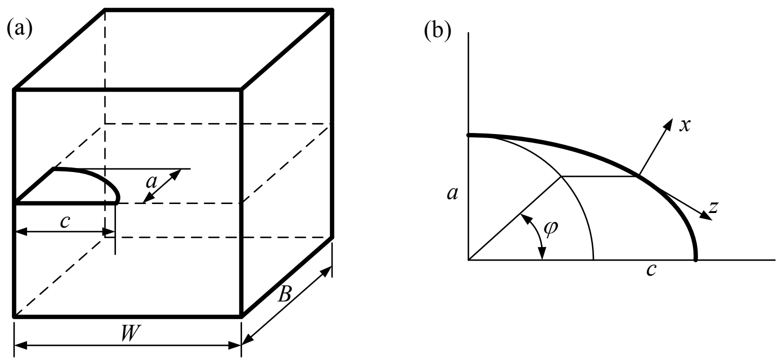


Fig.2 Geometry of a plate with a quarter elliptical crack under uniform tension. (a) The 3D geometry model. (b) Cracks with different a/c and the local rectangular coordinate system.

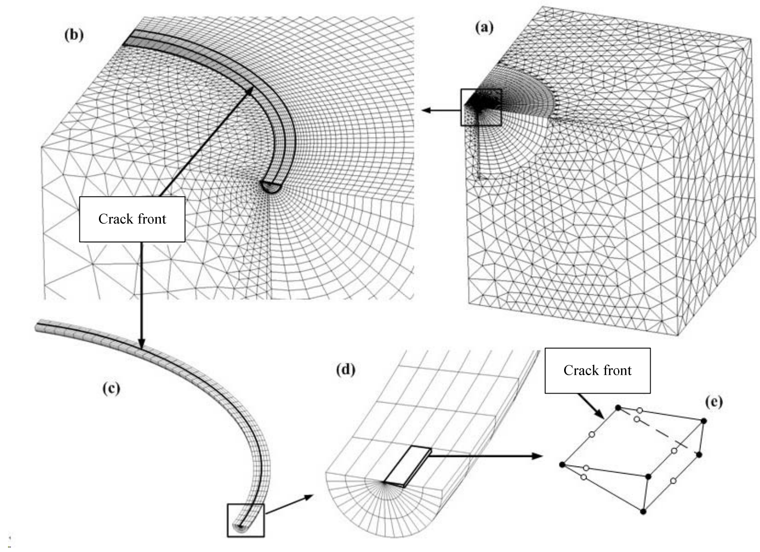
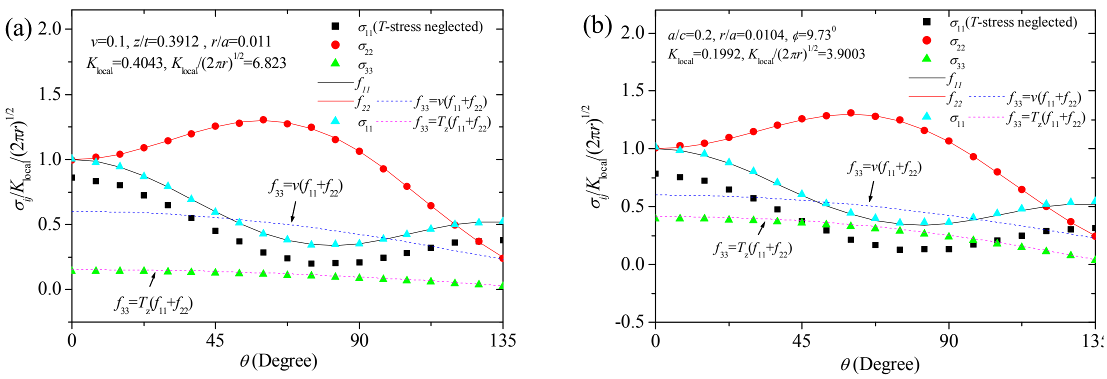


Fig.3 FE model of the quarter elliptic corner crack

The comparisons of the three-parameter principle K-T-T_z with the FE results are presented in Fig. 4.



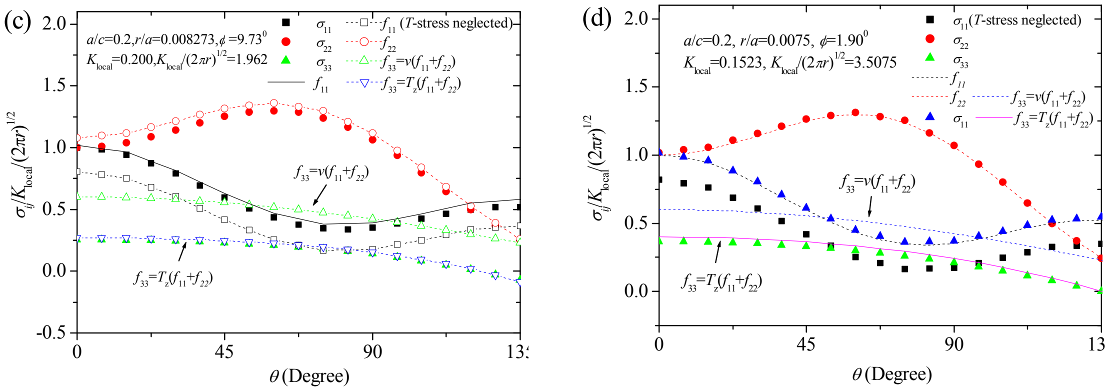


Fig.4 The angular distributions of stress components normalized by the local stress intensity factors in a normal plane of the quarter-elliptical corner crack front line. (a) through-the-thickness straight crack, (b) semi-elliptical surface crack, (c) quarter elliptical corner crack, (d) embedded elliptical crack.

As shown in Fig. 4, the angular distributions of stress components in a normal plane of various crack front lines are given in the local Cartesian coordinates. It can be seen that σ_{22} is in good agreement with f_{22}, while the differences between σ_{11} and f_{11} are great if the T-stress is neglected. When the T-stress is considered, the differences will become very small. In addition, the differences between σ_{33} and f_{33} for the plane strain state ($\nu(f_{11}+f_{22})$) are great. If the T_z factor is considered in f_{33}, f_{33} will be in good agreement with σ_{33}.

3. Three-parameter principle J–Q_T–T_z for the elastic-plastic material

The HRR stress components can be expressed as

$$\left(\sigma_{ij},\sigma_m,\sigma_e\right)=Kr^{-\frac{1}{n+1}}\left(\tilde{\sigma}_{ij}\left(\theta\right),\tilde{\sigma}_m\left(\theta\right),\tilde{\sigma}_e\left(\theta\right)\right) \tag{6}$$

where

$$K=\left(\frac{J}{\alpha\sigma_0\varepsilon_0 I\left(n\right)}\right)^{\frac{1}{n+1}} \tag{7}$$

σ_0 is the yield stress.

By considering the effects of geometry and size on crack-tip constraint, O'Dowd and Shih [8, 9] found that the near-tip stress field is governed by the two parameters of J and Q as follows:

$$\sigma_{ij}=\left(\frac{J}{\alpha\sigma_0\varepsilon_0 I\left(n\right)r}\right)^{\frac{1}{n+1}}\tilde{\sigma}_{ij}\left(\theta\right)+Q_{ij}\delta_{ij}\left(\frac{r}{J/\sigma_0}\right)^{\lambda}\sigma_0 \tag{8}$$

The first term is the HRR solution ($\left|\theta\right|<\pi/2$), Q is a function of the stress triaxiality achieved ahead of the plane strain cracks. The λ is set to zero, then $Q_{rr}=Q_{\theta\theta}$ and Q_{ij} is the form

$$Q_{ij}=\frac{\sigma_{ij}-\sigma_{ij}\big|_{HRR}}{\sigma_0}\qquad (\theta=0^{\circ},\ r=2J/\sigma_0) \tag{9}$$

The *J-Q* solution can effectively describe the influence of the in-plane stress parameters when the radial distances ($r/(J/\sigma_0)$) are relatively small, while the approach can hardly characterize it very well with the increase of $r/(J/\sigma_0)$ and strain hardening exponent n. On the other hand, it can hardly give a proper description of Von Mises equivalent stress σ_e because it seldom considers the out-of-plane stress constraint, so Guo and his collaborators proposed two 3D three-parameter principles of K-T-T_z and J-Q-T_z, combining with the in-plane constraint T or Q, for linear elastic and elastic-plastic materials.

Further researches by Guo [13-15, 20-22] show that I( ) is the function of n and T_z,

$$K=\left(\frac{J}{\alpha\sigma_0\varepsilon_0 I\left(n,T_z\right)}\right)^{\frac{1}{n+1}} \tag{10}$$

where

$$\begin{aligned}I\left(n,T_z\right)=\int_{-\pi}^{\pi}\{&\frac{n}{n+1}\tilde{\sigma}_e^{n+1}\cos\theta-\sin\theta\left[\tilde{\sigma}_{rr}\left(\tilde{u}_\theta-\frac{\partial\tilde{u}_r}{\partial\theta}\right)-\tilde{\sigma}_{r\theta}\left(\tilde{u}_r+\frac{\partial\tilde{u}_\theta}{\partial\theta}\right)\right]\\&-\cos\theta\left[n\left(s-2\right)+1\right]\left(\tilde{\sigma}_{rr}\tilde{u}_r+\tilde{\sigma}_{r\theta}\tilde{u}_\theta\right)\}d\theta,\end{aligned} \tag{11}$$

$$\tilde{u}_r=\frac{\tilde{\sigma}_e{}^{n-1}}{n\left(s-2\right)+1}\left(d_1\tilde{\Phi}+d_2\frac{\partial^2\tilde{\Phi}}{\partial\theta^2}\right), \tag{12}$$

$$\tilde{u}_\theta=\frac{1}{n\left(s-2\right)}\left(2d_s\tilde{\sigma}_e{}^{n-1}\frac{\partial\tilde{\Phi}}{\partial\theta}-\frac{\partial\tilde{u}_r}{\partial\theta}\right), \tag{13}$$

$$\frac{\partial\tilde{u}_r}{\partial\theta}=\frac{1}{n\left(s-2\right)+1}\left[\frac{\partial\left(\tilde{\sigma}_e{}^{n-1}\right)}{\partial\theta}\left(d_1\tilde{\Phi}+d_2\frac{\partial^2\tilde{\Phi}}{\partial\theta^2}\right)+\tilde{\sigma}_e{}^{n-1}\left(d_1\frac{\partial\tilde{\Phi}}{\partial\theta}+d_2\frac{\partial^3\tilde{\Phi}}{\partial\theta^3}\right)\right], \tag{14}$$

$$\frac{\partial\tilde{u}_\theta}{\partial\theta}=\tilde{\sigma}_e{}^{n-1}\left(d_3\tilde{\Phi}+d_4\frac{\partial^2\tilde{\Phi}}{\partial\theta^2}\right)-\tilde{u}_r. \tag{15}$$

The d_i is the function of T_z and n, $\tilde{\Phi}=\tilde{\Phi}\left(\Phi\right)$, which is same as that in HRR solution.

The Eq.(8) can be modified

$$\sigma_{ij}=\left(\frac{J}{\alpha\sigma_0\varepsilon_0 I\left(n,T_z\right)r}\right)^{\frac{1}{n+1}}\tilde{\sigma}_{ij}\left(\theta,T_z\right)+Q_{Tij}\delta_{ij}\left(\frac{r}{J/\sigma_0}\right)^{\lambda}\sigma_0 \tag{16}$$

where

$$Q_{Tij} = \frac{\sigma_{ij} - \sigma_{ij}\big|_{J-T_z}}{\sigma_0} \qquad (\theta=0^\circ \text{ and } r=2J/\sigma_0) \tag{17}$$

Combining Eq.(9), the relationship between Q_{Tij} and Q_{ij} is

$$Q_{Tij} = Q + \frac{\sigma_{ij}\big|_{HRR} - \sigma_{ij}\big|_{J-T_z}}{\sigma_0} \tag{18}$$

The comparisons of the three-parameter solution J–Q_T–T_z with the solutions of J-Q and HRR are shown in Fig.5. It is shown that the three-parameter approach J–Q_T–T_z can describe the 3D stress fields effectively.

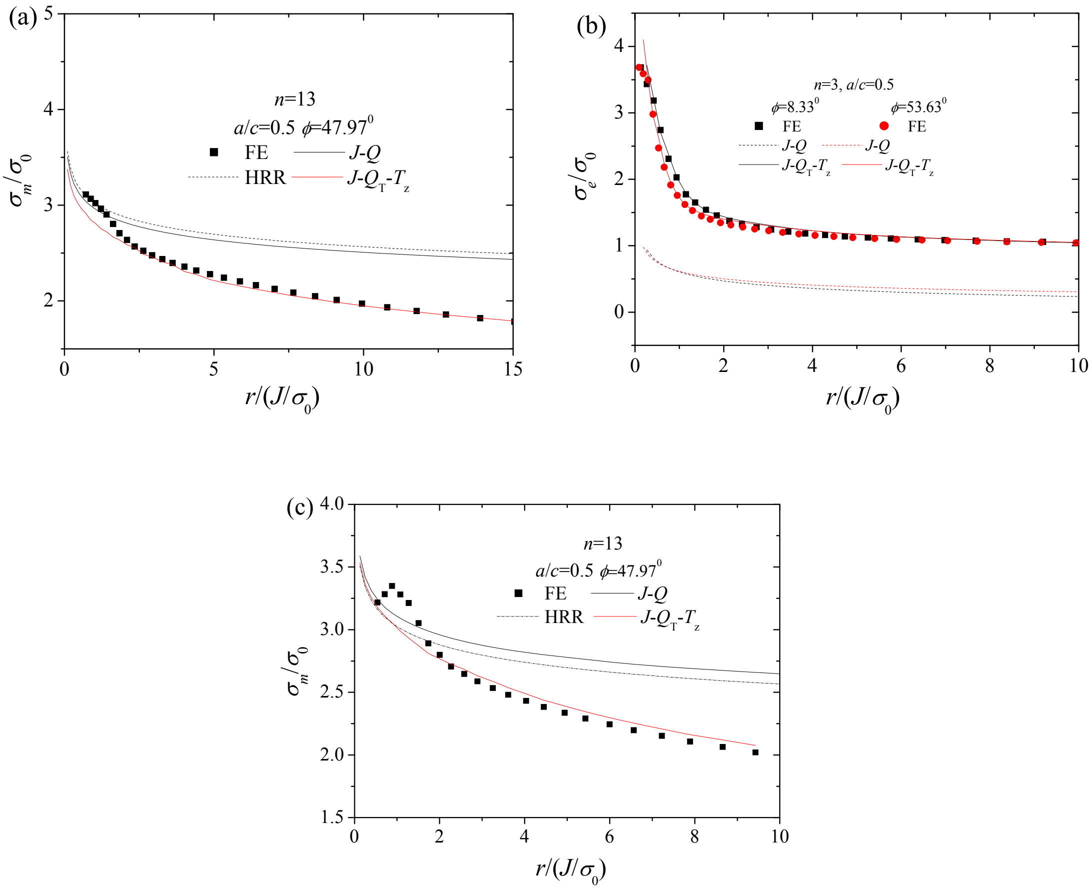


Fig.5 The radial distributions of the stress components. (a) Mean stress for a semi-elliptical surface crack, (b) Von Mises equivalent stress σ_e for a quarter elliptical corner crack, (c) Mean stress for an embedded elliptical crack.

4. Conclusions

Two three-parameter descriptions for the three-dimensional (3D) crack-tip stress fields have been introduced. The three-parameter solution K–T–T_z is developed to describe the linear elastic crack-tip stress state, and the J–Q_T–T_z is to elastic-plastic crack-tip field. The comparisons of the three-parameter solutions K–T–T_z, J–Q_T–T_z with the corresponding two-parameter solutions K-T, J–Q and single-parameter solutions K and HRR are presented. It is shown that the three-parameter approaches K–T–T_z, J–Q_T–T_z can describe the 3D stress fields effectively.

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