New Trends in the Fracture of Lightweight Structures

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Abstract Analysis of fractured lightweight structures has been performed in this paper. In particular a sandwich beam under three point bending containing a crack in the core material very close to the upper skin interface and parallel to the longitudinal beam axis is investigated. A numerical study of the fractured sandwich beam is developed calculating the stress intensity factors in order to investigate the fracture very close to upper skin interface under mixed-mode loading conditions. In addition a cohesive damage model is presented to simulate crack propagation and kinking into the core under mixed-mode loading conditions. The crack considered, is analyzed with static non-linear two- dimensional finite element analyses.

Keywords Lightweight structure, Crack Propagation, Stress Intensity Factor, Cohesive Model

1. Introduction

Lightweight composite structures are widely used in aerospace, marine and other modern industrial applications. Sandwich structures used in these applications consist of a lightweight foam core bonded to thin laminas to achieve high values of specific strength and stiffness. One of the main advantages of sandwich structures is their ability to provide increased flexural rigidity without an increase of the structural weight. The core is usually made of PVC, wood, a honeycomb material or lately carbon foam bonded to tough carbon fiber reinforced polyetheretherketone (PEEK) skins. In sandwich structures the foam is typically the weakest part and is the first to fail under static or cyclic loading because it transfers the applied loads as shear stresses. In addition a very critical problem in sandwich structures is the debonding problem between the face and core materials [1-5]. Unstable cracking propagation and kinking in core materials represents one of the weakest failure modes in sandwich composites. The fracture behavior in lightweight composites has been directed toward the understanding of crack propagation, and at the same time toward improving the durability of composites against fracture. A crack flaw may be introduced during processing or subsequent service conditions. It may result from low velocity impact, from eccentricities in the structural load path, or from discontinuities in structures, which induce a significant out-of-plane stress. In our paper a composite beam under three point bending and/or asymmetric three point bending is studied. In the core material of the beam an initially small crack is considered. In this study a computational analysis is developed based on an experimental investigation in composite beams under flexural loading [2]. From the numerical simulations stress intensity factors are calculated using the Finite element method and combined with crack propagation criteria predict the crack kinking into the core under flexural loading. On the other hand cohesive crack models [6, 7] are widely used to simulate crack growth and kinking phenomena. In order to develop numerical methodologies to simulate crack propagation in composite structures, cohesive damage models have attracted much interest [8-13] due to their well established advantages compared to the stress based

and fracture mechanics methods. Taking into consideration that the application of cohesive damage models in sandwich structures for the numerical simulation of the crack propagation is very limited [11-13]; a cohesive model in terms of cohesive parameters will also propose in this study in order to simulate crack propagation and kinking into the core of sandwich structures.
Thus the scope of this study is to numerically investigate the crack propagation and crack kinking into the core of lightweight structures under mixed mode loading conditions and to present a cohesive damage model appropriate to simulate crack propagation into the core. Results from the computational analysis predict the crack growth and kinking under flexural loading.

2. Specimens and Loading Material Testing.

The geometric and the loading conditions for the sandwich are shown in Fig. 1. The sandwich is composed of PVC-core and face sheets from isotropic glass (Table 1). The dimensions of the test specimen were $L = 228.6$ mm (support span) and $b = 63.5$ mm (width). The core thickness was $t_2 = 12.7$ mm and the face sheet thickness was $t_1 = 2.28$ mm. The overall thickness was 17.26 mm.

Static tests were first conducted to generate ultimate strength data [2]. Flexural fatigue tests on sandwich beams were performed at room temperature under load control at a stress ratio of 0.1, using a sinusoidal wave form. Fatigue data was generated for a minimum of three specimens at stress levels of 90%, 85%, 80%, 75%, 70%, 65% and 60% of the ultimate flexural strength. Three distinct damage events take place before the failure of the specimen.

At first crack initiation and propagation was observed on the compression side just below the top face sheet/core interface. This delamination crack was about 1-1.5mm below the interface. The crack runs parallel to the beam axis from the point of initiation towards the end support (Fig. 1). This first damage event occupies about 85% of the fatigue life. The propagated crack kinks at a certain distance and shears through the core thickness. The crack reaches the bottom face sheet/core interface. Finally, delamination takes place at bottom face/core interface using the separation of the core from the face sheet. This is also a rapid event and occupies the remaining 7-8% of fatigue life.

Table 1. Specimen properties

Material	E $\left(N/mm^2\right)$	ν
Face sheets (isotropic glass)	16300	0.3
PVC foam core (R75 by DIAB [5])	80	0.4

From the plot of the crack lengths via N (number of cycles) at different stress levels in [2], it is observed that the length of the crack depends on the stress level. Lower is the stress level; the longer is the crack in damage event-1. Thus, at lower stress levels $\left(r = 0.65, 0.70, 0.75\right)$ the first damage event dominates the fatigue life and consequently the crack propagation near the top face core-skin interface, may be used to develop a failure model for sandwich composites.

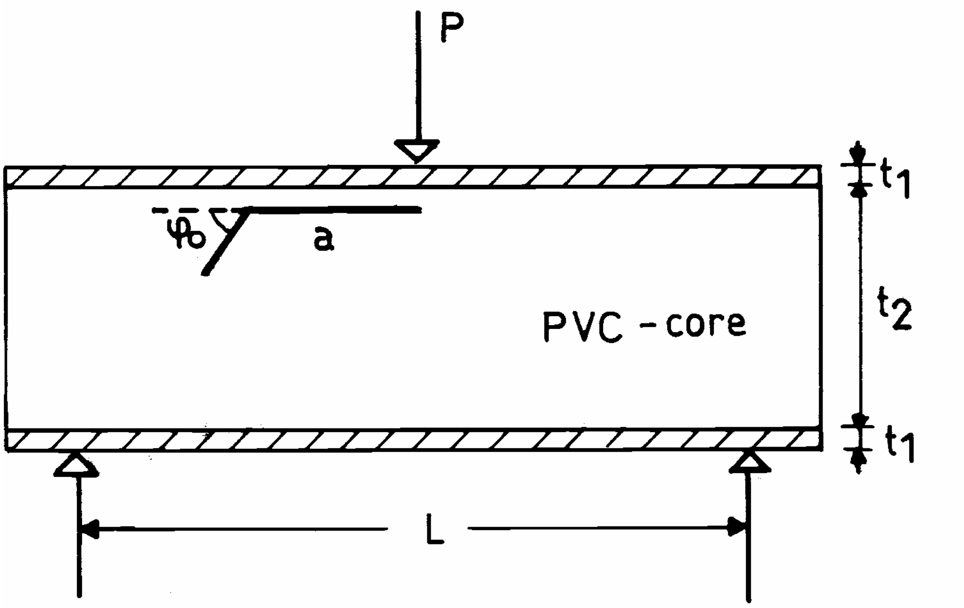

Figure 1. Crack propagation in the core of a sandwich beam under flexural loading

3. Crack Kinking Analysis.

The cracked sandwich beam is analyzed using the finite element method. Because of high stress gradients around the interface, fairly fine mesh consisting of two dimensional plane strain elements was used. The assumption of plane strain is justified throughout.

The finite element analysis is performed with the use of the general purpose finite element programs. The 6 node and 8 node two-dimensional plane strain triangular elements (plane 6) were used in order to model the beam. The frictionless contact area at the crack surfaces is modeled with 2-node linear contact elements, in order to prevent one surface from entering into the other. Singular elements (mid-side nodes at ¼) were used at the two crack tips.

In order to analyze the first damage event, a small crack is considered at about $1.5mm$ below the interface under the central load introduction and parallel to the beam axis. This small crack is considered propagating under the interface. For the different crack lengths the stress intensity factors,

$$\Delta K_1 = K_I^{\max} - K_I^{\min} , \quad \Delta K_{II} = K_{II}^{\max} - K_{II}^{\min} , \tag{1}$$

are calculated automatically from the finite element analysis for the left and right crack tip. The ΔK_I and ΔK_{II} ratios are given in Fig. 2, considering the lower stress level $r = 0.70$ and for the load P to be taken equal to $0.7P_{\text{ultimate}} \left(=943.46N\right)$, where P_{ultimate} the static failure load of the beam.

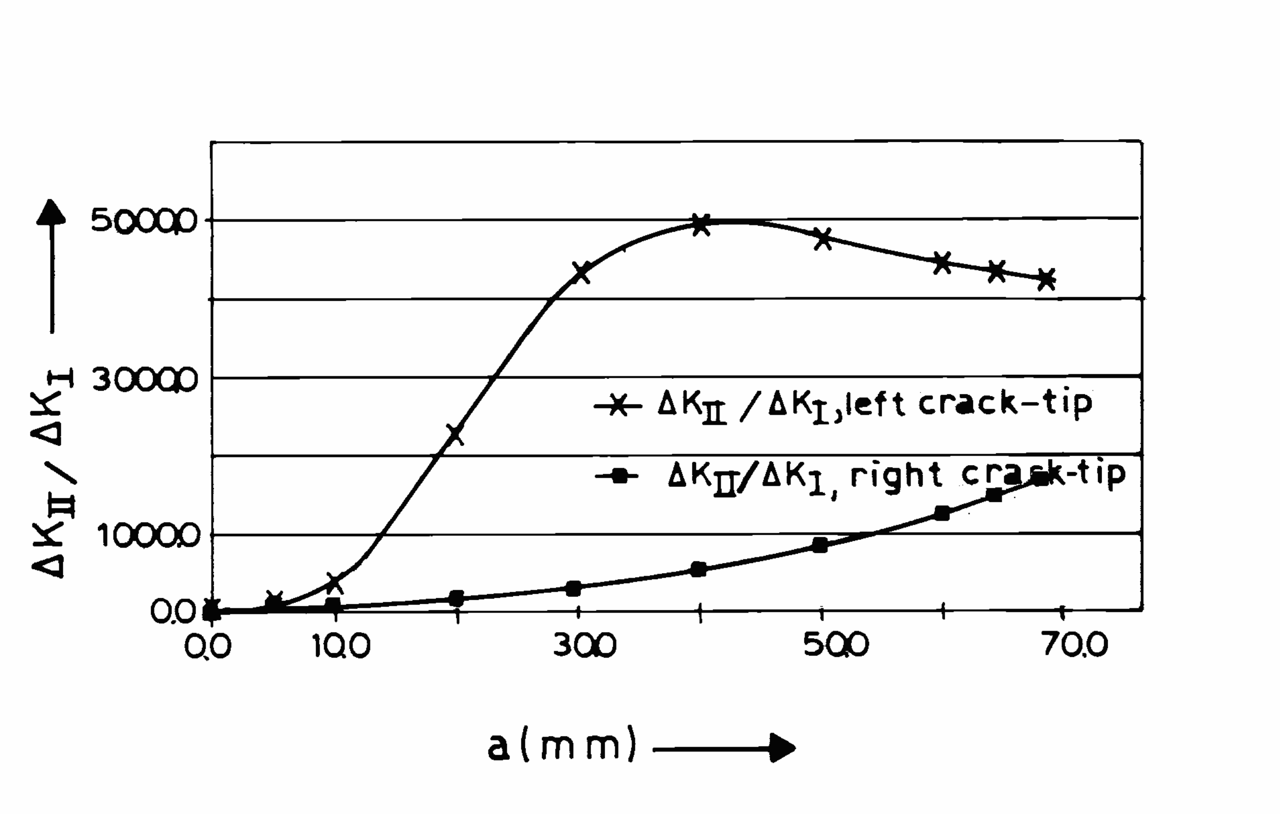

Figure 2. $\Delta K_{II}/\Delta K_{I}$ ratios for left and right crack-tip for different crack lengths

From the experimental investigation initiation of the crack kinking takes place for a crack length a about $65 \sim 70(mm)$, in an angle φ_0 (Figure 1) of $45^\circ \pm 5^\circ$. Two different crack propagation criteria have been examined to estimate which one should be used in outer to model crack kinking in cross linked PVC sandwich foam cores. At first the Energy density factor criterion [14] is used. According to this criterion, crack initiation occurs in the radial direction in which the local energy density possesses a stationary value. The energy W per unit area A, is expressed by:

$$\frac{dW}{dA} = \frac{S}{r}, \tag{2}$$

where r is the radial distance from the crack tip and S is the strain energy density factor which represents a material element in the distance r from the crack tip. The distance r from the crack tip is assumed to be small compared to the crack length a, and S is given by:

$$S = a_{11}K_I^2 + 2a_{12}K_I K_{II} + a_{22}K_{II}^2 \tag{3}$$

where

$$a_{11} = \frac{1}{16G}\left(\left(1+\cos(\varphi)\right)\left(\kappa-\cos(\varphi)\right)\right), \quad a_{12} = \frac{1}{16G}\sin(\varphi)\left(2\cos(\varphi)-(\kappa-1)\right)$$
$$a_{22} = \frac{1}{16G}\left((\kappa+1)(1-\cos(\varphi))+(1+\cos(\varphi))(3\cos(\varphi)-1)\right) \tag{4}$$

and, φ the angle related to the plane of the crack, G the shear modulus of the core material, $\kappa = 3-4\nu$ (plain strain).

The minimum energy density possesses a stationary value which is derived from

$$\frac{\partial S}{\partial \varphi} = 0 \quad \textit{at which} \quad \varphi = \varphi_0 \,, \tag{5}$$

where φ_0 is the crack propagation angle with respect to the plane of the crack with the minimum energy density.

From the numerical calculations and the core material considered (Table 1), we have:

$$\varphi_0 = \pm 80^\circ \,,$$

which is too high comparing with the kinking angle from the experimental investigation.

Secondly the maximum hoop stress is used [15]. The tangential stress component at the vicinity of the crack tip is given by:

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}}\cos\left(\frac{\varphi}{2}\right)\left\{\frac{1}{2}K_I\left(1+\cos(\varphi)\right)-\frac{3}{2}K_{II}\sin(\varphi)\right\} \tag{6}$$

where r is the distance from the crack tip and $\sigma_{\theta\theta}$ the tangential stress.

According to the maximum hoop stress criterion the crack will propagate in the direction in which $\sigma_{\theta\theta}$ obtains its maximum. The direction of $\max \sigma_{\theta\theta}$ can be found by the relations

$$\frac{\partial \sigma_{\theta\theta}}{\partial \varphi} = 0 \,, \quad \frac{\partial^2 \sigma_{\theta\theta}}{\partial \varphi} < 0 \tag{7}$$

From the numerical calculations, the following kinking angle is derived

$$\varphi_0 = 42^\circ ,$$

which is in agreement with the test results.

4. Proposed Damage Model

The proposed damage model [8, 11] based on interface finite elements is presented in order to simulate damage onset and growth into the core of a sandwich structure. A linear constitutive relationship between stresses σ and relative displacements δ between homologous points of the interface elements with zero thickness is considered. At first developing the pure-mode model, the constitutive equation before damage starts to grow, is given by:

$$\sigma = D\delta \tag{8}$$

where D is the diagonal matrix containing the penalty parameters (*d*) in each mode. The values of the penalty parameter must be quite high in order to hold together and prevent interpenetration of the element faces. According to a considerable number of numerical simulations [8, 11], it was determined $d = 10^6$N/mm^3. For this value the produced results converge and numerical problems during the nonlinear procedure have been avoided.

After the peak stress $(\sigma_{u,i})$ is reached a gradual softening process between the stress and the relative displacement is observed and is defined as:

$$\sigma = \left(I - E\right) D\delta \tag{9}$$

where I is the identity matrix and E is a diagonal matrix containing the damage parameter:

$$e = \frac{\delta_{u,i}(\delta_i - \delta_{o,i})}{\delta_i(\delta_{u,i} - \delta_{o,i})}, \quad i = I, II, \tag{10}$$

and, $\delta_{o,i}$, is the displacement corresponding to the onset of damage, and δ_i is the current relative displacement. In pure-mode loading, the strength along other directions is abruptly cancelled. The maximum relative displacement, $\delta_{u,i}$ for which complete failure occurs, is obtained by equating the area under the softening curve to the respective critical fracture energy:

$$G_{c,i} = \frac{1}{2}\sigma_{u,i}\delta_{u,i} \tag{11}$$

In thick composite structures under flexural loading such as in ship hulls, failure is more likely to occur under a mixed-mode situation. Therefore, a formulation for interface elements having zero thickness should include a mixed-mode damage model, which, in this case, is an extension of the pure mode model. Damage initiation may be predicted by using the following criterion [8, 11]:

$$\begin{aligned} &\left(\frac{\sigma_I}{\sigma_{u,I}}\right)^2 + \left(\frac{\sigma_{II}}{\sigma_{u,II}}\right)^2 = 1 && \text{if} \quad \sigma_I > 0 \\ &\sigma_{II} = \sigma_{u,II} && \text{if} \quad \sigma_I \le 0 \end{aligned} \tag{12}$$

where $\sigma_{u,I}$, $\sigma_{u,II}$ represent the ultimate normal and shear stresses, respectively and it is assumed that normal compressive stress does not induce damage. Providing an equivalent mixed-mode displacement:

$$\delta_e = \sqrt{\delta_I^2 + \delta_{II}^2} \tag{13}$$

and a mixed-mode ratio:

$$\beta = \frac{\delta_{II}}{\delta_I} \tag{14}$$

and taking into considering Eq. (8), we have from Eq. (12):

$$\left(\frac{\delta_{om,I}}{\delta_{o,I}}\right)^2 + \left(\frac{\delta_{om,II}}{\delta_{o,II}}\right)^2 = 1 \tag{15}$$

where $\delta_{om,i}(i = I, II)$ are the relative displacements at damage initiation, which correspond to the critical interface stresses $\sigma_{um,i}$. Combining Equations (13)-(15), the value of the equivalent mixed-mode displacement leading to damage initiation (δ_{om}) results:

$$\delta_{om} = \frac{\delta_{o,I}\delta_{o,II}\sqrt{1+\beta^2}}{\sqrt{\delta_{o,II}^2 + \beta^2\delta_{o,I}^2}} \tag{16}$$

The mixed-mode damage propagation is simulated considering the linear fracture energetic criterion:

$$\frac{G_I}{G_{Ic}} + \frac{G_{II}}{G_{IIc}} = 1 \tag{17}$$

The released energy in each model at complete failure can be obtained from the area of the triangle:

$$G_i = \frac{1}{2}\sigma_{um,i}\delta_{um,i} \tag{18}$$

being $\delta_{um,i}(i = I, II)$, the relative displacement in each direction for which complete failure occurs. From Equations (8), (11), (13), (14), (17) and (18), the mixed mode relative displacement leading to total failure (δ_{um}) can be obtained:

$$\delta_{um} = \frac{2(1+\beta^2)}{e\delta_{om}}[\frac{1}{G_{Ic}} + \frac{\beta^2}{G_{IIc}}] \tag{12}$$

The values of δ_e, δ_{om} and δ_{um} are introduced into Eq. (10), instead of δ_i, $\delta_{o,i}$ and $\delta_{u,i}$, thus setting the damage parameter under mixed mode. The mixed mode model proposed is general and it can be applied under any combination of modes.

The same sandwich beam considered in section 3 under three point bending with a crack parallel to the beam axis very close to the upper skin interface (Figure 1), is solved numerically. But in this case we adjust the model according to the demands of the previous analysis. Different models with different cohesive parameters, different crack lengths, different crack positions and orientations, may be confronted. The element type used in this analysis is the four-node two dimensional plane strain elements CPE4 [16]. Different mesh configurations were used in the vicinity of the crack tip and the cohesive layer in order the convergence of the solution to be succeeded. The cohesive layer is placed over the entire plane of crack propagation (Figure 3) by implementing the procedure given in Abaqus [15].

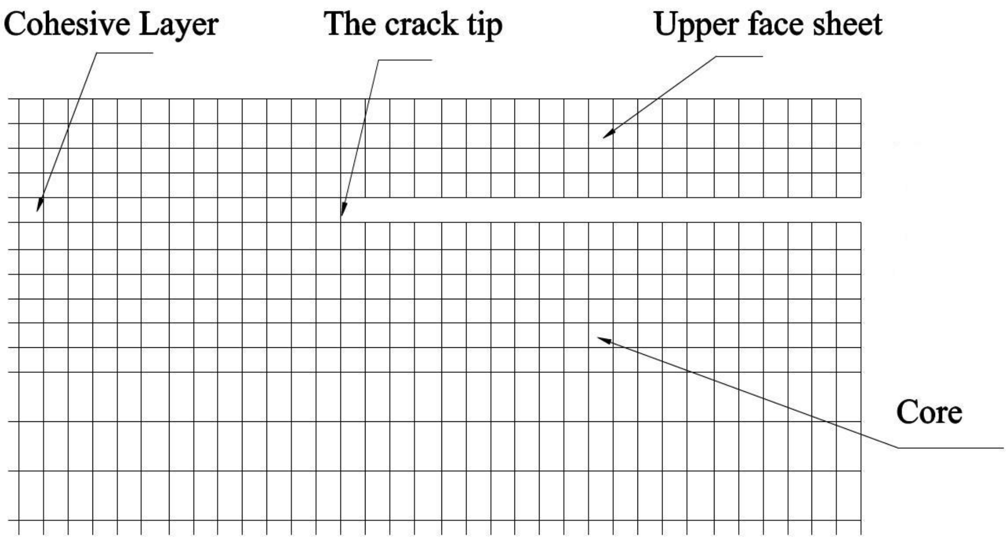

Figure 3. The finite elements mesh at the crack tip.

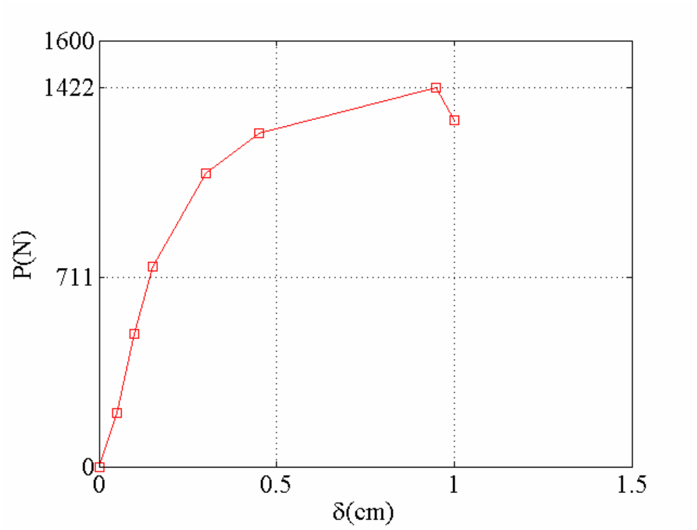

Figure 4. Numerical and experimental results for mixed mode loading

The fractured sandwich beam is modeled according to a mesh generator proposed in [5] which imposes the discretion of the areas of the beam regarding the mesh density and quality in three different regions according to the length of the crack and to crack distance from the upper interface [5]. This discretion is a prerequisite in order to avoid the worthless consumption of computational resources and time. The model shown in Fig. 3 corresponds to the maximum crack length $a \approx 70\text{mm}$ in which about 22500 elements have been used in the finite element analysis. The experimental data and the numerical predictions according to the proposed cohesive law are shown in Figure 4. At the load at failure ($P \simeq 1422\text{N}$) kinking of the crack into the core and catastrophic failure was observed.

5. Conclusions.

In this paper two procedures have been considered for lightweight structures in order to simulate crack propagation and kinking into the core of sandwich structures very close to the upper skin interface under mixed mode loading conditions. In the first one the crack kinking analysis based on linear fracture mechanics approach and to crack propagation criteria, was considered. Stress intensity factors were calculated and the crack kinking into the core was predicted. In the second one a cohesive damage model was developed to simulate crack propagation and kinking into the core in terms of cohesive parameters. The damage model was implemented in a finite element model. The numerical applications analyzed were in good agreement with the experimental results and in addition predict satisfactory the crack propagation and kinking into the core.

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