Optimal Design of Metallic Corrugated Sandwich Plates to Blast Loading

Jianxun Zhang[1], **Qinghua Qin**[1,*], **Weilong Ai**[1], **Tao Wang**[1, 2], **Tiejun Wang**[1,*]

[1] State Key Laboratory for Strength and Vibration of Mechanical Structures, Department of Engineering Mechanics, Xi’an Jiaotong University, Xi’an 710049, China
[2] Department of Engineering Physics, Xi’an Research Institute of High-Tech., Xi’an 710025, China
* Corresponding author: qhqin@mail.xjtu.edu.cn; wangtj@mail.xjtu.edu.cn

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Abstract Optimal designs of the resistance of metallic corrugated sandwich plates under blast loading are studied numerically and analytically. The dynamic response of metallic sandwich plates under blast loading is numerically analyzed. Comparisons among the corrugated sandwich plates made of the same material and equivalent weight are conducted. The study of weight optimization is carried out, such as the relative core density, core height, the ratio of core height to half-length of the unit, and the thickness ratio of the top face sheet to bottom one. Based on the optimal designs, the sandwich plates possessing better performance are obtained. Three-dimensional finite element (FE) simulations are performed and the FE predictions are in good agreement with the theoretical predictions.

Keywords Sandwich plate, Sinusoidal plate core, Dynamic response, Optimal design

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1. Introduction

Lightweight structures have been widely used in engineering, such as vehicles, ships, aircrafts and spacecrafts. Sandwich structures are typical lightweight structures with a number of advantages, e.g. ultralight, high ductility, high crashworthiness, high strength to weight ratio, high stiffness to weight ratio, multifunction. Lightweight metallic sandwich beams, plates and panels with various cores have received great attention. Several kinds of metallic cores are developed, such as metallic foams, lattice materials and corrugated cores [1-3].

In the past decades, some work has been devoted to analyzing the dynamic response of sandwich structures subjected to blast loading. Fleck and Deshpande [4] theoretically investigated the dynamic response of fully clamped metal sandwich beams under uniform transverse blast loading. Subsequently, Qiu et al. [5] developed an analytical model for dynamic response of fully clamped sandwich beams under impulsive loading over a central patch. More recently, Qin and Wang [6] and Qin et al. [7] derived new yield criteria for symmetric and geometrically asymmetric metal sandwich structures, in which the effect of core strength is incorporated. Based on the yield criterion, Qin and Wang [8] and Qin et al. [9] analytically studied the dynamic response of fully clamped metal sandwich beams subjected to impulsive loading using the membrane factor method, in which the interaction of bending and stretching is considered.

Xue and Hutchinson [10] numerically studied the dynamic response of metal sandwich plates subjected to air blast loading. The study of weight optimization was carried out for pyramidal truss, square honeycomb and folded plate with respect to the respective geometric parameters, including core and face sheet thickness, core member aspect ratios and relative density. Zhang et al. [11] numerically studied the resistance of the trapezoidal plate core sandwich plates, and the FE predictions were compared with analytical solutions, showing good agreement. In additions, numerical calculations were carried out to study the dynamic response of metal sandwich panels, beams and plates with different kinds of cores [12-14] subjected to underwater blast loading and air blast loading.

The objective of this study is to numerically and analytically investigate the optimal design of dynamic response of corrugated metal sandwich plates with sinusoidal plate core. This paper is organized as follows. Firstly, the problem is stated and the topology of sandwich plate with sinusoidal plate core is introduced in Section 2. The analytical solutions to predict the dynamic response of asymmetric sandwich plates are presented in Section 3. In Sections 4 and 5, the analytical solutions of the dynamic response of the sandwich plates under impulsive loading are compared with the finite element results. Furthermore, the study of weight optimization is carried out, such as relative core density, core height, the ratio of core height to half-length of the unit, and the thickness ratio of top and bottom face sheets. Finally, concluding remarks are presented in Section 6.

2. Statement of the problem

Here, we consider a metal sinusoidal plate core sandwich plate with infinite length in x direction under impulsive loading I, as shown in Figs. 1 and 2(a). Clamped conditions are imposed along the two sides in z direction. The top and bottom face sheets with thicknesses h_t and h_b are perfectly bonded to the sinusoidal plate core with the height of the core H_c, as shown in Fig. 2(b). The half-width of the sandwich plate and half-length of the unit cell are L and l, and the thickness of the core web is h_c. The relative core density $\bar{\rho} = \rho_c / \rho_s$ with ρ_c and ρ_s being the density of the core and core web material, respectively, can be calculated as $\bar{\rho} = h_c \cdot l_s \cdot \tan\phi / H_c^2$ [15], where l_s is the arc length of the core web in the half unit cell and $\tan\phi = H_c / l$.

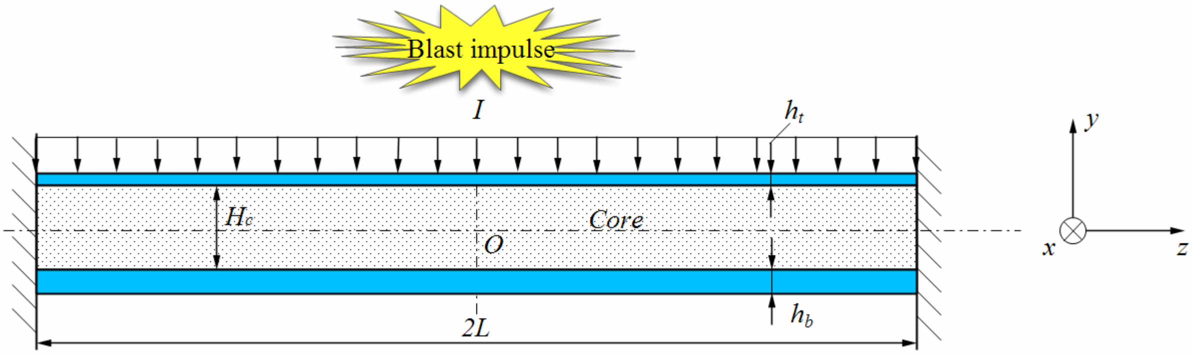
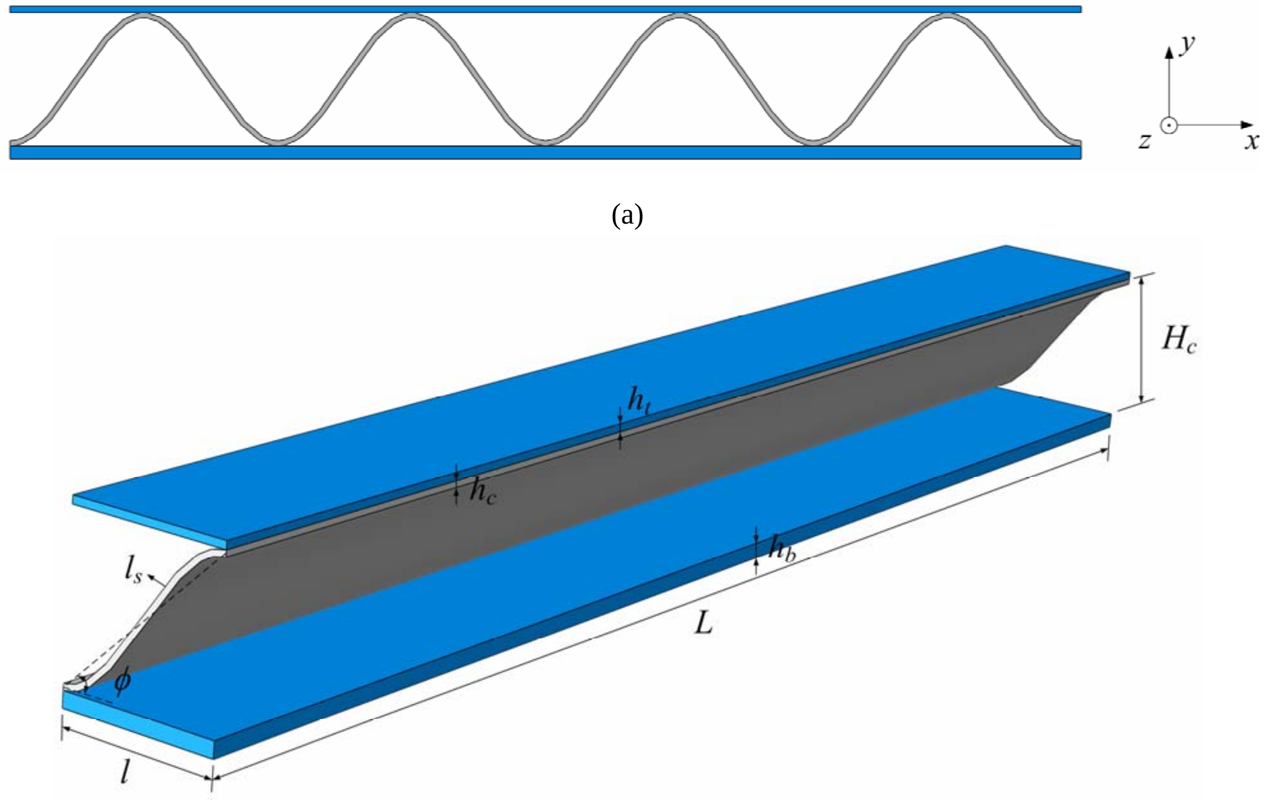


Figure 1. Sketch of a fully clamped sandwich plate subject to impulsive loading



(a)

(b)

Figure 2. Sketches of the metallic corrugated sandwich plates with sinusoidal plate core. (a) Topology of the sandwich plate, and (b) a quarter of unit cell for the sandwich plate

3. Analytical solutions for the dynamic response of asymmetric sandwich plates

In this section, employing the similar procedure to the plastic-string model [15] for the dynamic response of symmetric sandwich plates, the analytical solutions for dynamic response of the fully clamped asymmetric sandwich plates under the impulsive loading *I* are obtained shown in Fig. 1.

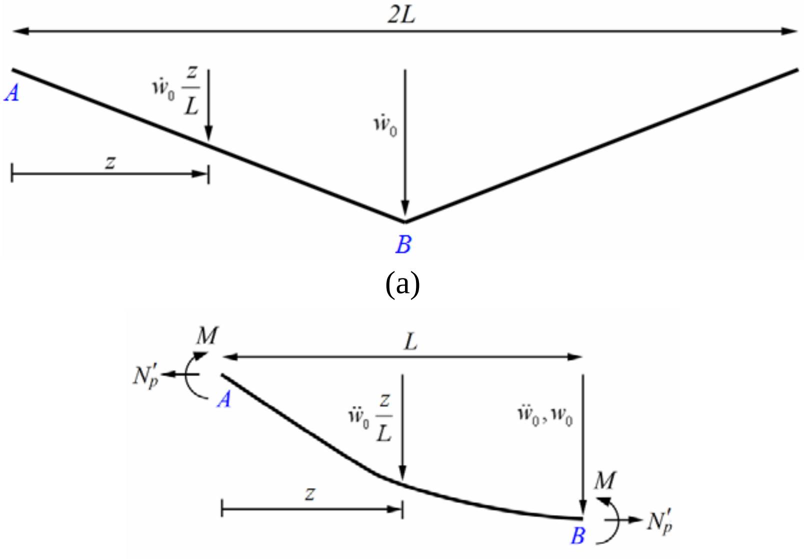
It is assumed that the top and bottom face sheets obey the rigid-perfectly plastic law with the yield strength σ_Y, and the metal sandwich core is modeled as a rigid-perfectly-plastic-locking (*r-p-p-l*) material with a plateau-stress level of σ_{nY} and a critical densification strain ε_D.

The phase of core compression is the same as that for dynamic response of symmetric sandwich beam [4]. It is assumed that the longitudinal plastic membrane force N_p is insensitive to the degree of the core compression [4], and then obtained

$$N_p' = N_p = \sigma_Y b\left(h_t + h_b\right) + \sigma_{lY} b H_c \tag{1}$$

where *b* is the length in *x* direction, σ_Y is the yield strength of face sheet material, and σ_{lY} is the longitudinal compressive strength of core.

The velocity field is assumed as $\dot{w} = \dot{w}_0 z/L$ when $0 \le z \le L$, where $\dot{w}_0$ is the velocity at the midspan of the plastic-string, shown in Fig. 3(a) for half of the plastic-string. In the phase of dynamic structural response for the plastic-string model [15], the dynamic response of symmetric sandwich beam is dominated by axial (membrane) force alone. However, the asymmetric sandwich beam has axial (membrane) force N_p' and bending moment *M*, shown in Fig. 3(b). This is because that the plastic neutral surface of asymmetric sandwich structure is usually different from the geometric surface, and the geometric and plastic neutral surfaces of the symmetric sandwich beam are coincident.



(a)

(b)

Figure 3. Sketches of the deformation process of a fully clamped plastic-string. (a) The velocity field and (b) a free body diagram of the half of the plastic-string.

Considering the conservation of the moment of momentum for half of the sandwich plate shown in Fig. 3(b) with respect to the fixed end support Point *A* at time *t*, we have

$$M + N_p' w_0 - M = -\frac{d}{dt}\int_0^L \left(\rho_s b h_t + \rho_s b h_b + \rho_c b H_c\right)\dot{w} z\, dz \tag{2}$$

where w_0 is the deflection at the midspan. The continuity conditions are $w_0(t=0)=0$ and $\dot{w}_0(t=0)=V_f$, where the final common velocity of the core and two face sheets at the end of the core compression stage $V_f = I/\left(\rho_s h_t + \rho_s h_b + \rho_c H_c\right)$ [4]. If $\dot{w}_0\left(t=T_f\right)=0$, the motion of the plastic-string ends. Then, the maximum deflection $w_1 = w_0\left(t=T_f\right)$ and structural response time T_f of the asymmetric sandwich plate can, respectively, be written as

$$w_1 = \frac{IL}{\sqrt{3\left(\sigma_Y h_t + \sigma_Y h_b + \sigma_{lY} H_c\right)\left(\rho_s h_t + \rho_s h_b + \rho_c H_c\right)}} \tag{3}$$

and

$$T_f = \frac{\pi}{2} L \sqrt{\frac{\rho_s h_t + \rho_s h_b + \rho_c H_c}{3\left(\sigma_Y h_t + \sigma_Y h_b + \sigma_{lY} H_c\right)}} \tag{4}$$

To simplify analysis, we introduce the following nondimensional geometric and property variables of the sandwich plate:

$\bar{H}_c = \dfrac{H_c}{L}$, $\bar{h} = \dfrac{h_t + h_b}{2H_c}$, $\alpha = \dfrac{h_t}{h_b}$, $\bar{w}_1 = \dfrac{w_1}{L}$, $\bar{I} = \dfrac{I}{\left(\rho_s h_t + \rho_s h_b + \rho_c H_c\right)\sqrt{\sigma_Y / \rho_s}}$, $\bar{\sigma}_l = \dfrac{\sigma_{lY}}{\sigma_Y}$ and $\bar{T}_f = \dfrac{T_f}{L\sqrt{\rho_s / \sigma_Y}}$.

Then, Eqs. (3) and (4) yield the following dimensionless maximum deflection and structural response time of the asymmetric sandwich plate,

$$\bar{w}_1 = \frac{\bar{I}\beta}{\sqrt{3}} \tag{5}$$

and

$$\bar{T}_f = \frac{\pi}{2\sqrt{3}}\beta \tag{6}$$

respectively, where $\beta = \sqrt{\dfrac{\bar{\rho} + 2\bar{h}}{\bar{\sigma}_l + 2\bar{h}}}$.

Substitution of the equation of normalized longitudinal compressive strength $\bar{\sigma}_l$ for sinusoidal plate core [15] into Eqs. (5) and (6) yields the following dimensionless maximum deflection and structural response time of the asymmetric sandwich plate with sinusoidal plate core,

$$\bar{w}_{1s} = \frac{\bar{I}}{\sqrt{3}} \tag{7}$$

and

$$\bar{T}_{fs} = \frac{\pi}{2\sqrt{3}} \tag{8}$$

Eqs. (5), (6), (7) and (8) can reduce the solutions for the dynamic response of symmetric sandwich plates under the impulsive loading [15].

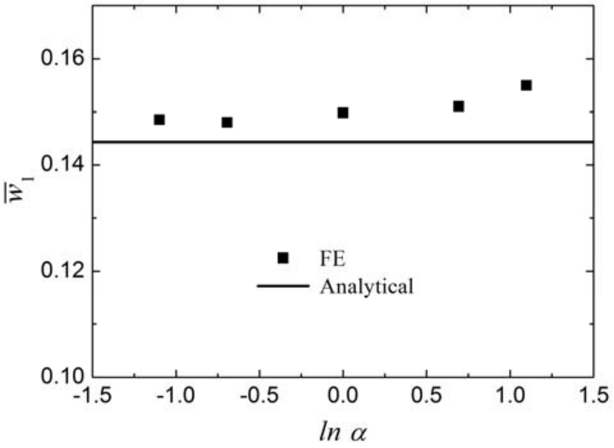
4. Numerical calculations

Finite element calculations are carried out to predict the dynamic response of the corrugated sandwich plates with sinusoidal plate cores. All simulations have been carried out using ABAQUS/Explicit code. Eight-node linear brick elements with reduced integration (Type C3D8R) are used to model the face sheets and core webs in ABAQUS/CAE software. The uniform distributed impulsive loading *I* per area is applied to the top face sheet of the sandwich plate as a uniform initial velocity $V_0 = I/\rho_s h_t$ [10,12]. Periodic boundary conditions are applied at each end of the repeating unit in *x* direction, and symmetrical boundary conditions about the centerline are adopted. The computational model of the corrugated sandwich plate is shown in Fig. 1(b). All the contact of the plates is modeled by using a general contact algorithm with a frictionless contact option in ABAQUS/Explicit. The vertical, horizontal and rotational displacements of nodes at the ends of the plate are zero. The face sheets and core web are made of stainless steel with yield strength $\sigma_Y = 400\text{MPa}$, Young's modulus $E_s = 200\text{GPa}$, yield strain $\varepsilon_Y = 0.2\%$, elastic Poisson's ratio $v_e = 0.3$, density $\rho_s = 7850\,\text{kg}/\text{m}^3$ and linear hardening modulus $E_t/\sigma_Y = 12$, respectively. It is assumed that the face sheets and core web materials have sufficient ductility to be able to sustain deformation without fracture. The face sheets and core webs are modeled as J_2 flow theory of plasticity.

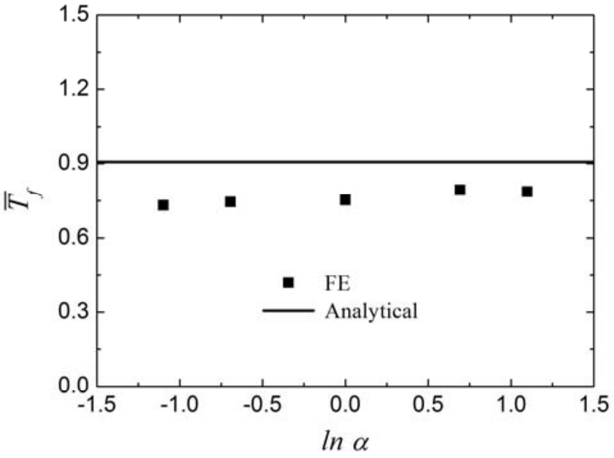
5. Results and discussion

5.1. Effect of the asymmetric factor

Comparisons of the analytical solutions and numerical results for the normalized maximum deflection $\bar{w}_1$ of the bottom face sheets and the nondimensional structural response time $\bar{T}_f$ for the asymmetric sandwich plates subjected to $\bar{I} = 0.25$ are shown in Figs. 4(a) and (b), respectively. The sandwich plates with sinusoidal plate core have $\bar{H}_c = 0.1$, $\bar{h} = 0.08$, $\phi = 45^\circ$, $\bar{\rho} = 0.055$ and $L = 1\text{m}$. In Fig. 4(a), it is seen the analytical solutions based on Eq. (7) are in good agreement with the numerical results and underestimates the numerical ones a little. Actually, the discrepancies between the analytical and numerical solutions lie in that the analytical procedure does not consider the effects of the wrinkling of face sheets and cores, the strain hardening of the metal material, the effect of shear force and the reduction in momentum provided by the supports in the core compression phase. The discrepancies in high impulsive loading may be due to the assumption that there is full densification of the core in the analytical solution while in the numerical solution there is no distinct densification in the core. In Fig. 4(a), it is seen that the numerical result for the smallest maximum deflection occurs in the case of $\alpha = 0.5$. Moreover, seen the deformed configurations at the impulse value $\bar{I} = 0.25$ in Figs. 5(a) and (b), the deformation of top and bottom face sheets in the case of $\alpha = 2$ both almost keeps horizontal at the midspan, while the top face sheet of the sandwich plate in the case of $\alpha = 0.5$ occurs more evident deformation in the location far away the core web relative to the location bonded to the core. In Fig. 4(b), the analytical solutions based on Eq. (8) for the structural response time agree well with the numerical results. The numerical results for the structural response time keep almost a constant for the several cases of α.



(a)



(b)

Figure 4. Analytical solutions and numerical results for (a) the normalized maximum deflection $\overline{w}_1$ of the bottom face sheets and (b) the nondimensional structural response time $\overline{T}_f$ for the asymmetric sandwich plates

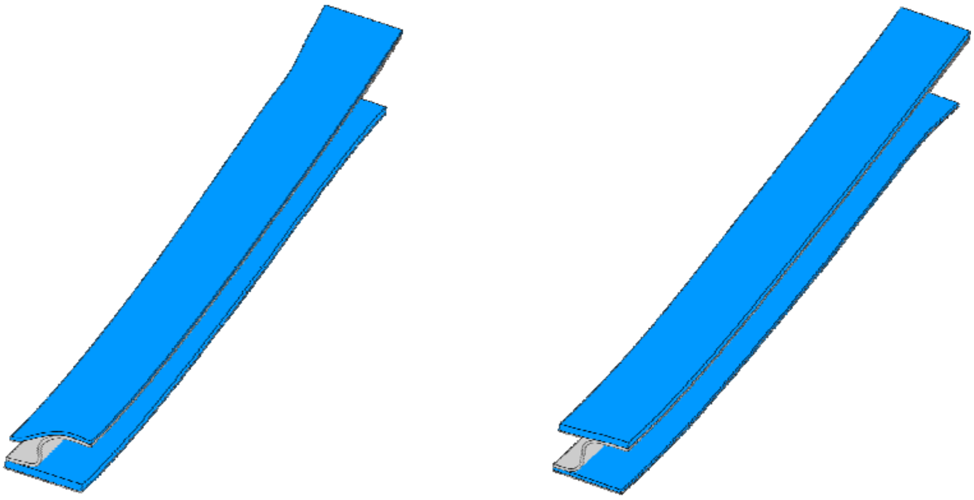
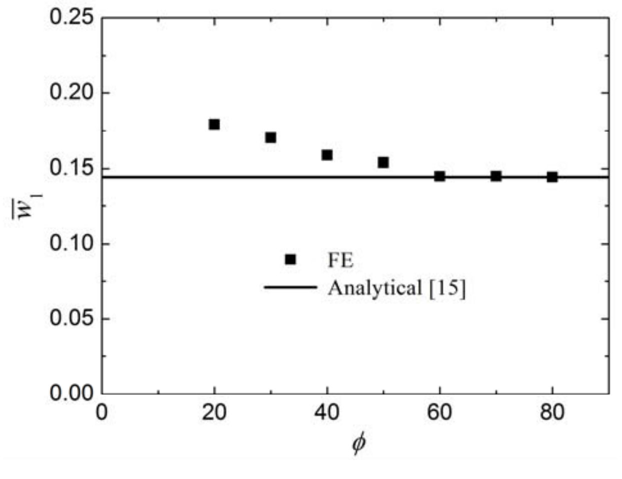


Figure 5. Deformed configurations of sinusoidal plate core sandwich plates subjected to $\overline{I} = 0.25$. (a) The case of $\alpha = 0.5$, (b) the case of $\alpha = 2$

5.2. Effect of the core height

Fig. 6(a) show the effect of the normalized core height $\bar{H}_c$ to the normalized maximum deflection $\bar{w}_1$ of the metal sandwich plates under $\bar{I} = 0.25$, in which $\alpha = 1$, $2\bar{h}\bar{H}_c + \bar{\rho}\bar{H}_c = 0.02$, $\phi = 45^{\circ}$, $\bar{\rho} = 0.04$ and $L = 1\text{m}$. Obviously, the analytical solutions [15] agree well with the numerical results for the moderate $\bar{H}_c$, and underestimate and overestimate numerical results for the cases of small and big $\bar{H}_c$. It is readily seen that the angle ϕ significantly affects the numerical results for maximum deflection of bottom face sheets in Fig. 6(a). The normalized maximum deflection of bottom face sheet has smallest value when $\bar{H}_c = 0.28$.

(a)



(b)

Figure 6. The effect of the (a) normalized core height $\bar{H}_c$ and (b) angle ϕ to the normalized maximum deflection $\bar{w}_1$ of the bottom face sheets of sandwich plates

5.3. Effect of the ratio of core height to half-length of the unit

Comparisons of the analytical [15] and numerical solutions for the normalised maximum deflection $\bar{w}_1$ of the bottom face sheets of the sandwich plates versus the angle ϕ are shown in Fig. 6(b). The angle ϕ is varied and other values ($\bar{I} = 0.25$, $\bar{H}_c = 0.1$, $\bar{h} = 0.08$, $\bar{\rho} = 0.04$, $\alpha = 1$ and

$L=1\mathrm{m}$) are fixed. The analytical solutions agree well with the numerical results for the case of big ϕ. The maximum deflection decreases with the increasing of ϕ, and remains more or less constant when ϕ exceeds 60^0. In other words, when ϕ is bigger than 60^0, the blast-resistance performances of the bottom face sheets of the sandwich plates are good.

5.4. Effect of the relative core density

The effect of relative core density $\bar{\rho}$ to the normalized maximum deflection $\bar{w}_1$ of the metal sandwich plates subjected to $\bar{I}=0.25$ is shown in Fig. 7, in which $\bar{H}_c=0.1$, $2\bar{h}\bar{H}_c+\bar{\rho}\bar{H}_c=0.02$, $\phi=45^{\circ}$, $\alpha=1$ and $L=1\mathrm{m}$. The analytical solutions [15] are a little smaller than numerical results in a whole, and the bottom face sheet has the smallest maximum deflection at a relative core density 5.5%.

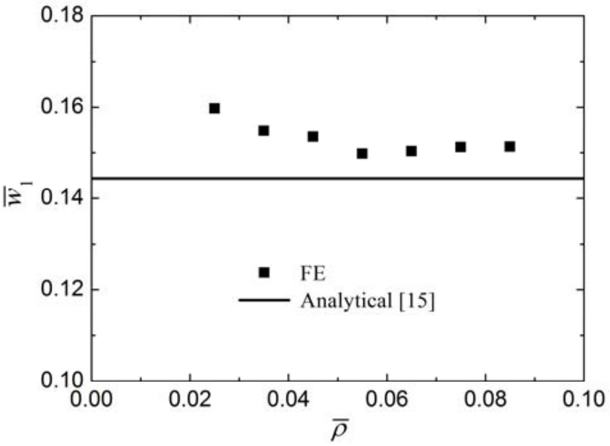


Figure 7. The effect of the relative core density $\bar{\rho}$ to the normalized maximum deflection $\bar{w}_1$ of bottom face sheets of the sandwich plates

6. Concluding remarks

The dynamic response of corrugated sinusoidal plate core sandwich plates under the impulsive loading has been investigated. Good agreement between the analytical predictions and numerical results is achieved. The optimizations of topology with respect to the respective geometric parameters have been obtained. It is shown that the axial force plays an important role in the dynamic large deflection response of corrugated metal sandwich plates and the present analytical procedure is simple and efficient.

Acknowledgements

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