Seismic Inversion using full wavefied data

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ABSTRACT

Most seismic migration methods stress primary reflection data, it means that multiples is considered as noise and need to be removed prior to migration process. In fact multiples contain important medium property information which can be utilized to improve the quality of image.

By introducing a background model, the difference between the known background properties and unknown real medium properties is inverted. Each gridpoint act as a scatterer in the background media. In our work, Finite-difference contrast source inversion method will be applied in seismic inversion with full wavefield data including surface-related and internal multiples, without including explicit computation of forward problem at each iteration step, this method can employ an inhomogeneous background medium. By solving an optimization problem to reconstruct the contrast function χ , which is a quantity having some connection with the real velocity distribution in the medium. Result of the process is an accurate angle-dependent reflectivity image as well as angle-dependent velocity model in subsurface.

Key words: migration, inversion, anisotropy, multiple, scatter

1. INTRODUCTION

Generally speaking, there are two possible approaches for velocity estimation from surface recorded seismic data, the two methods differ from the domain in which the information is used to update the velocity model. The first approach is formulated in the data space prior to migration, and it involves matching of recorded and simulated data using an approximate velocity model. The second approach is formulated in the image space after migration, in which it involves measuring and correcting image features that indicate model properties inaccuracies.

Nowadays, migration and inversion have been two different approaches to reveal the subsurface information [4,6,9-11], Their fundamental theory is very different. But we can treat migration and inversion as a same form in mathematics. Although they have different physical meaning. So we can incorporate migration and inversion process together to analyze the seismic data[1].

One of the well-known full nonlinear inversion methods based on the integral equation(IE) formulation is the contrast source inversion (CSI) method[7,8]. The main disadvantage of this approach based on integral equation formulation is that the background medium is usually a simple homogeneous medium.

Finite-difference based contrast source inversion(FDSCI) method is adaptive to the arbitrary inhomogeneous background medium[2,3,5,12], similar to the IE contrast source inversion method, the unknown contrast source and the unknown contrast function are updated alternately to reconstruct the scatters without requiring the solution of forward problem at each iteration step. An attractive feature of FDCSI approach is that the impendence matrix is only dependent on the background medium, which is invariant throughout the inversion process, Hence, the FD operator needs to be inverted only once and the results can be reused for multiple source positions.

2. THEORY

2.1 Joint migration and inversion

In the joint migration and inversion process, first extrapolate the receiver wavefield from depth level z_{m-1} to z_m, then inversion process is applied at depth z_m, and we will use the wavefield from all depth level which represents the internal and surface-related multiples. In full wavefield inversion process, the difference between the known background properties and the unknown real medium properties is inverted. Each gridpoint act as a point scatterer in the background medium, called as contrast source. The total wavefield in the true medium equals the sum of the wavefield of the primary sources in the background and the wavefield of all point scatterers in the background. The contrasts and the total wavefield are alternately updated until the total simulated wavefield matches the recorded wavefield at the detector positions. After that the full wavefield migration is applied to yield the angle-dependent reflectivity, which is solved as a constrained least-squares minimization problem. The two minimization process is in the same mathematic form[1]:

$$Q^{-}(z_m,z_0)-\left[U(z_m,z_m)P^{+}(z_m,z_0)+V(z_m,z_m)P^{-}(z_m,z_0)\right]=\min \tag{1}$$

In which P represents the incident wavefield and Q refers to response of depth z_m

In the migration process, the matrices U and V represent reflectivity:

$$U(z_m,z_m)=(R_1^{\mathrm{U}},R_2^{\mathrm{U}},\cdots R_k^{\mathrm{U}},\cdots) \tag{2}$$

$$V(z_m,z_m)=(R_1^{\mathrm{I}},R_2^{\mathrm{I}},\cdots R_k^{\mathrm{I}},\cdots) \tag{3}$$

In the inversion process, the matrices U and V are same and represent contrast:

$$U(z_m,z_m)=V(z_m,z_m)=(\chi_1,\chi_2,\cdots \chi_k,\cdots) \tag{4}$$

In the JMI process full wavefield extrapolation is applied from z_{m-1} to z_m, using the wavefield Q_j^+ and p_j^- at depth level z_{m-1}, this extrapolation step yields an estimate of the total wavefield, being denoted by $< p_j >$ at intermediate depth level z_n $\left(z_{m-1} < z_n < z_m\right)$:

$$< p_j(z_n, z_0) >= W_0(z_n, z_{m-1})Q_j^+(z_{m-1}, z_0) + W_0^*(z_n, z_{m-1})\, p_j^-(z_{m-1}, z_0) \tag{5}$$

W, W^* represent propagation operator and its conjugate complex respectively.

Next, a full waveform inversion step is applied:

$$p_j(z_n, z_0) =< p_j(z_n, z_0) > +\Delta p_j(z_n, z_0) \tag{6}$$

$$\Delta p_j(z_n, z_0) = \sum_{l=m-1}^{m} G_0(z_n, z_l) \sum_k \chi_k(z_l, z_l)\, p_{kj}(z_l, z_0) \tag{7}$$

yielding an update of the velocities in each gridpoint of layer $\left(z_{m-1}, z_m\right)$ and an update of the wavefield at z_m, G refers to Green's function:

$$p_j^+(z_m, z_0) = W_0(z_m, z_{m-1})Q_j^+(z_{m-1}, z_0) + \Delta p_j(z_m, z_0) \tag{8}$$

$$Q_j^-(z_m, z_0) = W_0^*(z_m, z_{m-1})\, p_j^-(z_{m-1}, z_0) + \Delta p_j(z_{m-1}, z_0) \tag{9}$$

After completion, the updated velocities in layer $\left(z_{m-1}, z_m\right)$ as well as the reflectivity and updated wavefield at depth level z_m are known, we can apply the following migration and inversion process at deeper depth recursively.

2.2 Contrast source inversion

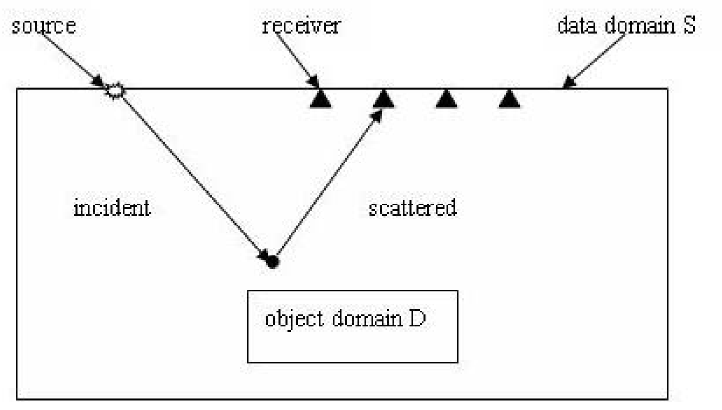

Figure 1. Configuration of the scattering problem

Denote the inversion domain as the object domain D and the data domain S as the domain where the sources and the receivers are located, combination of domain D and domain S are called total domain T (see Figure 1).

Total field $p_j(r)$ satisfies the Helmholtz equation[2,3]:

$$\left[\Delta^2 + k^2(r)\right] p_j(r) = -Q(r, r_j^s), \quad r \in T \tag{10}$$

Where Q is the source term.

We split the total field into its incident and scattered parts:

$$p_j = p_j^{inc} + p_j^{sct} \tag{11}$$

The incident field satisfies the equation:

$$\left[\nabla^2 + k_b^2(r)\right] p^{inc}{}_j(r) = -Q\left(r, r_j^S\right), \quad r \in T \tag{12}$$

Subtracting equation (12) from equation (10), then the scattered field satisfied the equation:

$$\left[\nabla^2 + k_b^2(r)\right] p^{sct}{}_j(r) = -k_b^2(r) w_j(r), \quad r \in T \tag{13}$$

Where $w_j(r)$ are the contrast sources, defined as:

$$w_j(r) = \chi(r) p_j(r) \tag{14}$$

In which the contrast function $\chi(r)$ is given by:

$$\chi(r) = \left[\frac{k(r)}{k_b(r)}\right]^2 - 1 = \frac{c^{-2}(r) - c_b^{-2}(r)}{c_b^{-2}(r)} \quad r \in D \tag{15}$$

Here, $c^{-2}(r) = \rho k(r)$ is the velocity of the scattering object, and $c_b^{-2}(r) = \rho k_b(r)$ is the velocity of the background medium.
Equation (13) can be written using operator notation:

$$H_b(p_j^{sct}(r)) = -k_b^2(r) w_j(r), r \in T \tag{16}$$

The solution of equation (16) can be written as:

$$p_j^{sct}(r) = H_b^{-1}\left[-k_b^2(r) w_j(r)\right] = L_b\left[w_j(r)\right], r \in T \tag{17}$$

Introducing an operator M^s that interpolates the field values defined at the finite-difference grids to the appropriate receiver position, the data equation for the contrast function χ can be written as:

$$p^{sct}{}_j(r) = M^s\{L_b[w_j(r')]\} \quad r \in S, r' \in T \tag{18}$$

Introducing an operator M^D that selects fields only on the object domain, we obtain the object equation for the contrast source w_j:

$$\chi p_j^{inc} = w_j - \chi M^D \left\{ L_b \left[w_j \right] \right\} \tag{19}$$

We handle the inverse problem as a minimization of a cost function, being a linear combination of errors in the data equation and the object equation, the method alternatively constructs sequence of contrast sources by a conjugate gradient iterative method such that the cost function is minimized. And the contrast function is then determined to minimize the error in the object equation. The cost function is a superposition of the errors in the data equations and errors in the object equations:

$$F(\chi, w_j) = F^S(w_j) + F^D\left(\chi, w_j\right) = \frac{\sum_j \left\| f_j - M^S \left\{ L_b \left[w_j \right] \right\} \right\|_S^2}{\sum_j \left\| f_j \right\|_S^2} + \frac{\sum_j \left\| \chi p_j^{inc} - w_j + \chi M^D \left\{ L_b \left[w_j \right] \right\} \right\|_D^2}{\sum_j \left\| \chi p_j^{inc} \right\|_D^2} \tag{20}$$

In which f_j is the scattered measured data.

The L_2-norms on domains S and domain D are defined as follow:

$$\left\| v_j \right\|_S^2 = \int_S v_j(r)\overline{v_j(r)}dr \qquad \left\| v_j \right\|_D^2 = \int_D v_j(r)\overline{v_j(r)}dr \tag{21}$$

Where the overbar denote the complex conjugate of a function.

We minimize the cost function (20) by conjugate gradient method, When the updated value of χ is obtained, we can produce the real velocity distribution of the medium. If the value of the cost function is not smaller than the prescribed error criterion, the update step will be repeated until convergence is achieved. The flow of the inversion and migration can be seen in Figure 2.

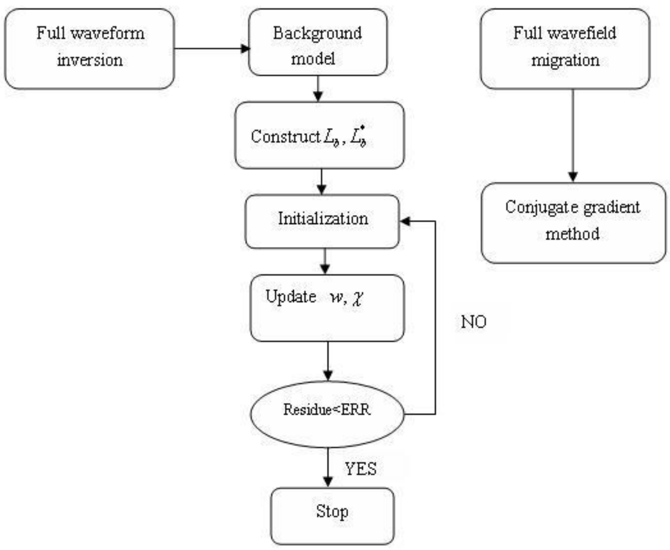

Figure 2. Flowchart of the full wavefield inversion and migration method

SUMMARY

Surface-related and internal multiples can be vital information for revealing the deep subsurface information, because the multiple wavefield can strength the illumination power towards the deeper subsurface. By applying the contrast source inversion method, I wish to produce the accurate velocity distribution and angle-dependent reflectivity of the subsurface model.

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