

Equivalent dynamic mechanical properties of grid cylindrical structure

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Abstract Grid cylindrical structure with infinite variety of cell shape and arrangement receives much concern, which results in many applications of these structural materials in aerospace and aviation engineering. However, the complexity of structure will bring high time-consuming in calculation and unanticipated self-contrast in researching dynamic performance. Equations were used to get the equivalent material properties by theoretical derivation. Then dynamic mechanical properties of both equivalent structure and grid cylindrical structure were investigated. In detail, the dynamic deformation and energy absorption characteristics of the structure under different lateral impact velocities were analyzed by software LS-DYNA. The relative density and the relative thickness of the grid cylindrical structure to equivalent performance effect were taken into account. The results show that when relative density and relative thickness are defined in certain range, the equivalent method has high accuracy. This equivalent method provides a reference for the impact properties of complex grid cylindrical structure with arrangement periodically.

Keywords Grid structure, Homogenization, Deformation mode, Energy absorption, Effective property

1. Introduction

Grid structure has been widely used in aerospace, automotive industry, civil materials, biological engineering, transportation and other applications because of excellent energy absorption, high load carrying capacity, high specific strength, high specific stiffness and low structural weight^[1,2]. It not only contains mesoscopic mechanics characteristics, but also has whole macro performance. These characteristics make the anti impact properties of structure greatly improved. In addition, the designability of cell geometric topological provides a broad platform for the development of grid structure. Therefore, the scientists and mechanical experts pay great attention to study grid structure in order to acquire better integrated design.

At present, impact resistance performance, energy absorption capability and dynamic mechanical properties of grid structure have been studied by many scholars. For example, The elastic response of triangular, hexagonal and quadrilateral grid structure has been obtained in by Hohe et al.^[3,4]. The deformation mode and load carrying capacity of hexagonal grid structure were studied by the numerical simulation in by Ruan et al.^[5,6] and Zou et al.^[7]. The formulas of bearing capacity and energy absorption ability were deduced through the deformation model of hexagonal grid structure in the high speed impact, and the energy distribution was discussed in Hu et al.^[8]. Researched in Hong^[9] and Zarei et al.^[10] is the impact direction angle on in-plane dynamics performance. The mechanical properties of grid structure filled with circle cells were researched through experiment and numerical simulation in Papka et al.^[11,12]. Nevertheless, all of the above researchers adopt discrete element method to solve the problem of the periodic grid structure, which not only costs a lot of computation resources, but also is infeasible sometimes^[13].

In order to simplify performance research of grid structure, the equivalent elastic constants have been explored over the last few years. Gibson and Ashby^[14] have investigated the equivalent elastic

constants of grid structure excluding the cell size factor. In other words, they calculated the equivalent elastic constants, which only considering the cell microstructure and volume fraction ratio. In addition, some correlative research works have been done by Andrews^[15], Yan and Cheng^[16]. Wei^[17] deduced the basic equation of elasticity of grid structure according to the principle of minimum potential energy and FEM, gave the elastic matrix based on the equivalent of virtual strain energy, and got the equivalent of density based on the equivalent of the kinetic energy. It is verified that the method has a good precision and is appropriate for such kind of analysis in the engineering field.

The grid cylindrical structure is equivalent to an isotropic shell with homogeneous thickness through the equivalent of unit cell mechanics properties. The energy absorption capacity, deformation model of grid cylindrical structure and equivalent continuum cylindrical structure were studied under the impact load. Meanwhile, the influence of the relative density and relative thickness to equivalent accuracy of grid cylindrical structure were considered.

2. Calculation model and method

Finite element models of grid cylindrical structure filled with stagger-arranged triangular cell and equivalent to continuum cylindrical structure with the same thickness were established as shown in figure 1. The dimensions of the two structures are: radius $R_1=100\text{mm}$, length $L_1=272\text{mm}$, and thickness T_1 . And the shape of cell is equilateral triangle with length $L_2=15.7\text{mm}$, height $H=13.6\text{mm}$ and thickness T_2 . The numbers of circumferential and axial cell are 40 and 20 respectively. For boundary conditions, all the nodes at upper and lower ends of cylindrical structure are constrained. Those two cylindrical structures are lateral impacted by rigid sphere with a certain speed, which radius $R_2=30\text{mm}$, horizontal direction $X_1=100\text{mm}$, vertical direction $Y_1=L_1/2$.

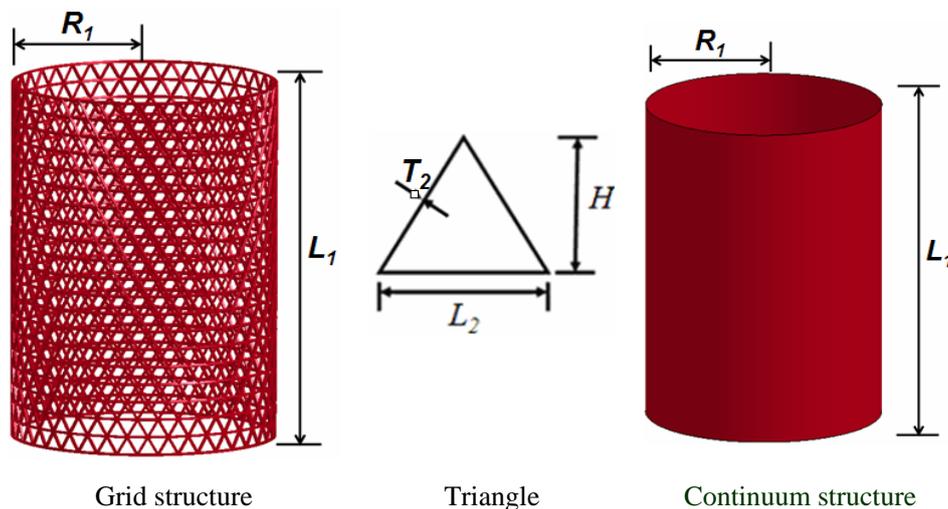


Figure 1. Finite element models of cylindrical structure

Cell-wall was modeled using Beam161 with Hughes-Liu element algorithm and rectangular cross section shape, and the material properties assumed to be perfect elastic. The number of the total elements is 24400 with each cell-wall divided into 10. In this numerical simulation, sphere was modeled using 3D Solid 164, of which material property is plastic-kinematic, and yield stress is 76Mpa. All degree of sphere freedom was confined except lateral impact direction. The density of

both cell-wall and sphere is 2700 Kg/m^3 , elastic modulus is 69Gpa , poisson's ratio is 0.3 . Mesh sensitivity studies revealed that additional mesh refinements did not improve the accuracy of the calculations appreciably. The automatic single-surface contact type with the principle simple, rarely arouse the hourglass effect, no numerical noise and the momentum conservation accurate penalty function method is adopted.

3 Equivalent processing

Cell-wall was assumed to be 'two force rod' model as shown in Figure 2. Young's modulus and density of cell-wall matrix are E_s and ρ_s . The tensile load and internal force distribution on X_1 and X_2 direction can be found out through the force equilibrium condition. Then according to harmonize deformation of cell-wall, the relation of equivalent stress and strain can be established.

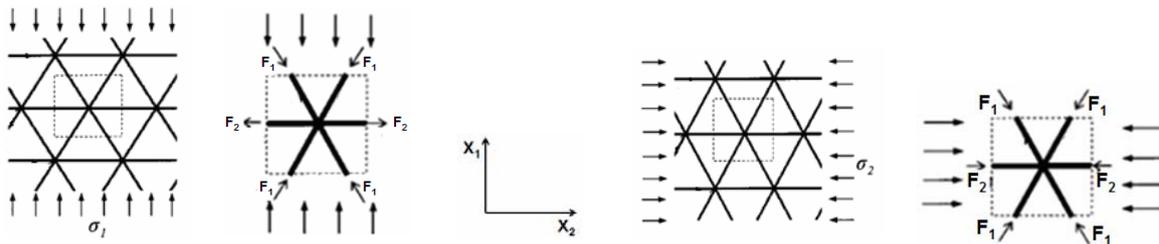


Figure 2. Triangle-grid structure under two cases of compression load^[18]

The relative density is defined as $\rho = \rho^* / \rho_s$, where ρ^* is the equivalent density of grid structure, and ρ_s is the density of cell-wall matrix.

The material parameters of equivalent homogeneous shell^[18] are:

Relative density: $\rho = \frac{\rho^*}{\rho_s} = \frac{3T_2}{L_2 \sin 60} = \frac{2\sqrt{3}T_2}{L_2}$, Young's modulus: $E_1 = E_2 = \frac{\rho}{3} E_s = \frac{2\sqrt{3}T_2}{3L_2} E_s$, Poisson's

ratio: $\nu_{12} = \nu_{21} = \frac{1}{3}$, Shear modulus: $G_{12} = \frac{\rho}{8} E_s = \frac{\sqrt{3}T_2}{4L_2} E_s$

4 Result analysis and discussion

When structure is subjected to an impact load, the governing equations can be expressed as following^[19]:

Equation of mass conservation:

$$\rho' V_{ij} = \rho_0 V_0 \quad (1)$$

where ρ' denotes the current density, V_{ij} represents the current volume, and ρ_0 denotes the initial density.

Equation of momentum conservation:

$$\sigma_{ij,j} + \rho' f_i = \rho' \ddot{u}_i \quad (2)$$

where σ_{ij} represents the Cauchy stress, f_i represents the body force density, and \ddot{u}_i denotes the acceleration.

Equation of energy conservation:

$$\dot{E} = V s_{ij} \dot{\epsilon}_{ij} - (p + q) \dot{V} \quad (3)$$

where s_{ij} and p denote the deviatoric stresses and hydrostatic pressure, respectively. q represents the bulk viscosity, δ_{ij} denotes the Kronecker delta (if $i = j$; otherwise $\delta_{ij}=0$), and ε_{ij} denotes the strain rate tensor.

The proportion of hourglass energy has a crucial effect on energy conservation. Hughes-Liu integral was used in Beam element calculation, and full integral was adopted in the process of homogenization equivalent for continuous shell element. The relationship of internal energy and hourglass energy of grid cylindrical structure and equivalent continuum cylindrical structure are shown in figure 3(c-continuum cylindrical structure, g-grid cylindrical structure). It can be seen that the hourglass energy proportion is far less than 5%, which is required in simulation calculation. If the hourglass energy having a good control, we believe that energy is conservative, and calculation accuracy is high in the whole impact process.

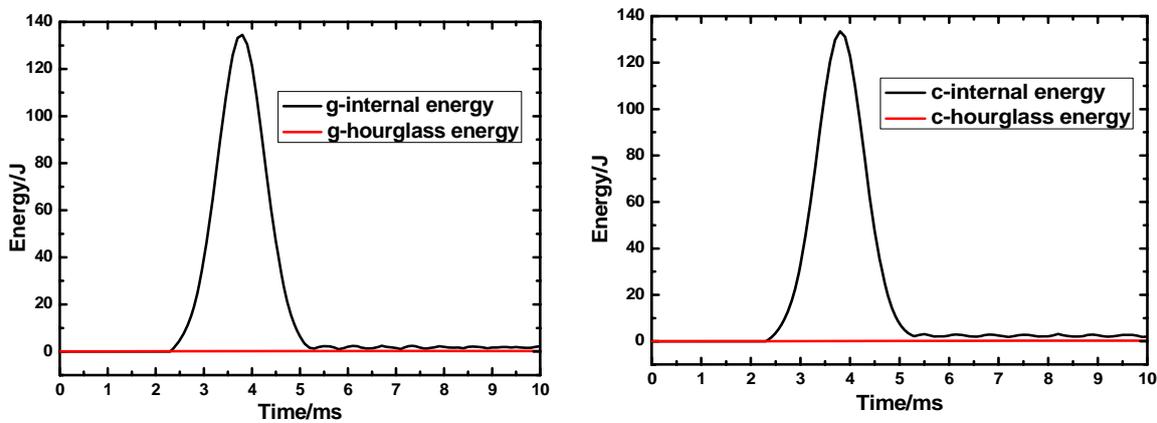


Figure 3. The relationship of internal energy and hourglass energy

4.1 The effect of the impact velocity

There are two main influence factors to impact resistant ability of cylindrical structure: structure deformation and energy absorption. When the sphere impacts to two kinds of structure at the velocity of 30 m/s, the displacement variation of structure is acquired before the sphere reflected back as shown in figure 4 (c-continuum cylindrical structure, g- grid cylindrical structure). It is obvious that the deformation of structure is basically identical before 3.1ms. With the increase of impact, the deformation error of structure is gradually increase. And the deformation of continuum cylindrical structure is larger about 6% than that of grid cylindrical structure at this stage.

This is because that the energy is absorbed through global deformation of the structure in the initial stage of impact. At this time, the difference of the two kinds of structure is not obvious and the deformations are approximately the same. With the proceeding of impact, the local deformation of structure becomes gradually obvious and concentrates to the impacted area. The influence of grid performance and arrangement mode to the impact resistance properties of structure is gradually reflected. Some energy is additionally absorbed by distortion of grid. Therefore, deformation of the continuum cylindrical structure is larger than that of grid cylindrical structure at this stage.

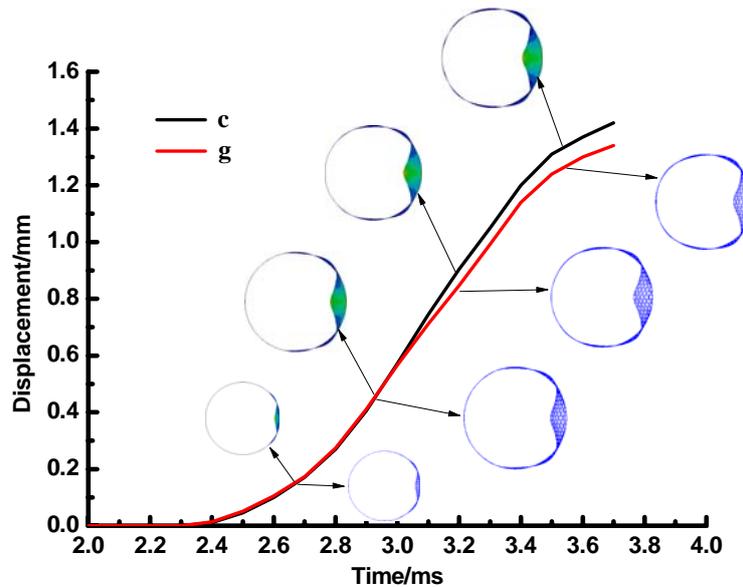


Figure 4. The deformation comparison of the two kinds of structures

Figure 5 depicts the deformation comparison of the two kinds of structure under different velocity. As can be seen from the graph, with the increase of impact velocity, deformation of structure is heightened, and deformation of continuum cylindrical structure is a little higher than that of grid cylindrical structure. It is obvious that the deformations of the two structures are agreement better at great impact velocity. The reason is that the coupled interaction between cells of grid cylindrical structure is small at the lower impact velocity. But cell shape and configuration mode has great influence on the structural deformation, which leads to the grid cylindrical structure can absorb more energy than the continuum cylindrical structure in small deformation. However, the coupling effect between grids will lead to the deformation more integrated and more closer to the deformation of the continuum cylindrical structure with the increase of impact velocity. So the deformation process is more similar and equivalent effect is better at higher velocity.

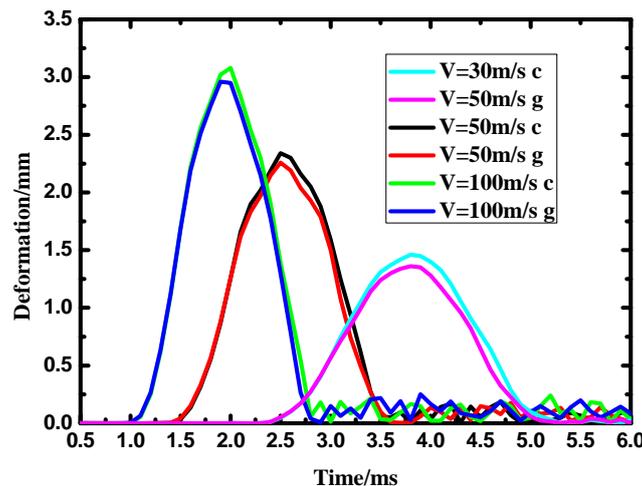


Figure 5. The deformation comparison of two kinds of structure under different velocities

4.2 The effect of the relative density

The relative density of grid cylindrical structure is changed by cell-wall thickness T_2 , and other

parameters are constant. Take the relative density were 0.11, 0.22, 0.27, 0.3, 0.44, 0.88. When the impact velocity of the rigid ball is 10m/s, the deformation variation curve of structure is shown in figure 6. It is revealed that the homogenization equivalent method of grid structure is reasonable in a certain relative density range. When the relative density ranges from 0.2 to 0.4, the equivalent effect of structure is good, and the error is controlled within 10%. But, when the relative densities are 0.11 and 0.88, the errors are 30% and 104%, respectively. The impact performance of grid cylindrical structure has not been reflected really. Then, this kind of equivalent method is failure.

The relative density has a great influence on equivalent properties of structure. Grid cylindrical structure has a tendency to local deformation, and grid topology structure form has greater influence on deformation of structure. But the topology form has no influence on equivalent continuum cylindrical structure, which is the major reason to cause different deformations of structure.

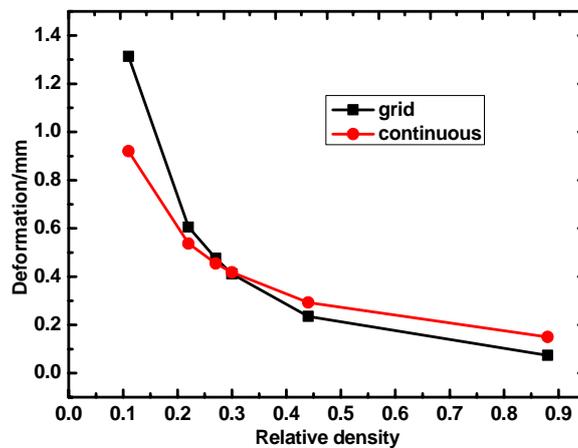


Figure 6. The relationship of relative density and deformation

4.3 The effect of the relative thickness

The relative thickness is defined as T_2/T_1 , which is changed by thickness of the cylinder structure T_1 . Take the relative density were 0.41, 0.61, 1.22, 2.44 respectively. When the relative density is 0.27, the largest deformation change of structure with the increase of relative thickness is shown in figure 7. When the relative thickness less than 1.2, the maximum deformations of the two structure are almost to be equal. But the relative error has reached 23% at the relative thickness up to 2.44. It can be seen that with invariable thickness of grid cylindrical structure, the thicker the cylindrical structure is, the better the equivalent result is. However, the out-of-plane performance of grid cylindrical structure will gradually highlight with the increase of the thickness of the cylindrical structure, which will result in the failure of the equivalent method.

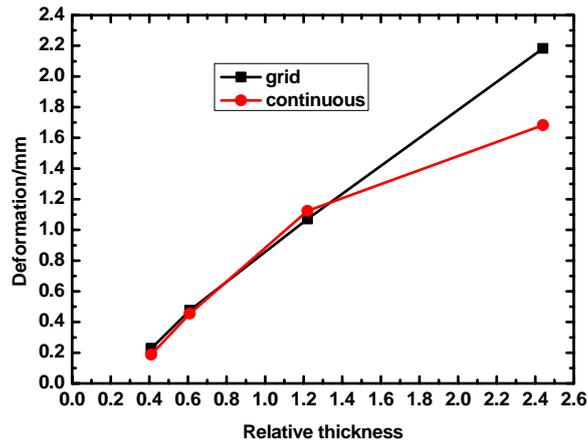


Figure 7. The relationship of the relative thickness and deformation

5. Concluding remarks

Based on the results obtained in the present work, the following main conclusions can be drawn:

- (1) The relative density of grid cylindrical structure plays an important role in the homogenization process.
- (2) It can be seen that the equivalent method is credible in a certain dimensionless parameter of relative density or relative thickness range.
- (3) The deformation of grid cylindrical structure is smaller than that of the homogeneous equivalent for continuous shell structure under impacting, but the biggest error is controlled within 10%. So the equivalent method is reasonable in a certain range.

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