Threshold Fracture Energy for Differently Shaped Particles Impacting Halfspace (Erosion-Type Fracture)

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Abstract Energetic aspects of erosion fracture are studied. Threshold (minimal) energy needed for initiation of fracture caused by a particle impact is estimated using the incubation time criterion. The dependences of the fracture threshold energy on impact duration are calculated for two different shapes of the impacting particle: sphere and cylinder. The difference in the threshold energy behavior between these two cases is demonstrated and discussed.

Keywords Incubation time criterion, erosion, threshold energy

Introduction

In the present work the problem of dynamic impact of spherical and cylindrical solid particle on elastic half-space is studied. This problem is considered within the framework of fracture mechanics by neglecting of the heat transfer process and wave-process origination. Also there is an assumption that the impact is quasistatic. It permits to use the solution of the problem of quasistatic of the indenter pressing in [1].

This problem plays an important role in practical application, since the short impacts are typical for such industry process as ultrasonic assisted machining [2,3].

Threshold Energy of a Sphere Particle

The normal impact of the spherical rigid particle on the elastic half-space is considered. Hertz solution for the contact problem gives the following temporal dependence of the contact force P on the distance h between the bodies approaching each other:

$$P(t) = k[h(t)]^{3/2}, \tag{1}$$

where $k = 4\sqrt{R}E/[3(1-\nu^2)]$, R - particle radius, E and ν - elastic constants of the half-space. If m is the mass of the particle and V is its initial velocity then the time dependence of the approach $h(t)$ can be determined by solving the equation of motion:

$$m\frac{d^2h(t)}{dt^2} = -P(t). \tag{2}$$

The solution of the differential equation (2) can be approximated with high precision by the following expression:

$$h(t) = h_0 \sin\left(\frac{\pi t}{t_0}\right), \tag{3}$$

where $h_0 = \left(\frac{5}{4k}mV^2\right)^{2/5}$ is maximum approach and $t_0 = 2.94\frac{h_0}{V} \approx 3.2\left(\frac{m^2}{Vk^2}\right)^{1/5}$ is the contact time. Also Hertz solution gives the following expression for the tensile stresses originated on the half-space surface:

$$\sigma_r(t) = -\sigma_\theta(t) = \frac{1-2\nu}{2}\frac{P(t)}{\pi a^2(t)}, \tag{4}$$

$$a(t) = \left(3P(t)(1-\nu^2)\frac{R}{4E}\right)^{1/3} = \left(P(t)\frac{R^{3/2}}{k}\right)^{1/3}. \tag{5}$$

Parameter $a(t)$ defines the radius of the contact area and its value depends on the contact time. Values $\sigma_r(t)$ and $\sigma_\theta(t)$ correspond to maximum values of main stresses in cylindrical coordinates. Therefore, the radial component of stresses $\sigma_r(t)$ is considered, since usually tensile stresses lead to fracture. Below $\sigma_r(t)$ is denoted by $\sigma(t)$.

The expressions (3)-(5) can be used to determine the expression for the threshold energy of the spherical particle. By threshold energy we mean the minimum quantity of energy, which is necessary to spend on initiation of the threshold fracture pulse during impact intersection. In this work the incubation time criterion is applied for fracture prediction [4]:

$$\frac{1}{\tau}\max_t \int_{t-\tau}^{t} \sigma(R,V,s)ds = \sigma_c \ . \tag{6}$$

The static strength of the material σ_c can be experimentally measured. Parameter τ corresponds to incubation time of the fracture. It characterizes the time period for preparing the media to fracture or phase transformation. The incubation time is a material strength constant and its value can be measured experimentally or derived by computational and experimental methods. In the work [5] different interpretations of the incubation time are shown for various problems.

The application of the incubation time criterion caused by this criterion takes into account the process dynamics. The dynamic strength properties of materials appear when the loading duration has a similar value as the incubation time [6,7]. When the time of loading significantly exceeds the incubation time, the media resistance is specified by static strength properties.

After the substitution of the expressions (1), (3) and (5) into (4), the fracture criterion becomes:

$$\frac{1-2\nu}{2\pi}\frac{k}{R}\sqrt{h_0}\max_t \int_{t-\tau}^{t} \sqrt{\sin\left(\frac{\pi s}{t_0}\right)}ds = \tau\sigma_c \ . \tag{7}$$

The integral in the expressions (7) takes the maximum value at the time $t = (t_0 + \tau)/2$. The impact duration, radius and velocity of the particle should be introduced in dimensionless form:

$$\lambda = \frac{\tau_0}{\tau}, \quad R_d = \frac{R}{c_p \tau}, \quad V_d = \frac{V}{c_p} .$$

From now on $c_p = \sqrt{E(1-\nu)/\rho_m(1+\nu)(1-2\nu)}$ is the propagation velocity of the dilatational wave in the elastic media with density ρ_m.

Then, the accepted criterion (7) gives the following expression for calculation of the threshold velocity in dimensionless form:

$$\alpha_d V_d^{2/5} \int_{\frac{\lambda-1}{2}}^{\frac{\lambda+1}{2}} \sqrt{\sin\left(\frac{\pi s}{\lambda}\right)}\left[H(s) - H(s-\lambda)\right]ds = 1 , \tag{8}$$

where $\alpha_d = c_p^{2/5}\left(E^4\rho\right)^{1/5}(2\pi\sigma_{cr})^{-1}(1-2\nu)\left(4/3\left(1-\nu^2\right)\right)^{4/5}(5\pi/3)^{1/5}$ is dimensionless parameter, ρ is material density of the particle, $H(s)$ is the Heaviside function.

The value of the threshold radius can be calculated from following expression for the impact duration:

$$\lambda = \frac{t_0}{\tau} \approx 3.2\frac{1}{\tau}\left(\frac{\pi\rho\left(1-\nu^2\right)}{E}\right)^{2/5}\frac{R}{V^{1/5}} .$$

Then, the threshold radius is determined in dimensionless form by:

$$R_d = \frac{\lambda}{\beta_d} V_d^{1/5}, \tag{9}$$

where $\beta_d = 3.2\left(\pi\left(1-\nu^2\right)\rho c_p^2 / E\right)^{2/5}$ is dimensionless parameter. If the threshold velocity (8) and radius (9) are known, then the threshold energy of the particle can be calculated as:

$$W = \frac{2}{3}\pi\rho R^3 V^2. \tag{10}$$

In dimensionless form, (10) is given by the formula:

$$W_d = \frac{W}{\omega} = R_d^3 V_d^2, \tag{11}$$

where the parameter $\omega = 2\pi\rho\tau^3 c_p^5 / 3$ has the dimension of energy and is determined by material constants. Thus, the quantity W_d determines the minimum dimensionless value of particle energy required for half-space fracture. Figure 1 presents the graphs of dependence of the energy (11) on the impact duration (Fig. 1*a*) and radius (Fig. 1*b*), where the half-space material is zinc and parameters the value $\rho = 3200\, kg/m^3$.

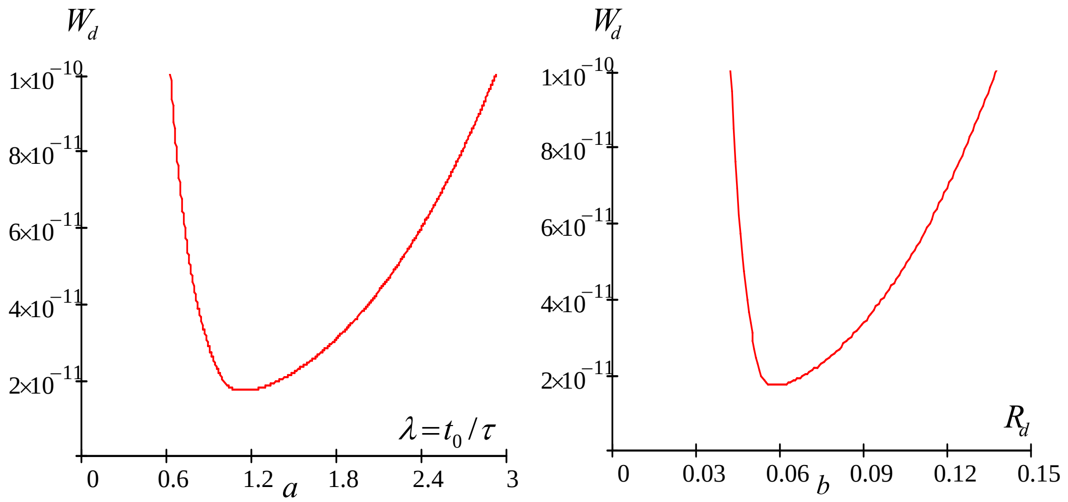

Figure 1. Dependence of the threshold energy of the spherical particle on: *a* – the impact duration, *b* – particle radius

These graphs show that the value of the threshold energy has the marked minimum distinct from equal zero. Hence, it is possible to decrease the energy costs for fracture by controlling such parameters as radius and velocity of particle.

Threshold Energy of a Cylindrical Particle

The similar analysis can be provided for cylindrical particles with a constant circle section of radius R. The cylinder height is equal to $H = 4R/3$, that follows from the assumption that the cylinder mass is equal to the ball mass. In the cylindrical case the contact force $P(t)$ and approach $h(t)$ are related as follows:

$$P(t) = kh(t), \tag{12}$$

where $k = 2RE/\left(1-\nu^2\right)$.

In this case the equation of motion (2) has the exact solution [8]:

$$h(t) = h_0 \sin\left(\frac{\pi t}{t_0}\right), \tag{13}$$

where $h_0 = \sqrt{\frac{mV^2}{k}}$ and $t_0 = \sqrt{\frac{m}{k}}$.

Also, there is the similar expression for maximum tensile stress:

$$\sigma(t) = \frac{1-2\nu}{2}\frac{P(t)}{\pi R^2}, \quad r \to R+0 \tag{14}$$

After substituting the expression of tensile stress (14) into criterion (6), the integral attains its maximum value at time $t = (t_0 + \tau)/2$.The dimensionless quantities are introduced by analogy with the case of the spherical particle. And the following expressions of dimensionless velocity can be obtained by using criterion (8) and formulas (12) - (14):

$$\alpha_d V_d \int_{\frac{\lambda-1}{2}}^{\frac{\lambda+1}{2}} \sin\left(\frac{\pi s}{\lambda}\right)[H(s) - H(s-\lambda)]ds = 1, \tag{15}$$

where $\alpha_d = (1-2\nu)\left(8E\rho c_p^2 / 3\pi(1-\nu^2)\right)^{1/2}(2\sigma_{cr})^{-1}$ is dimensionless parameter, ρ - is material density of the particle. From the expression of the impact duration it is possible to calculate the value of radius in dimensionless form:

$$R_d = \frac{\lambda}{\beta_d}, \tag{16}$$

where $\beta_d = \pi\left(2\pi\rho c_p^2(1-\nu^2)/3E\right)^{1/2}$ is dimensionless parameter.

The threshold energy can be calculated by formula (10) and in dimensionless form by formula (11), where cylinder velocity and radius are determined by (15) and (16) respectively.

Figure 2 presents the graphs of dependence of the energy on impact duration (Fig. 2*a*) and radius (Fig. 2*b*), where the half-space material is zinc and parameters the value $\rho = 3200\,kg/m^3$.

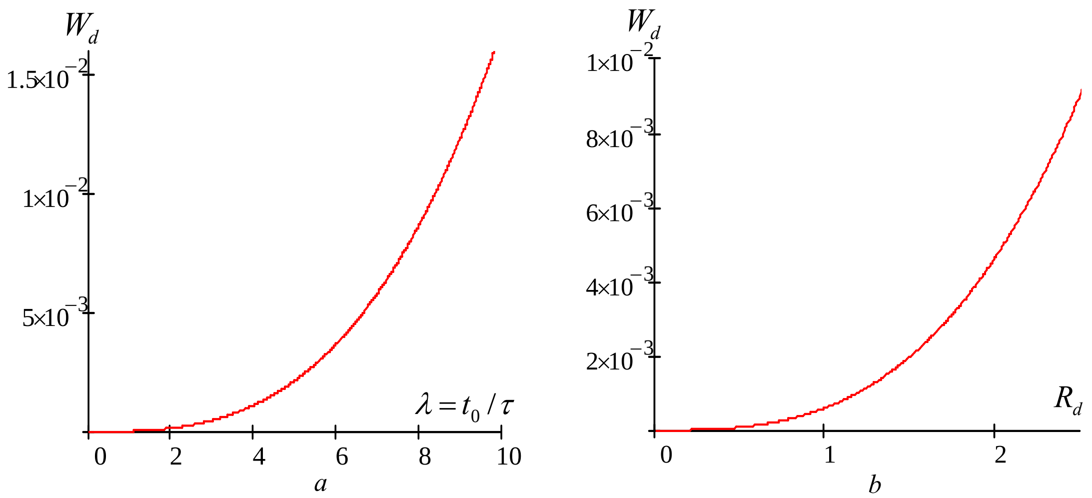

Figure 2. Dependence of the threshold energy of the cylindrical particle on *a* – the impact duration, *b* – particle radius

Fig. 2 shows that in case of cylindrical particle the threshold energy dependence is a monotone increasing function in contrast with the case of the spherical particle.

Results and Discussion

For the explanation of the difference in threshold energy behavior between both cases the following should be marked. The problems of the penetration with spherical and cylindrical indenters are fundamentally different. In case of the cylindrical particle, the contact area is a constant value during the penetration process, whereas for spherical particle, the contact area is a variable quantity. Also, in cylindrical case the points belonging to contact area border are singular and the stresses take out infinity in these points. But sphere indentation is attended with the finite values of stresses in all points of the contact region. The fact that the energy is zero for the zero impact duration and radius in the cylindrical case (Fig. 2) can be explained by the fact that the problem of penetration of a cylinder into a half-space is an idealized problem. When considering small particles, it is impossible to neglect the roundedness of the cylinder angles near the basis, and this model becomes unsuitable.

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