# Study on the fatigue crack growth behavior of $\mathbf{3 0 C r M n S i A}$ Straight Attachment Lugs 

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#### Abstract

The finite element model of straight attachment lug subjected to axial or oblique loading is built by using finite element software, a cosine pin-bearing pressure distribution is applied on the pin-hole squeeze surface as a boundary condition. The stress intensity factor (SIF) expressions for single through-the-thickness crack in straight attachment lug that subjected to axial or oblique pin-load less than 45 degrees are determined and validated. The fatigue crack growth rate $\mathrm{d} a / \mathrm{d} N$ and the stress intensity factor range $\Delta K$ are obtained by using the fatigue crack growth test data and the SIF expressions. The fatigue crack growth model of the typical straight lugs is established by using the Paris law, offering an analytical as well as experimental method for assessing and designing damage tolerant attachment lugs.


Keywords attachment lug, finite element method, fatigue crack growth, stress intensity factor, the Paris law

## 1. Introduction

Attachment lugs are one of the most fatigue-and fracture-critical components in modern engineering structures ${ }^{[1]}$. In aircraft structures, lug-type joints are frequently used to connect major structural components or in linkage structure, theirs failure can result in disastrous accidents. In the study of fatigue crack growth and fracture behavior of attachment lugs, an accurate calculation of the stress intensity factor is essential. There are a number of different methods for determining SIF, $K$, for crack in aircraft attachment lug.
Over the years, several extensive studies have been made on lug fatigue performance, involving both experimental and analytical means. Liu and $\mathrm{Kan}^{[2]}$ and Kirkby and Rooke ${ }^{[3]}$ used the simple compounded solution method which involves superimposing known solutions, such as in Reference [4] to estimate the stress intensity factors. Schijve and Hoeymakers ${ }^{[5]}$ and Wanhill ${ }^{[6]}$ derived empirical K-solutions from the growth rate data for through cracks under constant amplitude loading using a backtracking method such as that proposed by James and Anderson ${ }^{[7]}$. Pian, et al ${ }^{[8]}$, used the hybrid finite element method to compute the K-values for cracks oriented in various angles from the axial direction of straight lugs. Aberson and Anderson ${ }^{[9]}$ used a special crack-tip singularity element to compute the stress intensity factors for a crack in a nonsymmetrical aircraft lug of an engine pylon. Impellizzeri and Rich ${ }^{[10]}$ modified the exact weight function derived by Bueckner ${ }^{[11]}$, for an edge crack in a semi-infinite plate, to include a series of geometry correction factors. Then they computed the K-values using the weight function method. However, at the aspect of damage tolerance design and analysis, only the situation of straight lug subjected to axial pin loading can be solved. For the straight lug and symmetric tapered lug, which are subjected to axial, oblique and transverse pin loadings, there is a lack of methods for ascertaining the crack propagation characteristics and residual strength. So, it is important to develop analytical as well as experimental procedures for assessing and designing damage tolerant attachment lugs to ensure the operational safety of aircraff ${ }^{[12]}$, and to calculate the stress intensity factors (SIFs) in different geometric parameters and load conditions.
Most of the researches made the assumption that the assumed or computed pin-bearing pressure distribution for an uncracked case remains unchanged even after the crack has initiated and propagated. Based on the parametric study conducted in Reference [8], it was found that, for any given crack length, the difference in the SIF computed using the uniform and cosine pin-bearing pressure distributions was as much as 30 percent. Therefore, it is salient that the correct representation of the pin-bearing pressure distribution during the crack growth process is essential to the calculation of accurate stress intensity factors.
This paper presents a systemic study of the 30 CrMnSiA straight attachment lug's fatigue crack growth behavior using the finite element software ANSYS. The finite element model of the lug is established
and the effect of geometric parameters on SIF is calculated and analyzed by the finite element method (FEM) with linear elastic assumptions, as in essence, the fatigue crack growth is brittle and it is often based on Linear Elastic Fracture Mechanics (LEFM) assumption ${ }^{[13]}$. The fatigue crack growth test and residual strength test are carried out by the material test system MTS-810, the test data validate the finite element analysis and are used to determine the constants C\&m of the Paris Law. The fatigue crack growth model of the typical straight lugs is established, offering an analytical as well as experimental method for assessing and designing damage tolerant attachment lugs.

## 2. Finite Element Modeling and Analyzing

### 2.1. Finite element modeling and meshing

Figure 1 shows the size of the straight lug discussed in this paper, millimeter is used as the length unit. The length of the single through-the-thickness crack is $a$. The model is meshed with PLANE 183 triangular element, the real constant for "plane stress with thickness" is set to 6.7. The elastic modulus is 200 Gpa and the Poisson ratio is 0.3 . The node at the crack tip is defined as a singular point and there are 12 triangular elements around it. The radius of the 1 st row of elements is $a / 10$. The global elements edge length is set to 2 . Figure $2 \&$ Figure 3 show the mesh result.


Figure 1. Size of the straight lug


Figure 2. Mesh of the straight lug


Figure 3. Mesh of the crack tip

### 2.2. Setting boundary conditions and applying cosine distributed load

Pin load is transferred by squeezing between the lug and the pin matched. Load and stress are supposed to keep unchangeable along the thickness of the hole. The pressure distributed on the squeeze surface follows the cosine distribution law, and the radial load $P_{\mathrm{i}}$ at the point $i$ on the squeeze surface is equal to $P_{0} \cos \gamma_{i}{ }^{[14]}$, as shown in Figure 4.
The x and y axis components of the load $P_{\mathrm{i}}$ are as follows.

$$
\begin{align*}
& P_{i x}=P_{i} \sin \gamma_{i},  \tag{1}\\
& P_{i y}=P_{i} \cos \gamma_{i} . \tag{2}
\end{align*}
$$

The resultant force along y axis is shown as follow.

$$
\begin{equation*}
P=\sum P_{i y}=\sum P_{i} \cos \gamma_{i}=\sum P_{0} \cos ^{2} \gamma_{i} . \tag{3}
\end{equation*}
$$

The cosine distributed nodal force is applied on the nodes on the surface ABC. All the nodes on the symmetry plane of the lug are selected and the displacement values are set to zero. Figure 5 shows the boundary conditions.


Figure 4. Pin-loading on the squeeze surface
Figure 5. Boundary conditions

### 2.3. Results analysis

Figure 6 and Figure 7 respectively show the Von Mises stress distribution around the lug's hole and the crack tip.


Figure 6. Stress distribution around the lug's hole


Figure 7. Stress distribution around the crack tip

SIFs of cracks with different lengths are calculated by three methods. Case 1 uses the equation that is used for estimating the straight lugs' critical load in [15] (see formula (4)) to calculate the SIFs. Case 2 uses the configuration factor $J_{41 \mathrm{~L}}$ in [15] (see Figure 8) to get the SIFs' values. This paper calculates the SIFs by finite element method, and the results are shown as case 3 in Figure 9 to compare with the other two cases.
In reference [15] (page 270), it is indicated that the critical load [ $P]_{\mathrm{c}}$ of the attachment lug subjected to axial pin-load is

$$
\begin{equation*}
[P]_{C}=\frac{2 R_{2} t K_{C}}{\beta \sqrt{\pi a}} \tag{4}
\end{equation*}
$$

Where

$$
\begin{gather*}
\beta=\left(\frac{2 W}{1.971 D} \beta_{1}+\frac{1}{2} \beta_{2}\right) \cdot \beta_{f w},  \tag{5}\\
\beta_{1}=\frac{0.31711}{\left(0.3857+\frac{a}{R_{1}}\right)^{1.33734}},  \tag{6}\\
\beta_{2}=\frac{0.8734}{0.3246+\frac{a}{R_{1}}}+0.6762,  \tag{7}\\
\beta_{f w}=\sqrt{\sec \left(\frac{\pi}{2} \cdot \frac{2 R_{1}+a}{2 R_{2}-a}\right)} . \tag{8}
\end{gather*}
$$

In reference [15] (page 139), the SIF of attachment lug subjected to axial pin-load is equal to

$$
\begin{equation*}
K=J_{41 L} \cdot \sigma \cdot \sqrt{\pi a} . \tag{9}
\end{equation*}
$$

where the configuration factor $J_{41 \mathrm{~L}}$ can be determined by using Figure 8 .


Figure 8 . The configuration factor $J_{41 \mathrm{~L}}$ curves


Figure 9. Comparison of the stress intensity factors

## 3. Calculation of the SIF equations

The geometric parameters of the lug are wrote to the APDL program text file and executed by ANSYS to solve the SIFs automatically and efficiently. The case of the straight attachment lug subjected to axial pin-load is taken as an example for illustrating the approach. Figure 10 shows the parameterized geometry of the lug.


Figure 10. Geometry of the straight attachment lug
The influence law of the dimensionless crack length $\left(a / R_{1}\right)$, the ratio of outer radius to inside radius $\left(R_{2} / R_{1}\right)$ and the inside radius $\left(R_{1}\right)$ on the SIF are studied, and then, the equation (4) is modified to fit the SIF equation for the straight attachment lug with a single through-the-thickness crack.
Table 1 shows the solving idea. $K, K_{1}, K_{2}, K_{3}$ stand for the SIFs calculated by equation (4) and represent the values that need to be modified by the influence law of $a / R_{1}, R_{2} / R_{1}$ and $R_{1}$ respectively. $K^{\prime}$ stands for the SIF calculated by ANSYS as the reference exact value for modifying $K_{1}, K_{2}, K_{3}$.

Table 1. Solving the SIF equation of the straight lug subjected to axial pin-load

| Variate | Invariants | K | $K^{\prime}$ | $\left(K^{\prime}\right)_{\mathrm{i}} / K_{\mathrm{i}}$ | Calculating process |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\left(a / R_{1}\right)_{1}$ | $R_{2} / R_{1}, R_{1}$ | $\left(K_{1}\right)_{1}$ | $\left(K^{\prime}\right)_{1}$ | $\left(K^{\prime}\right)_{1} /\left(K_{1}\right)_{1}$ | $\left(a / R_{1}\right)_{\mathrm{i}}$ is set as the variate, while $\left(K^{\prime}\right)_{i} /\left(K_{1}\right)_{\mathrm{i}}$ as the dependent variable, function $g\left(a / R_{1}\right)$ can be fitted as below.$\begin{equation*} K_{2}=K_{1} \cdot g\left(a \mid R_{1}\right) \tag{10} \end{equation*}$ |
| ! | : | : | ! | $\vdots$ |  |
| $\left(a / R_{1}\right)_{\mathrm{n}}$ | $R_{2} / R_{1}, R_{1}$ | $\left(K_{1}\right)_{\mathrm{n}}$ | $\left(K^{\prime}\right)_{\mathrm{n}}$ | $\left(K^{\prime}\right)_{\mathrm{n}} /\left(K_{1}\right)_{\mathrm{n}}$ |  |
| $\left(R_{2} / R_{1}\right)_{1}$ | $a / R_{1}, R_{1}$ | $\left(K_{2}\right)_{1}$ | $\left(K^{\prime}\right)_{1}$ | $\left(K^{\prime}\right)_{1} /\left(K_{2}\right)_{1}$ | $\left(R_{2} / R_{1}\right)_{\mathrm{i}}$ is set as the variate, while $\left(K^{\prime}\right)_{\mathrm{i}} /\left(K_{2}\right)_{\mathrm{i}}$ as the dependent variable, function $\not\left(R_{2} / R_{1}\right)$ can be fitted as below.$\begin{equation*} K_{3}=K_{2} \cdot \not\left(\left(R_{2} / R_{1}\right)=K_{1} \cdot g\left(a / R_{1}\right) \quad\left(R_{2} / R_{1}\right)\right. \tag{11} \end{equation*}$ |
| ; | : | : | : | $\vdots$ |  |
| $\left(R_{2} / R_{1}\right)_{\mathrm{n}}$ | $a / R_{1}, R_{1}$ | $\left(K_{2}\right)_{\mathrm{n}}$ | $\left(K^{\prime}\right)_{\mathrm{n}}$ | $\left(K^{\prime}\right)_{\mathrm{n}} /\left(K_{2}\right)_{\mathrm{n}}$ |  |
| $\left(R_{1}\right)_{1}$ | a/ $R_{1}, R_{2} / R_{1}$ | $\left(K_{3}\right)_{1}$ | $\left(K^{\prime}\right)_{1}$ | $\left(K^{\prime}\right)_{1} /\left(K_{3}\right)_{1}$ | $\left(R_{1}\right)_{\mathrm{i}}$ is set as the variate, while $\left(K^{r}\right)_{\mathrm{i}} /\left(K_{3}\right)_{\mathrm{i}}$ as the dependent variable, function $h\left(R_{1}\right)$ can be fitted. <br> Finally the straight attachment lug's SIF equation can be obtained as below. $\begin{align*} K=K_{3} \cdot h\left(R_{1}\right)= & K_{1} g\left(a / R_{1}\right) \cdot f\left(R_{2} / R_{1}\right) \cdot h\left(R_{1}\right) \\ & =K_{1} \mathrm{~F}\left(a / R_{1}, R_{2} / R_{1}, R_{1}\right) \tag{12} \end{align*}$ |
| ! | : | ; | ; | : |  |
| $\left(R_{1}\right)_{\mathrm{n}}$ | a/ $R_{1}, R_{2} / R_{1}$ | $\left(K_{3}\right)_{\mathrm{n}}$ | $\left(K^{\prime}\right)_{\mathrm{n}}$ | $\left(K^{\prime}\right)_{\mathrm{n}} /\left(K_{3}\right)_{\mathrm{n}}$ |  |

The value of $a / R_{1}$ is increased from 0.1 to 1.6 by 0.1 while the others are fixed, $K_{1}$ can be modified as $K_{2}=K_{1} * g\left(a / R_{1}\right)$, where

$$
\begin{equation*}
g\left(a / R_{1}\right)=-0.1285 *\left(a / R_{1}\right)^{2}+0.3059 *\left(a / R_{1}\right)+0.8228 \tag{13}
\end{equation*}
$$

The value of $R_{2} / R_{1}$ is increased from 1.5 to 3 by 0.1 while the others are fixed, $K_{2}$ can be modified as $K_{3}=K_{2}{ }^{*} f\left(R_{2} / R_{1}\right)$, where

$$
\begin{equation*}
f\left(R_{2} / R_{1}\right)=-0.1654 *\left(R_{2} / R_{1}\right)^{4}+1.6326 *\left(R_{2} / R_{1}\right)^{3}-6.0293 *\left(R_{2} / R_{1}\right)^{2}+9.9046 *\left(R_{2} / R_{1}\right)-5.1399 \tag{14}
\end{equation*}
$$

The value of $R_{1}$ is increased from 5 to 20 by 1 while the others are fixed, the function $h\left(R_{1}\right)$ can be obtained as $h\left(R_{1}\right)=0.99$, it is obvious that the effect of $R_{1}$ on SIF is less than the other parameters.
Finally we can obtain the modified straight attachment lug's SIF equation as $K=K_{3}{ }^{*} h\left(R_{1}\right)$, the SIF equation of straight lug (Figure 10) is obtained as blows.

$$
\begin{equation*}
K=F \cdot \sigma \cdot \sqrt{\pi \cdot a} . \tag{15}
\end{equation*}
$$

Where

$$
\begin{gather*}
\sigma=\frac{P}{2 \mathrm{c} t},  \tag{16}\\
c=R_{2}-R_{1},  \tag{17}\\
F=f_{a r} \cdot f_{f w} \cdot f_{r},  \tag{18}\\
f_{a r}=\frac{1.98 R_{2}}{1.971 R_{1}} \cdot \frac{0.31711}{\left(0.3857+\frac{a}{R_{1}}\right)^{1.33734}+\frac{1}{2} \cdot(0.8734 /(0.3246+a l R)+0.6762)},  \tag{19}\\
f_{f w}=\sqrt{\sec \left(\frac{\pi}{2} \cdot \frac{2 R_{1}+a}{2 R_{2}-a}\right)},  \tag{20}\\
f_{r}=\left(-0.1654 \cdot\left(\frac{R_{2}}{R_{1}}\right)^{4}+1.6326 \cdot\left(\frac{R_{2}}{R_{1}}\right)^{3}-6.0293 \cdot\left(\frac{R_{2}}{R_{1}}\right)^{2}+9.9046 \cdot\left(\frac{R_{2}}{R_{1}}\right)-5.1399\right) . \tag{21}
\end{gather*}
$$

Scope of application:

$$
0.1 \leq \frac{a}{R_{1}} \leq 1.6, \quad 1.5 \leq \frac{R_{2}}{R_{1}} \leq 3, \quad 5 \leq R_{1} \leq 20 .
$$

In the same way, considering the load degrees $\beta$ additionally, the SIF equation of the straight attachment lugs subjected to oblique pin-load less than 45 degrees can be obtained as below.

$$
\begin{equation*}
K=F \cdot \sigma \cdot \sqrt{\pi \cdot a} \tag{22}
\end{equation*}
$$

where

$$
\begin{gather*}
\sigma=\frac{P}{2 \mathrm{c} t},  \tag{23}\\
c=R_{2}-R_{1},  \tag{24}\\
F=f_{\beta} \cdot f_{a r} \cdot f_{a r}^{\prime} \cdot f_{f_{w}} \cdot f_{r r \prime},  \tag{25}\\
f_{\beta}=-3.7107 \times 10^{-6} \cdot \beta^{3}+8.4026 \times 10^{-5} \cdot \beta^{2}+0.0092469 \cdot \beta+1.05,  \tag{26}\\
f_{a r}=\frac{1.98 R_{2}}{1.97 r_{1}} \cdot \frac{0.31711}{\left(0.3857+a / R_{1}\right)^{1.33734}+}+\frac{1}{2} \cdot\left(0.8734 /\left(0.3246+a / R_{1}\right)+0.6762\right),  \tag{27}\\
f_{a r}^{\prime}=-0.063305 \cdot\left(\frac{a}{R_{1}}\right)^{2}+0.067224 \cdot \frac{a}{R_{1}}+0.94853, \tag{28}
\end{gather*}
$$

$$
\begin{gather*}
f_{f w}=\sqrt{\sec \left(\frac{\pi}{2} \cdot \frac{2 R_{1}+a}{2 R_{2}-a}\right)},  \tag{29}\\
f_{l r}=\left(-0.1654\left(\frac{R_{2}}{R_{1}}\right)^{4}+1.6326\left(\frac{R_{2}}{R_{1}}\right)^{3}-6.0293\left(\frac{R_{2}}{R_{1}}\right)^{2}+9.9046\left(\frac{R_{2}}{R_{1}}\right)-5.1399\right) . \tag{30}
\end{gather*}
$$

Scope of application:

$$
5^{\circ} \leq \beta \leq 45^{\circ}, \quad 0.1 \leq \frac{a}{R_{1}} \leq 1.6, \quad 1.5 \leq \frac{\mathrm{R}_{2}}{\mathrm{R}_{1}} \leq 3, \quad 5 \leq R_{1} \leq 20 .
$$

## 4. Fatigue crack growth test and residual strength test

### 4.1 Test pieces

Figure 11 shows the dimensional drawing of the straight attachment lug piece. There is a small slot made by electrical discharge machine (EDM) besides the lug hole with an angle of $\theta=90^{\circ}$ or $\theta=60^{\circ}$. The slot length $a_{0}$ is approximately equal to 1 mm . The lugs pieces subjected to axial $\left(\theta=90^{\circ}\right)$ or 30 degrees oblique ( $\theta=60^{\circ}$ ) pin-load are all made of 30 CrMnSiA material, 4 pieces for each case.


Figure 11. Geometry of the test piece

### 4.2 Test equipments and loading requirement

The tests are carried out on the material test system MTS-810 (load relative error $< \pm 2 \%$ ) with a constant amplitude loading spectrum of sine waveform. The length of the crack is observed and measured by JXD- 250 mm scale microscope with a precision of 0.01 mm .
Surface around test piece's wire-cutting slot tip is polished by a sand paper along the direction perpendicular to the slot direction. Peak load $P_{\mathrm{pp}}$ in the crack growth preparation test is 13.85 KN , valley load $P_{p \nu}$ is $0.83 \mathrm{KN}, R=0.06$, load frequency is 5 Hz . A crack with a length of 0.314 mm was observed when fatigue cycle reaches about 15000 , and grows to 0.612 mm when 21000 cycles. The $a-N$ data shows that the crack is growing steadily under the peak load of the fatigue crack growth test $P_{\mathrm{p}}=20.77 \mathrm{KN}$ and the valley load $P_{\mathrm{v}}=1.246 \mathrm{KN}$.
Table 2 shows values of the test load.
Table 2. The test load values

| Case | $P_{\mathrm{pp}} /(\mathrm{KN})$ | $P_{\mathrm{pv}} /(\mathrm{KN})$ | $P_{\mathrm{p}} /(\mathrm{KN})$ | $P_{\mathrm{v}} /(\mathrm{KN})$ |
| :---: | :---: | :---: | :---: | :---: |
| $\theta=90^{\circ}$ | 13.85 | 0.83 | 20.77 | 1.25 |
| $\theta=60^{\circ}$ | 13.85 | 0.83 | 20.77 | 1.25 |

### 4.3 Fatigue crack growth test

Figure $14 \&$ Figure 15 show the experimental scene.


Figure 14. Experimental field of the case of $\theta=90^{\circ}$


Figure 15. Experimental field of the case of $\theta=60^{\circ}$

The micro objective is aimed at the wire-cutting slot tip for detecting the fatigue crack in time. The frequency is changed to 1 Hz every other $1000 \sim 2000$ cycles in order to observe the crack length and record the $a-N$ data.

### 4.5 Residual strength test

After obtaining enough $a-N$ data, the test pieces with different residual fatigue crack lengths are loaded with a sustained increasing load till the test pieces fracture, the corresponding residual strength data are recorded in Table 3 \& Table 4.

Table 3. The residual strength test results of the lugs subjected to axial pin-load

| test piece num. | No.1 | No.2 | No.3 | No.4 |
| :--- | :---: | :---: | :---: | :---: |
| crack length $a /(\mathrm{mm})$ | 7.109 | 8.127 | 8.972 | 10.115 |
| residual strength $P /(\mathrm{KN})$ | 91.456 | 83.242 | 78.051 | 72.947 |

Table 4. The residual strength test results of the lugs subjected to 30 degrees oblique pin-load

| test piece num. | No.1 | No. 2 | No.3 | No. 4 |
| :--- | :--- | :--- | :--- | :--- |
| crack length $a /(\mathrm{mm})$ | 10.58 | 10.99 | 11.45 | 9.98 |
| residual strength $P /(\mathrm{KN})$ | 58.54 | 42.38 | 40.24 | 58.8 |

The data of crack length $a$ and corresponding residual strength $P$ in Table $4 \&$ Table 5 are used to substitute the $a \& P$ in formula (15) and formula(22) to calculate the SIFs. The SIFs of the axial pin-load case are: $164.30,162.47,166.37,184.04$, while the other case's SIFs are: 214.91, 162.08, 171.77, 179.69, the unit is $M P a \cdot \sqrt{m}$. As the fracture toughness of the material 30 CrMnSiA with thickness 6.7 mm is around $180 \mathrm{MPa} \cdot \sqrt{m}{ }^{[16]}$, it shows the SIFs calculated by formula (15) and formulas (22) are consistent with the true values.

## 5. Establishment of the fatigue crack growth model

The fatigue crack growth data are fitted by using least square method, the curves are shown in Figure 16 and Figure 17.


Figure 16. $a-N$ curves of the case of axial pin-load


Figure 17. $a$ - $N$ curves of the case of $30^{\circ}$ pin-load

The $a$ - $N$ data are calculated by the seven points incremental polynomial method ${ }^{[16]}$. The SIF range value $\Delta K$ and $\mathrm{d} a / \mathrm{d} N$ can be obtained by the fitted crack length value $a^{*}$. Table 5 shows the data processing of the test piece No.1.

Table 5. Data processing of the test piece No. 1

| $N /($ cycle $)$ | $a /(\mathrm{mm})$ | $a^{*} /(\mathrm{mm})$ | $\mathrm{d} a / \mathrm{d} N$ | $\Delta K /\left(\mathrm{MPa} \cdot \mathrm{m}^{1 / 2}\right)$ | $\lg (\mathrm{d} a / \mathrm{d} N)$ | $\lg (\Delta K)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 1.539 |  |  |  |  |  |
| 2000 | 1.967 |  |  |  |  |  |
| 4000 | 2.276 |  |  |  | 1.424 | -6.746 |
| 6000 | 2.506 | 2.579 | $1.79 \mathrm{E}-7$ | 26.556 | 1.436 | -6.732 |
| 8000 | 2.958 | 2.927 | $1.85 \mathrm{E}-7$ | 27.292 | 1.447 | -6.697 |
| 10000 | 3.307 | 3.316 | $2.01 \mathrm{E}-7$ | 28.026 | 1.459 | -6.669 |
| 12000 | 3.767 | 3.752 | $2.14 \mathrm{E}-7$ | 28.781 | 1.469 | -6.665 |
| 14000 | 4.159 | 4.186 | $2.16 \mathrm{E}-7$ | 29.497 | 1.480 | -6.649 |
| 16000 | 4.652 | 4.629 | $2.24 \mathrm{E}-7$ | 30.218 | 1.490 | -6.636 |
| 18000 | 5.091 | 5.074 | $2.31 \mathrm{E}-7$ | 30.956 | 1.502 | -6.614 |
| 20000 | 5.503 | 5.543 | $2.43 \mathrm{E}-7$ | 31.771 | 1.512 | -6.608 |
| 21800 | 5.981 | 5.976 | $2.47 \mathrm{E}-7$ | 32.577 | 1.520 | -6.585 |
| 23000 | 6.291 | 6.278 | $2.59 \mathrm{E}-7$ | 33.180 |  |  |
| 24000 | 6.578 |  |  |  |  |  |
| 25000 | 6.756 |  |  |  |  |  |
| 26000 | 7.109 |  |  |  |  |  |

The $\mathrm{d} a / \mathrm{d} N$ data of the other three test pieces can be obtained by the same method. The data points of region II are fitted by the least square method to get the $\lg (\mathrm{d} a / \mathrm{d} N) \sim \lg (\Delta K)$ curve. Figure 18 shows the fitted line of the data points.


Figure 18. $\lg (\mathrm{d} a / \mathrm{d} N) \sim \lg (\Delta K)$ curve of the axial pin-load case

The equation of the line is shown as below.

$$
\begin{equation*}
\lg (d a / d N)=1.4231 \cdot \lg (\Delta K)-8.7221 \tag{31}
\end{equation*}
$$

The Paris equation is

$$
\begin{equation*}
d a / d N=C(\Delta K)^{m} . \tag{32}
\end{equation*}
$$

In order to get the values of $C$ and $m$, the paper takes the logarithm on both sides of the equation (32).

$$
\begin{equation*}
\lg (d a / d N)=m \cdot \lg (\Delta K)+\lg C . \tag{33}
\end{equation*}
$$

The values of the Paris constants $C$ and $m$ are obtained by comparing the equation (31) with (33).

$$
C=1.896 \mathrm{E}-09, m=1.4231 .
$$

The $\lg (\mathrm{d} a / \mathrm{d} N) \sim \lg (\Delta K)$ curve of region II of the lugs subjected to 30 degrees oblique pin-load can be got by the same method. Figure 19 shows the $\lg (\mathrm{d} a / \mathrm{d} N) \sim \lg (\Delta K)$ curve.


Figure 19. $\lg (\mathrm{d} a / \mathrm{d} N) \sim \lg (\Delta K)$ curve of the 30 degrees oblique pin-load case
The equation of the line and the values of the Paris constants $C$ and $m$ are shown as below.

$$
\begin{gather*}
\lg (d a / d N)=0.7649 \cdot \lg (\Delta K)-7.8902  \tag{34}\\
C=1.2877 \mathrm{E}-08, m=0.7649 .
\end{gather*}
$$

## 6. Conclusions

Analytical and experimental investigations for fatigue crack growth behavior of 30 CrMnSiA straight attachment lugs were performed. From this investigation followings are concluded.

1. The boundary conditions of the finite element model is consistent with the true condition, cosine distributing load is the key of ensuring precision of FEM analysis.
2. The expressions of the SIFs of straight lugs subjected to axial or less than 45 degrees oblique pin-load are determined and validated. Based on this, the stress intensity factor range $(\Delta K)$ can be calculated.
3. The fatigue crack growth model of the typical straight lugs is established, offering an analytical as well as experimental method for assessing and designing damage tolerant attachment lugs in engineering.

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