Experimental Investigation of Mode I Fatigue Crack Growth Behavior of Titanium Foils

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Abstract A low-cost experimental apparatus has been developed to investigate the Mode I fatigue crack growth behavior of thin metallic foils and sheets. The apparatus utilizes magnetic coupling between a ceramic magnet and a rotating steel disk to induce cyclic tensile loads in notched rectangular test specimens.To illustrate the testing apparatus, Mode I fatigue crack growth in 30 μm thick high purity titanium foils was studied. Experiments were performed at ambient temperature using a loading frequency of 2 Hz and a nominal stress ratio of 0.1. The cyclic crack growth data could be fit to a Paris relationship between crack growth rate and stress-intensity range. The stress intensity factor exponent, *m*, in the Paris relationship was between 4 and 5, which is comparable to the relatively high values found in the literature for the tension-tension fatigue of other metallic bulk materials. Self-similarity analysis was used to explain the observed higher m values for thin metallic foils.

Keywords Fatigue testing, Mode I crack growth, foil

1. Introduction

Foil thickness materials, with a thickness in the range of 10 μm to 250 μm are used in a variety of applications, including micro-electro-mechanical systems (MEMS), integrated circuits, fuel cells and printers. The progressive trend of miniaturization of structural components leads requires knowledge regarding the mechanical behavior and properties of thin metallic materials. As discussed below, the tensile, fracture and fatigue behavior of foil-thickness materials can be very different from that observed in bulk materials. In addition to influencing the tensile properties (ductility and strength) of thin foils, grain size and grain orientation can have a major effect on crack initiation, crack growth and fracture toughness. Compared to bulk materials, microstructural inclusions or precipitates in thin foils will also have a larger impact on fracture toughness and crack growth. Surface damage, in the form of roughness, oxidation or corrosion will also have a much larger effect on the mechanical behavior of thin metallic foils.

Several experimental and theoretical investigations of the mechanical properties of foils have been carried out during last two decades. Klein *et al* [1] studied the stress-strain behavior of Cu and Al with thicknesses ranging from 10 and to 250 μm and observed a thickness effect on fracture strain, which decreased with decreasing specimen thickness. In experiments with thin Cu films, Zhang *et al* [2] observed an increase in yield strength and a decrease in fatigue life as thickness was decreased. In another study, Zhang *et al* [3] found that although the mode of fatigue damage in 25 μm thick 304 stainless steel followed that of the bulk material, the fatigue strength was higher for the 25 μm foils; the increase in fatigue strength was attributed in part to the smaller grain size of the foils

Fracture-mechanics-based test results. In a review of fatigue test techniques and experimental results for materials used in MEMS, Sharp and Turner found that the majority of available fatigue data for foils was in the form of S-N curves obtained from bending fatigue. Limited fatigue data was found for uniaxial loading with a positive mean stress [4]. The cyclic crack growth behavior from a pre-existing notch in free-standing thin foils is still a relatively unexplored area of research.

Holmes *et al* [5, 6] studied the mode I fatigue crack growth behavior of 250 μm thick center-notched Ni-base specimens, which were subjected to tension-tension fatigue at a loading frequency of 2 Hz. Using the Paris relationship, it has been found that the stress intensity factor exponent, *m* was significantly higher than commonly reported for bulk Ni-base specimens [7, 8]. A similar trend was also observed in Ti-6Al-4V foils. The high cyclic crack growth rate in thin foils was attributed to the lower Mode I lower fracture toughness commonly observed for thin metallic foils [5, 6, 9]. In a very informative study, Meirom *et al* [9] observed that ~500 nm thin films exhibit low fracture toughness and inferior resistance to fatigue crack growth when compared to conventional micro-scale-grained bulk forms of the materials. Their studies showed that thickness had a substantial effect on the Paris power law exponent and the estimation of fracture toughness. At first glance, the high crack growth rates found during Mode I testing of thin metallic foil [5, 6] is at odds with observations of higher fatigue strength found from standard (S-N) tests with un-notched thin metallic foils [2,3]. Standard fatigue life (S-N) tests include cycles to crack initiation, which may be strongly influenced by yield strength and residual stresses as well as surface finish and microstructural features such as grain size. However, fracture toughness can play an important role in the cyclic crack growth behavior of materials. For certain metals, fracture toughness exhibits a “bell-shaped” dependence on thickness, with fracture toughness reaching a maximum at intermediate thickness. In experiments with Cu, Wang *et al* [10] observed that J_c initially increased with increasing thickness, reached a maximum at a thickness of about 0.3 mm, and decreased at larger thicknesses.

Because of the very low applied loads, the cyclic crack growth behavior of metallic foils requires specialized testing approaches. For lower frequency testing (up to 1000 Hz with specially designed load frames) servo hydraulic and electro-mechanical load frames can be used for tension-tension fatigue loading histories [11, 12]. However, because of the low load levels required, the use of servo hydraulic load frames for fatigue testing is generally limited to foil thicknesses above 50 μm. Moreover, because of high cost, servo hydraulic and electro-mechanical load frames are generally not used for long duration testing of materials. The cost to perform long duration fatigue and fatigue crack growth experiments is of particular concern for low loading frequencies in the range of 0.01 to 5 Hz, which is a frequency range of general interest for many practical devices which utilize foil-thickness materials, including MEMS, fuel cells with metallic interconnects, and printer drives.

The objective of the present investigation was to develop a reliable low-cost experimental apparatus to study the low frequency high cycle Mode I cyclic crack propagation behavior of thin foils under tension-tension loading. The approach, which involves use of magnetic coupling between a clamped rectangular specimen and a rotating steel disk, is well suited for studies of Mode I fatigue crack growth. In order to illustrate the test apparatus, the cyclic crack growth rate of 30 μm thick edge-notched high-purity (99.6%) annealed titanium (Ti) foils was investigated at RT.

2. **Development of Testing Apparatus**

The overall design goal was to develop fatigue test apparatus using commercially available components and conventional machining. As shown in Figures 1, this was accomplished by magnetic coupling between a magnet attached to a sliding grip and a rotating eccentric steel disk mounted to a DC motor. As shown in Fig. 1, the specimen is clamped in face-loaded (friction) grips. One of the specimen grips is attached to a linear bearing slide, which is free to translate parallel to the specimen centerline; motion perpendicular to the centerline is constrained by the linear rail. The grip at the other end of the specimen was fixed to a load cell (Honeywell Model 41) which was rigidly fixed to the test frame. A ceramic magnet attached to the end of the translating specimen grip provides magnetic coupling with a rotating eccentric steel rotor. The strength of the couple, and hence tensile force in the specimen, is controlled by the magnet strength and the air gap between the rotor and magnet. The air gap varies with the rotation of the eccentric rotor; therefore, the force varies in a cyclic manner. A mean tensile stress is achieved by use of a weight applied through a cable and pulley attached to the sliding grip/magnet assembly. To control the cyclic loading frequency, the eccentric steel rotor is driven by variable speed DC motor. For the test apparatus shown, the DC motor had a power rating of ½ HP, and a maximum rotational speed of 34 RPM.

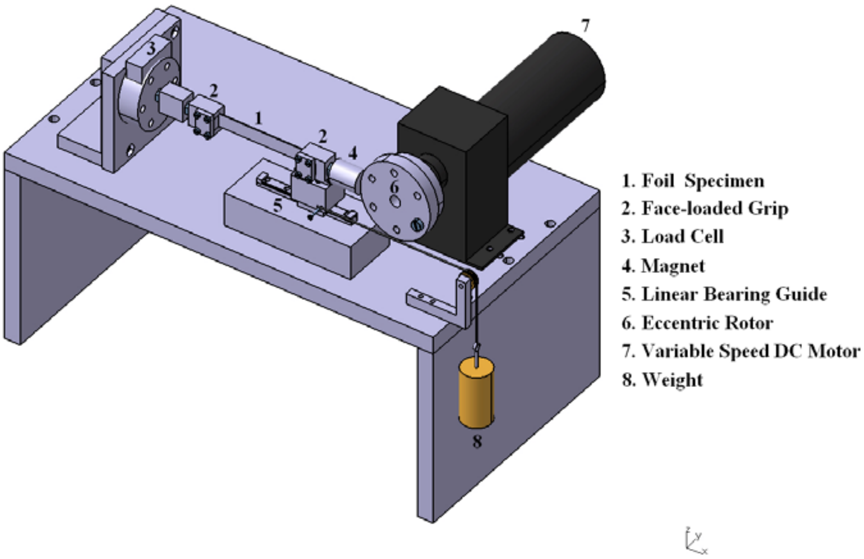

Figure 1. Schematic illustration of the fatigue testing apparatus.

For a given magnet strength and rotor speed, the waveform of the applied axial load is determined by the geometry of the rotor (shape and number of lobes). For the current test configuration, the motor speed was adjusted to provide a loading frequency of 2 Hz. Although a simple eccentric rotor was used in the present investigation, a multi-lobed rotor can also be used to increase the fatigue loading frequency or provide different peak and mean load levels within a given cycle. The shape of the rotor lobe (s) can also be changed to produce other load *versus* time waveforms. Alternatively, the shape or number of magnets that interact with the rotor can be altered to achieve a desired waveform.

Signal monitoring. An impulse counter was used to record the number of fatigue cycles. The load cell and impulse counter signals were monitored by a data acquisition board and DASYLAB software.[1] By monitoring the load cell signal, the DA board also enabled stopping the motor when the specimen failed. Figure 2 shows a close-up view of the actual fatigue testing apparatus with a 30 μm thick specimen installed.

3. Experimental Procedure

3.1. Material and Specimen Preparation

The material used for this study, commercially available annealed Ti foil (99.6% purity), was obtained as 30 μm thick sheets.[2] General information regarding the composition and mechanical

[1]Measurement Computing Corporation, Norton, MA USA.
[2]Goodfellow Corp., 125 Hookstown Grade Road, Coraopolis, PA 15108-9302 USA.

behavior of 99.6% purity Ti foil can be found in Refs [13, 14]. Single-edge-notched specimens were cut from the foil sheets using a single-blade paper cutter. To determine if differences in specimen geometry with the same notch to width ratio would affect test results, specimens with widths of 12 mm and 10 mm were tested. The first type of specimen (Group A) had a nominal thickness of 30 μm, a width of 12 mm and an overall length of 120 mm. Group A specimens had a notch length of 1.6 mm along one edge of the specimen. A second group of specimens (Group B), with the same thickness and length, but a smaller 10 mm width and shorter 1.35 mm long edge-notch were also tested. Note that both specimen groups had the same notch-to-width ratio of 0.13.

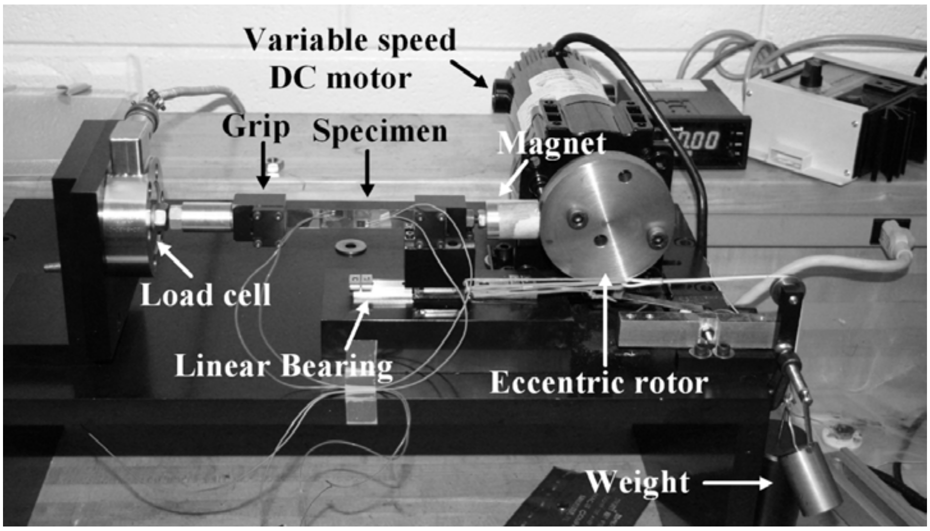

Figure 2. Close-up of the experimental setup

3.2 Experimental Procedure

All fatigue experiments were performed at room temperature at a nominal stress ratio of 0.1 and loading frequency of 2 Hz. At the start of each experiment, and with a specimen installed, the distance between the rotor and magnet was adjusted to achieve the desired maximum stress level in the specimen; in parallel, the tensile mean stress (and therefore the minimum load level) was adjusted by use of a weight applied through the cable attached to the moving grip/magnet assembly.

Table 1. Overview of the test conditions used to investigate the Mode I fatigue crack growth rate of thin titanium foils

	Type A	Type B
Loading mode	Tension-Tension	Tension-Tension
Stress ratio, R	0.1	0.1
Frequency, f	2 Hz	2Hz
Max stress, σ_{max}, MPa	111~124	122~ 124

Table 1 summarizes the specimens and test conditions that were used in the investigation. Two types of specimens having different width. For specimen in fatigue tests, crack length was measured with a 20X optical microscope attached to a video camera and digital micrometer. During the fatigue tests, crack length was measured at the end of each fatigue loading block. The optical microscope had a resolution of 0.002 mm and a repeatability of 0.005 mm. At the completion of testing, fracture surface characteristics were examined by optical and scanning electron microscopy.

4. Data analysis

As discussed in the results section, the cyclic crack growth behavior of the Ti foils could be described by the Paris relation between cyclic fatigue crack growth rate (*da/dN*) and applied stress intensity, ΔK :

$$\frac{da}{dN} = C\Delta K^{m} \tag{1}$$

C and m are obtained from a linear curve fit to the experimental data. For a single-edge-notched specimen, the applied stress intensity range, ΔK, is related to the far field stress amplitude, $\Delta\sigma$ and the crack length, a, by

$$\Delta K = Y\Delta\sigma\sqrt{\pi a} \tag{2}$$

Y is a geometry correction factor defined in terms of the ratio of the crack length a to the specimen width w. The determination of the geometry correction factor depends on the specimen's geometry and the applied loading type. For a single-edge-notched specimen, *Y* is given by

$$Y = 1.12 - 0.231\left(\frac{a}{w}\right) + 10.55\left(\frac{a}{w}\right)^{2} - 21.72\left(\frac{a}{w}\right)^{3} + 30.39\left(\frac{a}{w}\right)^{4} \tag{3}$$

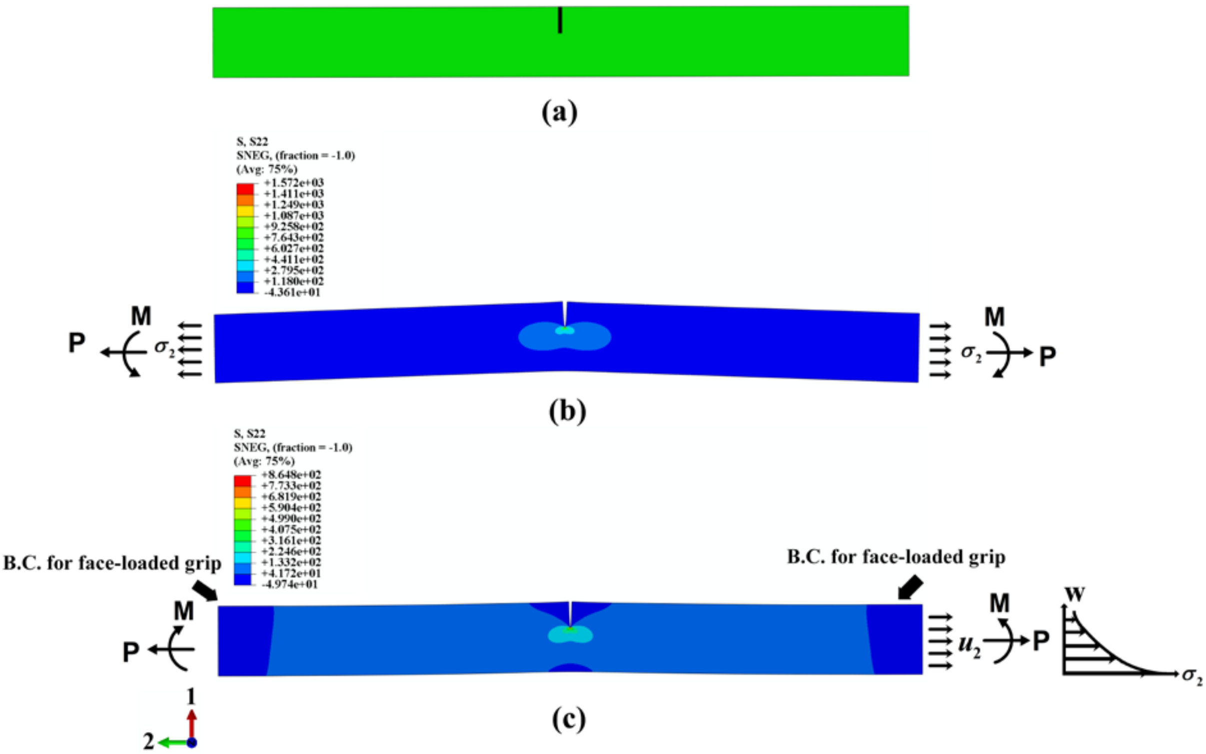

Figure 3. FEM models of the foil specimen. (a) FEM model of unloaded foil specimen. (b) Foil specimen under a uniform stress, which represents the condition used for the intensity factor given by Eq. 3. (c) Foil specimen loaded through the face-loaded grips with a uniform displacement u_2 boundary condition is applied along the right grip.

Correction of the effective stress intensity factor

Eq.3 is limited to loading configurations which can provide a uniform stress throughout the specimen cross section. In the present investigation, the foil specimen is constrained to move parallel to the applied tensile load by the linear bearing; this most closely represents a fixed-displacement end condition. The application of Eq. 3 to a fixed-end displacement loading configuration can overestimate the applied stress intensity factor [15]. The center line of the specimen shift can cause a bending moment which increases the applied Mode I stress intensity factor. This overestimation is of particular importance as a crack grows and the crack-depth-to-width ratio increases.

To estimate a corrected K_I, a finite element analysis of the end conditions for the magnetic-driven fatigue apparatus was performed. The foil, which is rigidly clamped by the face-loaded grips along its ends, is under a uniform fixed-end displacement boundary condition (Figure 3c). Due to the non-uniform tensile stress σ_2 distribution along the specimen width, a closing moment is imposed on the specimen, which reduces the effective stress intensity at the crack tip. A summary of the results of the analysis for the loading cases examined is provided in Figure 4. The corrected stress intensity factors obtained from the FEM analysis were used in the subsequent analysis of ΔK versus crack growth rate presented below.

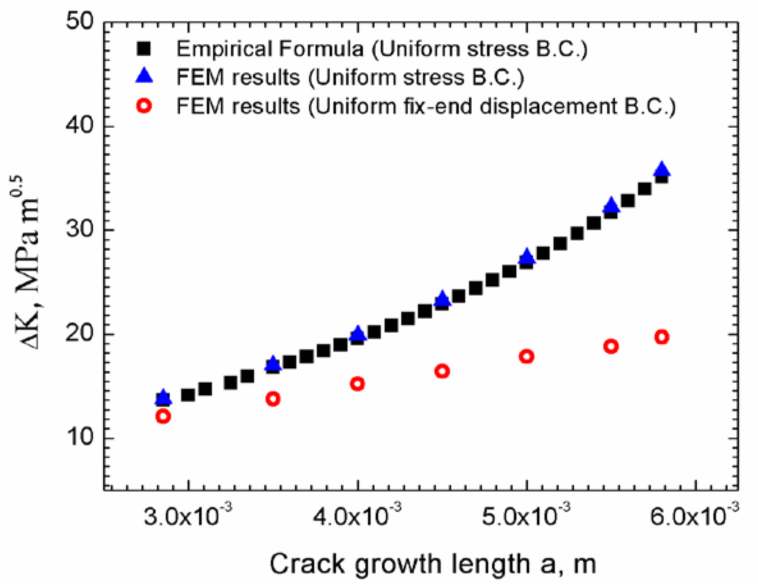

Figure 4. Variation of stress intensity factor K_I with the crack length. The data defined by solid squares represents K_I calculated from the empirical Eq. 2; the triangular data point represents K_I is determined by FEM analysis. A comparison of the data shows that the overestimation of K_I by Eq. 2 increases as crack length increases.

5. Results and discussion

5.1 Results

All fatigue cracks started from the root of the machined notch; cyclic crack propagation behavior, which remained perpendicular to the applied load, was similar for all specimens, indicating adequate alignment of the specimen was achieved during the experiments. Figure 5 shows typical results for crack length versus number of cycles. All specimens behaved in a similar manner, and the amount of scatter was limited for similar test parameters.

Figure 6 shows crack growth rate *vs.* corrected stress intensity for all specimens. The double-logarithmic plot shows that the data can be fit reasonably well using a simple Paris-equation relationship between crack growth rate and applied stress intensity range (corrected with FEM analysis). The measured stress intensity factor exponent m ranged from 3.94~5.08 for the eight specimens tested (Table 2). The small change in specimen width from 12 mm to 10 mm (Group A and Group B), did not have a measurable influence on the test results. The crack growth rate data for thin foils is quite limited and the Paris “m” value for similar pure Ti foils is not available. However, the m values found in the current investigation are similar to elevated m values of around

4~5 reported for other metals [5,6].

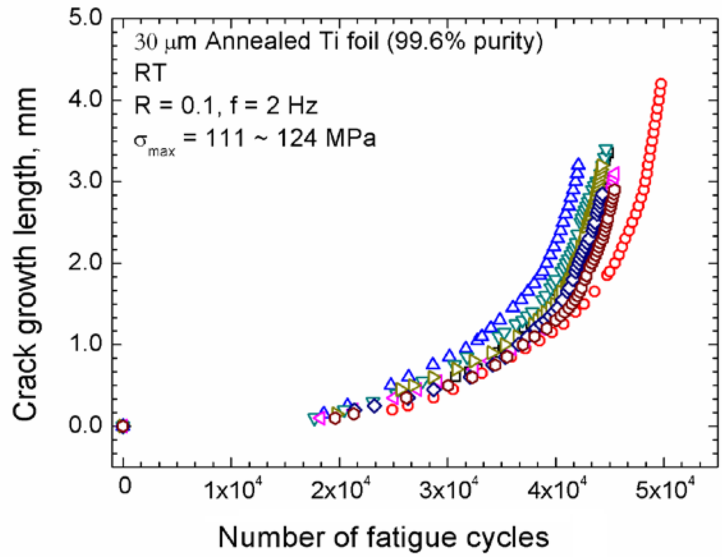

Figure 5. Crack growth length versus fatigue cycles for 30 μm thick single-edge notch Ti foil specimen foil specimens subjected to tension-tension fatigue at RT. The data scatter is very limited for similar experimental conditions.

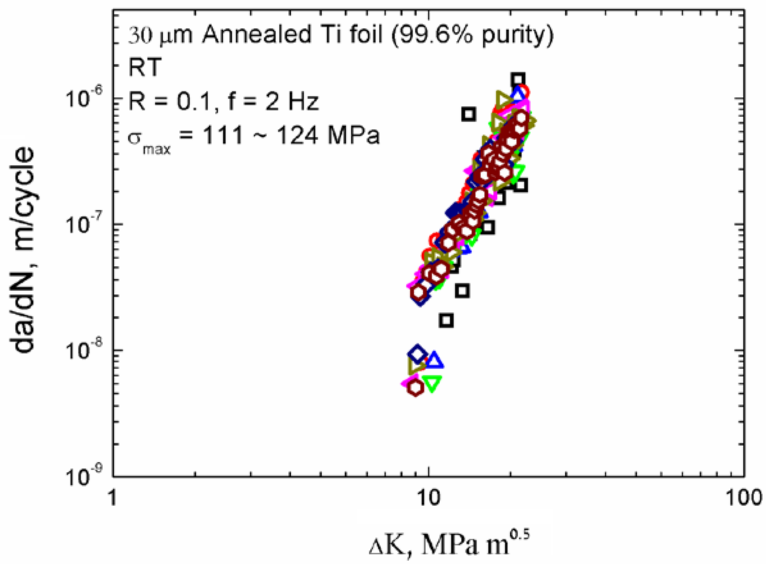

Figure 6. Cyclic crack growth rate data for 30 μm thick Ti foil specimens subjected to tension-tension fatigue loading at room temperature.

The typical crack growth path is observed after specimen failure; these images show the side view of the flat fracture surface in crack growth region II, and the corrugated facture surface in the high crack growth rate region prior to failure. The crack growth was generally perpendicular to the tensile loading direction until a final shear slip forms during failure. According to experimental measurements, the stable crack growth length was around 7~8 mm, which covers two crack growth regimes. For instance, in crack growth region II, the applied ΔK is lower than 30 MPa $m^{1/2}$ and the crack growth rate is less than 10^{-6} m/cycle. In this crack growth regime, the crack growth rate *vs.* applied ΔK can be described by the Paris relationship with the exponent value m between 4~5. With the crack growth length increasing, the applied ΔK increases and the crack propagation fit in very high crack growth regime (crack growth region III). The angle of the "knife edge" fracture surface ranges between 44° and 48°. To evaluate the morphological characteristics of Ti foil fracture surface, fractography for foil specimens was investigated by scanning electron microscopy (SEM). All SEM images exhibited similar fracture surface appearances. Representative SEM images taken of specimens that failed during cyclic loading are shown in Fig. 7. These images show the complicated fracture surface appearance for crack growth within the Paris region with occasional striations visible. Fractography features also indicate a ductile, void rupture along with indications of transverse (through thickness) necking.

Table 4. Overview of the experimental conditions and test results for the cyclic fatigue crack growth behavior 30 □m thick Ti.

ID	□□, MPa	Corrected □K_{max}, MPa $m^{1/2}$	Corrected □K_{min}, MPa $m^{1/2}$	a_i mm	N_{tot}	C	m
						correlated with FEM results	
A-1	111	19.5	11.4	1.6	44,941	2.55E-13	4.96
A-2	100	19.7	9.46	1.6	49,751	8.62E-13	4.72
A-3	112	19.1	10.4	1.6	42,104	3.34E-13	4.9
A-4	110	19.7	10.2	1.6	44,667	9.25E-13	4.58
B-1	109	19.9	8.9	1.35	45,476	4.48E-13	4.80
B-2	109	20.3	9.04	1.35	44,234	2.31E-13	5.01
B-3	110	19.0	9.22	1.35	44,330	3.61E-13	5.02
B-4	112	19.6	9.07	1.35	45,449	4.84E-14	5.45

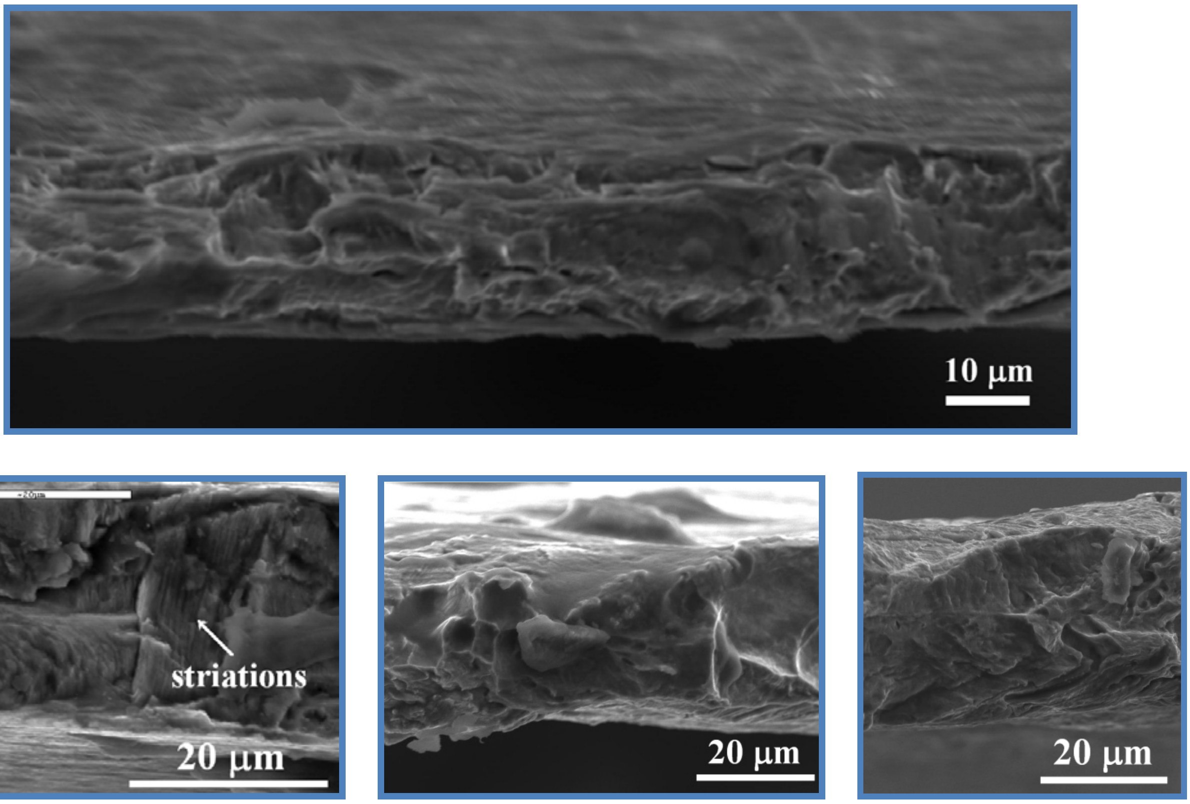

Figure 7. SEM fractography images fracture surfaces of a typical specimen subjected to tension-tension fatigue until failure. The fracture surface exhibited complicated fracture features including striations and transverse necking.

5.2 Similarity Analysis

Incomplete self-similarity of higher crack growth rate for foils

The crack growth rate can be correlated with the corrected stress intensify factor ΔK_I by a power-law relationship (Paris relationship), which is a notable scaling law [16], between the fatigue-crack growth rate per cycle, *da/dN*, and the stress intensity factor range, ΔK. This relationship has been shown to apply over a wide range of cyclic crack growth rates for metallic, polymeric and ceramic materials. For the limited number of foil-thickness materials that have been studied, the m value of is higher than that commonly found for bulk (thicker) samples [5,6].

The Paris relationship (Eq. 1) is an empirical equation that provides little information of the effect that other parameters might influence cyclic crack growth behavior. Obviously, the average crack growth depends on (i) fatigue loading parameters, *i.e.* loading ratio *R* and frequency *f*, (ii) material properties, such as yield strength, σ_y and fracture toughness, K_c and (iii) specimen size. To examine the effect of various parameters on fatigue crack growth behavior, following the mathematical similarity approach from the work of Barenblatt *et al* [17] and Ritchie [18] crack growth rate can be expressed as:

$$\frac{da}{dN} = \left(\frac{\Delta K}{\sigma_y}\right)^2 \Phi\left(\frac{\Delta K}{K_c}, R, z, f\,t\right) \tag{4}$$

z is the basic similarity parameter, given in terms of material constants and a characteristic specimen length scale *h*:

$$z = \frac{\sigma_y \sqrt{h}}{K_c} \tag{5}$$

If incomplete similarity is assumed in the parameter $\Delta K << K_c$, then

$$\Phi = \left(\frac{\Delta K}{K_c}\right)^{\alpha} \Phi_1(R,z), \quad \frac{da}{dN} = \frac{(\Delta K)^{2+\alpha}}{\sigma_y^2 K_c^{\alpha}} \Phi_1(R,z) \tag{6}$$

Comparison with the Paris relation (Eq. 1) gives,

$$C = \frac{\Phi_1(R,z)}{\sigma_y^2 K_C^{\alpha}}, \quad m = 2 + \alpha(R,z) \tag{7}$$

where α is a function of loading ratio R and similarity parameter *z*. From Eqs. 6 and 7, the incomplete self-similarity infers that the constants C and m in the Paris power law relationship depend not only experimental parameters, such as ΔK, and material parameters, but also (through the similarity parameter *z*) on a specimen length scale h. For a set of loading parameters, the exponent α (and therefore m in Eq. 7) depends only on the parameter *z*.

It has been first reported by proposed *et al* [17] and interpreted by Ritchie [18] that there is a trend of increasing Paris exponents *m* with increasing *z* corresponding to the transition from ductile (low *m* value) to brittle fracture (high *m* value). For the fatigue crack growth behavior of the Ti foils investigated, the experimentally measured Paris constant *m* varied between 4 to 5, which is slightly higher than the *m* value typically associated with bulk metal materials. The result implies accelerated fatigue crack growth rate due to foil thickness effect. According to incomplete self-similarity method, higher *m* value occurs with increasing *z* value and it corresponds to a transition from ductile fatigue fracture (m = 2~4) to more brittle fatigue fracture ($m > 4$). The higher *m* value induced by higher *z* value results from the competing mechanism between decreasing sample characteristic dimension *h*, fracture toughness K_c and increasing yield strength σ_y in the similarity parameter of *z*. Taking specimen thickness as the characteristic length scale *h*, would lead to the conclusion that foil thickness materials should have smaller z values, however compared to the dependence on yield strength and fracture toughness, z has a weaker, square-root, dependence on h. Although the mode I fracture toughness K_{Ic} and yield strength σ_y of 30 μm pure Ti foil were not determined in the present investigation, it can be expected, in line with results for foil-thickness metals, that the mode I fracture toughness K_{Ic} *decreases* with decreasing sample thickness [10]. Therefore it is indicated from the slightly higher value of m that although the sample thickness h decreases for foils, the effect of σ_y/K_{Ic} on the similarity parameter of z is more significant. The foil samples investigated also demonstrated complicate ductile combined brittle fatigue fractures features (Fig. 7). The fatigue behavior of the Ti foils, which was characterized by slightly elevated m values and both ductile and brittle fracture surface characteristics, is consistent with the speculation that the higher m value induced by higher z value with decreasing sample characteristic dimension h, fracture toughness K_c and increasing yield strength σ_y.

To summarize, a preliminary incomplete self-similarity analysis, indicates that that the slightly higher m values observed for the Ti foil specimens, and in other metal foils [5, 6] is related to a the reduction in the fracture toughness and the rising in the yield strength with decreasing metal thickness. It can also be inferred from the incomplete similarity that with a further decrease in fracture toughness with decreasing thickness even higher values of m indicative brittle crack growth could occur.

6. Conclusions

A new experimental approach to study the tensile fatigue behavior of ultra-thin metal foils was developed. The technique, which involves magnetic coupling between an eccentric steel rotor and a ceramic magnet attached to a specimen grip mounted to a linear bearing slide. Cyclic tensile loads are induced in the specimen during rotation of the eccentric disk which is attached to a variable speed DC motor. The apparatus can be readily fit with a small furnace for testing at elevated temperatures. For the proof-of-concept experiments, fatigue crack growth tests were performed with commercially pure 0.03 mm thick Ti foils. Based on the observations, test results and a preliminary similarity analysis, the following conclusions can be made:

1. The test apparatus shows considerable promise as a low-cost method to study tensile fatigue and mode I cyclic crack growth of thin foils and wires at ambient and elevated temperatures.

2. Analysis showed that a shift in the centerline of loading can occur when testing edge-notched specimens, which induces an opening moment at the crack tip. If not accounted for, this effect can lead to overestimation of the mode I stress intensity factor.

3. Crack growth data for specimens with the same notch-to-width ratios, exhibits similar trends and fatigue crack growth parameter values. For all test cases, stable crack growth was observed.

4. The effective stress intensity factor ΔK_I corrected by FEM analysis for face-loaded specimens were used to correlate the crack growth rate *da/dN*. The crack growth rate can be described by a Paris relationship with a m value between 4~5. The results are consistent with other studies of thin metal foils where high m values are commonly observed.

An incomplete self-similarity theory was used to gain additional insight to the higher m value in Paris law and possible trends with decreasing specimen thickness. It is proposed that due to a significant reduction in fracture toughness K_{Ic} for foil thickness metals, the basic similarity parameter z increases, which leads to an increasing m value. This also infers that there is a transition from ductile fracture to brittle facture behavior with decreasing thickness. It is suggested that with continuing decreasing specimen thickness, *m* could increase until a constant maximum value is achieved and brittle fatigue fracture behavior dominates the crack growth process.

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