Several kinds of Calculation Methods on the Crack growth Rates for Elastic-Plastic Steels

Yangui Yu*

Zhejiang GuangXin New Technology Application Academy of Electromechanical and Chemical Engineering
Hangzhou, 310007, China
* gx_yyg @126.com

Abstract Research interdependent correlations among various calculations models, relationship between their calculations parameters, and relationship between their materials constants in equations on strengths and life predictions, which are elastic-plastic steels with cracks. Thereby obtain and suggest to adopt several new calculations models and their calculations methods on crack propagation rates, which are respectively suitable for calculations of crack propagation rates under different stress ratio loading, or are suitable for calculations under symmetrical and unsymmetrical cyclic loadings, the work stresses less than or greater than the yield limit. Also the material constants B_2 in new models are defined as comprehensive material constants and give also out their physical meaning and geometric significances that are having a function relation with other parameters. Moreover provide yet a calculation example of crack propagation rate. Anticipate that the works will be having practical significances for fatigue - damage - fracture tests to save experimental times, fund inputs and to expand engineering applications.
Keywords Elastic-Plastic-materials, Comprehensive material constants, Crack growth rate, Calculation methods

1. Introduction

In the engineering fields, loading conditions are very complicated. In the engineering designs and calculations, there are a lot of calculating models are applied. In traditional design methods, for their calculating parameters, for their material constants adopted in the models, in addition to some key calculating parameters, in addition to some limited materials constants which are showing the material properties, they must be by means of the experiments to take data. However there are also some of the traditional calculating models, their calculating parameters and material constants had been already extensively applied, which had been derived usually from the relationships between various calculation parameters or from the relationships between all kinds of material constants, afterwards again applied to engineering design calculation and analysis.
The same, for including cracks structures and materials undergone to very complicated loadings, the calculations on their strengths, crack propagation rates and service life, in all kinds of calculation models also must refer to many calculation parameters and material constants, but these required parameters of designs and calculations got from experiments are always limited. In fact, between these calculations parameters or between these materials constants each other, there are also usually some functional relations. So that to research and analyze these calculations models, their calculations parameters and materials constants, of course, some new models and their parameters and constants can also be derived from them and would be applied to designs and calculations in engineering.
The author researches various strengths, crack propagation rates and life calculations models, to analyze and find interdependent relationships between the various calculation parameters and all kinds of material constants, proposes respectively several new calculation models and calculation methods for the elastic-plastic steels with cracks. But also puts forward specially the new concepts of comprehensive material constants and their calculation methods, its purpose is in combination

with limited experimental data and conventional material constants, thereby to realize the calculations for the crack growth rates and to do the life predictions which are in different stress ratio under complicated loading conditions. Moreover, provides yet homologous calculation examples. Anticipate that the works will be having practical significances for fatigue - damage - fracture tests to save experimental times, fund inputs and to expand engineering applications.

2. The calculations of crack growth rate for elastic-plastic steels

In references [1，2] provide the relation between the fracture life N_{2fc} and half-cycle of amplitude value $\Delta\delta_t/2$ of crack tip open displacement at crack growth stage,

$$\frac{\Delta\delta_t}{2} = \delta_{fc} \times 2N_{2fc}^{c'_2} \tag{1}$$

But if it is expressed to adopt one-cycle of crack tip open displacement range $\Delta\delta_t$, it become as following form

$$\Delta\delta_t \times N_{2fc}^{-c_2} = B_2^{c_2} \tag{2}$$

And relation between the crack growth rate da_2/dN_2 and one-cycle of crack tip open displacement range $\Delta\delta_t$ is as discussed [3~4],

$$da_2/dN_2 = B_2 \Delta\delta_t^{\lambda_2} \tag{3}$$

Where the equations (1) and (2) can be described as the reversed beeline C_2C_1 in bidirectional combined coordinate system and bidirectional curves in the whole process(Fig.1) ［5~7］, that is predigested by logarithm transacting just is showed the relation between the life and the crack tip open displacement. And the equations (3) can be described as the positive direction beeline C_1C_2, it just is showed for the relation between the crack growth rate and the crack tip open displacement.

Research and discover that in fact the relations among equations (1), (2) and (3) are consistent. Here the parameters λ_2 , c_2 and B_2 are all material constants. λ_2 and c_2 are exponents of equations, $\lambda_2 = -1/c_2$, its geometrical meaning is the slope of force triangle bevel edge. Of course some key material constant in the engineering calculations must be obtained from experimentations. But actually in a number of calculation models, there are functional relationps between some parameters or between some material constants each other, so that we can use the kinds of having important significances of correlations, can derive a number of beneficial formulas, obtain some calculation methods on the strengths, crack propagation rates and life predictions. For example between the material constant B_2 and other constants are having functional relations as formulas below,

$$B_2 = 2(2\delta_{2fc})^{-\lambda_2}(a_{2c} - a_{02})/(N_{2c} - N_{02})\ (\sigma_m = 0) \tag{4}$$

$$B_2 = 2\left(2\delta_{2fc}(1 - \frac{\delta_m}{\delta_{2fc}})\right)^{-\lambda_2} \times (a_{2c} - a_{02})/(N_{2c} - N_{02}), (\sigma_m \neq 0) \tag{5}$$

Where δ_{2fc} in (4~5) are critical value of the crack tip open displacement, $\delta_{2fc} = \delta_c$, the B_2 is defined as comprehensive material constants. Its physical meaning is a concept of power, is a maximal increment value, and also is given out energy in one cycle made specimens to failure. Its geometrical meaning is a maximal micro-trapezia area to approximate to beeline, and is also an intercept between $O_3 - O_4$ (Fig.1). And slope of micro-trapezium bevel edge just is corresponding to the exponent λ_2 of formula (4, 5). a_{2c} is a critical crack size, a_{02} is initial (or precrack) crack size. N_{02} is initial life, $N_{02} = 0$; N_{2c} is a critical life. δ_m is a mean value of the crack tip open displacement, $\delta_m = (\delta_{\max} + \delta_{\min})/2$, here $\delta_{\max}$ and $\delta_{\min}$ are the maximal and minimal values of the crack tip open displacement. Moreover if its unit of the parameter B_2 is calculated by 'mm', then the value of B_2 must be again multiplied by 1000.

For the sake of safety, the comprehensive constant B_2 should be calculated as following form,

$$B_{2eff} = 2\left(2\delta_{2eff}(1 - \frac{\delta_m}{\delta_{2fc}})\right)^{-\lambda_2} \times (a_{2eff} - a_{02})/(N_{pv} - N_{02}) \tag{6}$$

Where B_{2eff} is also an effective comprehensive material constant, its physical meaning is a given out effective energy in one cycle made specimens before failure. Here $(a_{2eff} - a_{02})/(N_{pv} - N_{02}) = v_{pv}$, v_{pv} is defined to be the virtual rate, $v_{pv} = v^*(m/cycle)$ [6,8]. $v_{pv} \approx 3\times10^{-8} \sim 3\times10^{-7} = v^*, (m/cycle)$. a_{eff} is an effective size during steady growth course. N_{pv} is a virtual life. δ_{2eff} is an effective value of crack tip open displacement, that is the parameters during steady growth course, they must be all obtained from experiment. Here suppose that

$\delta_{2eff} = (0.3 \sim 0.4)\delta_{\mathrm{c}}$.

It should point that for elastic-plastic materials, the calculating validity of the crack tip open displacement is restricted by different work stresses σ, their calculation expressing forms would be different for work stress under $\sigma < \sigma_{\mathrm{s}}$ or $\sigma > \sigma_{s}$ condition. Such, calculations of the crack growth rate in equation (3), in addition to directly calculate from the experiment achieved data, and can also use the following varied calculation methods to calculate it.

2.1 Calculations for crack growth rate under work stress $\sigma < \sigma_s$ condition

According to the D-M' as discussed by [9] mathematical model to calculate the crack tip open disparagement δ_t, it can be adopted as following equation.

$$\delta_t = \left(\frac{8\sigma_s a_2}{\pi \cdot E} \ln \sec \frac{\pi \times \sigma}{2\sigma_s} \right) \tag{7}$$

Under $\delta_t -> \delta_c$, $K_1 -> K_{1c}$ condition, and the work stress $\sigma / \sigma_s << 1$($\sigma \leq 0.5\sigma_s$), for the D-M' model is made by simplify treatment, it is

$$\delta_t = \beta \frac{y_2 K_2^2}{E \cdot \sigma_s} \tag{8}$$

Here y_2 is correction coefficient related to crack size and shape. The coefficient β is $\beta = 1$ under plane stress condition, and $\beta = (1 - \nu^2)/2$ under plane strain condition. ν is Poisson's ratio. If $\beta = 1$ and under cyclic loading, the crack tip open displacement can adopt three of kinds calculations methods as following forms.

(1) The calculations methods used by the stress intensity factor $K_2 (= K_{\mathrm{I}})$

For this method, the comprehensive material constant B_{2eff} and the crack tip open displacement $\Delta\delta_t$ range in the crack propagation rate equation da_2 / dN_2 are all to adopt the stress intensity factor to express. Here the crack tip open displacement range $\Delta\delta_t$ can be [9],

$$\Delta\delta_t = \frac{y_2 \Delta K_2^2}{2E \cdot \sigma_s} \tag{9}$$

$$\frac{da_2}{dN_2} = B_{2eff} \times v_{pv} \times \left(\frac{y_2 \Delta K_2^2}{2E \cdot \sigma_s}\right)^{\lambda_2}, (m/cycle), (\sigma_m = 0) \tag{10}$$

Here for $\sigma < \sigma_s$, and under the symmetrical cyclic loading ($R_\delta = \delta_{\min}/\delta_{\max} = -1$),

$$B_{2eff} = 2\left(\frac{K^2{}_{eff}}{2E \cdot \sigma_s}\right)^{-\lambda_2} \times v_{pv} \tag{11}$$

But for $R_\sigma = \sigma_{\min}/\sigma_{\max} \neq -1$ (or $R_\delta = \delta_{\min}/\delta_{\max} \neq -1$), for $\beta = 1$, the parameters δ_m in equations (6) should be also changed as follow forms,

$$B_{2eff} = 2\left\{\frac{K^2{}_{eff}}{2E \cdot \sigma_s} \times \left(1 - \frac{K_{1\max}^2 + K_{1\min}^2}{2K_{1c}^2}\right)\right\}^{-\lambda_2} \times v_{pv} \tag{12}$$

$$K_{eff} \approx \sqrt{K_{th} K_{1c}} \quad \text{or} \quad K_{eff} \approx (0.25 \sim 0.55) K_{1c} \tag{13}$$

Where K_{eff} is an effective stress intensity factor, K_{th} is threshold stress intensity factor. $K_{1\max}$ and $K_{1\min}$ are respectively the Maximums and minimums value of stress intensity factor. E is a modulus of elasticity. In fact the (12) is consistent with (6).

(2) The calculation methods used by the stress σ

For this method, the comprehensive material constant B_{2eff} and the crack tip open displacement $\Delta\delta_t$ range in the crack propagation rate equation are all to use the stress to express. Here the crack tip open displacement δ_t and $\Delta\delta_t$ are as below [10]

$$\delta_t = \pi \sigma_s y_2 (\sigma/\sigma_s)^2 a_2 / E, (m) \tag{14}$$

Under cyclic loading, and for stress ratio $R_\delta = \delta_{\min}/\delta_{\max} = -1$, the crack tip open displacement range is,

$$\Delta\delta_t = \frac{y_2 \Delta\sigma^2 \pi a_2}{4\sigma_s E}, (m) \tag{15}$$

Then equation of crack growth rate is

$$\frac{da_2}{dN_2} = B_{2eff} \times \left(\frac{y_2 \Delta\sigma^2 \pi a_2}{4\sigma_s E} \right)^{\lambda_2}, (m / cycle) \tag{16}$$

And for different stress ratio, the B_{2eff} are respectively,

$$B_{2eff} = 2\left[4(\pi\sigma_s (\sigma_{meff} / 2\sigma_s)^2 a_{meff} / E)\right]^{-\lambda_2} \times v_{pv}, (\sigma_m = 0) \tag{17}$$

$$B_{2eff} = 2\left\{4(\pi\sigma_s (\sigma_{meff} / 2\sigma_s)^2 a_{meff} / E)\left(1 - \frac{a(\sigma_{\max}^2 + \sigma_{\min}^2)}{2a_{meff}\sigma_s^2}\right)\right\}^{-\lambda_2} \times v_{pv}, (\sigma_m \neq 0) \tag{18}$$

Where σ_{meff} is maximal effective stress, $\sigma_{meff} = (0.5 \sim 0.55)\sigma_s$. a_{meff} is a maximal effective crack size,

$$a_{meff} = \frac{K_{eff}^2}{\sigma_s^2 \pi} = \frac{(0.5 \sim 0.55)K_{1c}^2}{\sigma_s^2 \pi}, (m) \tag{19}$$

(3) Calculation method of combination of the stress and the stress intensity factor

For this method, the comprehensive material constant B_{2eff} in the crack propagation rate equation adopts the stress intensity factor to express, and the crack tip open displacement $\Delta\delta_t$ range adopts the stress to express. So their forms are as bellows

$$\frac{da_2}{dN_2} = 2\left(\frac{K^2{}_{eff}}{2E \cdot \sigma_s}\right)^{-\lambda_2} \times v_{pv} \times \left(\frac{y_2 \Delta\sigma^2 \pi a_2}{4\sigma_s E}\right)^{\lambda_2} \ (\sigma_m = 0) \tag{20}$$

$$\frac{da_2}{dN_2} = 2\left(2(K_{eff} / \sigma_s)^2 \times \frac{\sigma_s}{E}\left[1 - \frac{K_{2\max}^2 + K_{2\min}^2}{2 \cdot K^2{}_{1c}}\right]\right)_{eff}^{-\lambda_2} v_{pv} \times \left(\frac{y_2 \Delta\sigma^2 \pi a_2}{2\sigma_s E}\right)^{\lambda_2} \ (\sigma_m \neq 0) \tag{21}$$

It must point that the equations (10), (16), (20) and (21) can also only apply to work stress $\sigma/\sigma_s << 1$($\sigma \le 0.5\sigma_s$).

2.2 Calculations for crack growth rate under work stress $\sigma => \sigma_s$ condition

On the other hand, for work stress $\sigma => \sigma_s$ condition, it must be changed to use as following calculating equations. Here there are also two kinds of methods

(1) The calculation methods used by the stress σ

It also can directly use the stress σ to calculate the δ_t and $\Delta\delta_t$ [10]. Here the crack tip open displacement δ_t and $\Delta\delta_t$ are as bellows [10]

$$\delta_t = \frac{0.5\pi\sigma_s y_2(\sigma/\sigma_s + 1)a_2}{E} \quad (m) \tag{22}$$

$$\Delta\delta_t = \frac{\pi\sigma_s y_2(\Delta\sigma/2\sigma_s + 1)a_{02}}{E}, (m) \tag{23}$$

$$\frac{da_2}{dN_2} = B_{2eff} = \left(\frac{\pi\sigma_s y_2(\Delta\sigma/2\sigma_s + 1)a_2}{E}\right)^{\lambda_2}, (m/cycle) \tag{24}$$

$$B_{2eff} = 2\{2[(\pi\sigma_s(\sigma/\sigma_s + 1)a_{meff}/E]\}^{-\lambda_2} \times v_{pv} \ (\sigma_m = 0) \tag{25}$$

$$B_{2eff} = 2\left\{[2(\pi\sigma_s(\sigma/\sigma_s + 1)a_{meff}/E] \times \left(1 - \frac{a(\sigma_{\max}^2 + \sigma_{\min}^2)}{2a_{meff}\sigma_s^2}\right)\right\}^{-\lambda_2} \times v_{pv} \ (\sigma_m \ne 0) \tag{26}$$

$$a_{meff} = \frac{E\delta_{eff}}{\pi\sigma_s(\sigma/\sigma_s + 1)}, (m) \tag{27}$$

Where $\delta_{eff} = (0.3 \sim 0.35)\delta_c$.

(2) The calculation methods combined by stress and strain

Its method is that the comprehensive material constant B_{2eff} to adopt the δ_{eff} to express, and the crack tip open displacement to adopt the stress σ to express, its form as follow

$$\frac{da_2}{dN_2}=2\left(2\delta_{eff}(1-\frac{\delta_m}{\delta_c})\right)^{-\lambda_2}\times v_{pv}\left(\frac{y_2\pi\sigma_s(\Delta\sigma/2\sigma_s+1)a_2}{E}\right)^{\lambda_2},(\sigma_m\neq 0) \tag{28}$$

Here can take $\delta_m \approx 0$.

3. Calculation example

A pressure vessel is made with steel 16MnR, its strength limit of material $\sigma_b = 573MPa$, yield limit $\sigma_s = 361MPa$, modulus of elasticity $E = 200000MPa$. Critical stress intensity factor $K_{2c} = K_{1c} = 92.7\mathrm{MPa}\sqrt{\mathrm{m}}$. Suppose shape correcting coefficient of long crack $y_2 = 1.01$. Its local stress $\sigma_{\max} = 450\,MPa$ at stress concentration point, $\sigma_{\min} = 0$. Other computing data is all in table 1. Try to calculate the growth rate da_2/dN_2 at crack size $a_2 = 0.001m$ under $\sigma = 450MP > \sigma_s = 391\quad MPa$ conditions.

Table 1. Calculation data

$K_{1c}, MPa\sqrt{m}$	$K_{eff}, MPa\sqrt{m}$	$K_{th}, MPa\sqrt{m}$	v_{pv}	m_2	δ_{2fc}, m	λ_2	y_2	β
92.7	28.23	8.6	2×10^{-7}	3.91	0.00018	2.9	1.01	1

(1) Calculations for relevant parameters

1) According to table 1, take the virtual rate, $v_{pv} = 2\times10^{-7}$ (m/Cycle).

2) Take effective crack tip open displacement,

$\delta_{eff} = 0.35\times\delta_c = 0.35\times0.00018m = 6.3\times10^{-5}(m)$; Take $\delta_m \approx 0$.

3) Calculation for effective crack size

$$a_{eff} = \frac{\delta_{eff}E}{\pi\sigma_s(\sigma/\sigma_s+1)} = 4.94537\times10^{-3}(m)$$

4) According to (26), calculation for comprehensive material constant B_{2eff}

$$B_{2eff} = 2\left[2\times(\pi\sigma_s(\sigma/\sigma_s+1)a_{eff}/E)\right]^{\lambda_2}\times v_{pv}$$
$$= 2\left[2\times3.1416\times361(450/361+1)\times0.00494537/200000\right]^{-2.9}\times2\times10^{-7} = 84151(m^{\lambda_2}\cdot m/cycle)$$

The experiment value of Steel 16MnR, $B_2 = 84570$, and here its calculation value 84151, so that it is close to experimental data.

5) According to equation (23), computing for crack tip open displacement range $\Delta\delta_t$

$$\Delta\delta_t = \frac{y_2\pi\sigma_s(\Delta\sigma/2\sigma_s+1)a_2}{E} = \frac{1\times3.1416\times361(450/2\times361+1)0.001}{200000} = 9.204\times10^{-6},(m)$$

(2) Calculation for crack growth rate under $\sigma = 450MPa > \sigma_s$ condition.

According to equation (24), at $a_2 = 0.001m$, its crack growth rate is

$$\frac{da_2}{dN_2} = 2\left[2\times(\pi\sigma_s(\sigma/\sigma_s+1)a_{eff}/E)\right]^{-\lambda_2}\times v_{pv}\left[\frac{\pi\sigma_s y_2(\Delta\sigma/2\sigma_s+1)a_2}{E}\right]^{\lambda_2}$$
$$= 2\left[2\times3.1416\times361(450/361+1)\times0.00494537/200000\right]^{-2.9}\times2\times10^{-7}$$
$$\times\left[\frac{3.1416\times361(450/2\times361+1)0.001}{200000}\right]^{2.9} = 84151\times2.4869\times10^{-15} = 2.062\times10^{-10},(m/cycle)$$

4. Conclusions

(1) The correlating equation $\Delta\delta_t/2 = \delta_{fc}\times2N_{2fc}^{c'2}$ between half-cycle of crack tip open displacement amplitude $\Delta\delta_t/2$ and life N_{2fc}, the correlating equation $\Delta\delta_t\times N_{2fc}^{-c_2} = B_2^{c_2}$ between one-cycle of crack tip open displacement range $\Delta\delta_t$ and life N_{2fc} and the correlating equation $da_2/dN_2 = B_2\Delta\delta_t^{\lambda_2}$ between one-cycle of crack tip open displacement range $\Delta\delta_t$ and crack growth rate da_2/dN_2, which the interrelations among three of kinds relation- expressions actually are consistent.

(2) The crack growth rate equations (10), (16), (20), (21), (24) and (28) of suitable for elastic-plastic steels are also consistent with the rate equation (3) in [3, 4]. But the material constant B_2 in latter (3) is obtained from experiment under constant stress ratio condition, so that only applying to corresponding loading. And the former, they can respectively do calculations for cracks growth rates of suitable for different stress ratio and loading conditions which they are only in combination with a small number of experiment data, Therefore they can expand the application ranges, and increase the calculating reliabilities and safeties.

(3) The comprehensive materials constants B_2 and B_{2eff} in crack growth rate equations are to have interdependent functional relationships with parameters δ_{eff}, λ_2, δ_m and v_{pv}. Its physical

meaning of the B_{2eff} is to give out effective value of energy in one cycle made specimens before to failure. Its geometrical meaning is a maximal micro-trapezia area to approximate to beeline.

(4) The crack growth rate equations (24) and (28) are generally suitable for under $\sigma => \sigma_s$; and the (10), (16), (20) and (21) can only applied to under $\sigma <= 0.5\sigma_s$. If the (10), (16), (20), (21) are applied to under $\sigma > 0.5\sigma_s$ and $\sigma < \sigma_s$, their calculation error are larger.

Acknowledgments

Author thanks sincerity the Zhejiang Province Natural Science fund Committee to support and provide subsidization of research funds. Numbers of supporting project is Y1111267.

References

[1] Yu Yangui, Tan Jianrong. The calculations in whole process of fatigue-damage-fracture for components. Advances in fracture research. ICF9, PERGAMON, Sydney, 1997, pp.1401–1409.

[2] Yu Yangui. The Correlation among Each Parameter in Some Equation on Crack Growth Stage, Advances in Fracture Research, ICF9, Sydney, Volume 3. PERGAMON. pp.1395-2002. 1997.

[3] W. D. Dover, Engineering Fracture Mechanics, Vol. 5, Nol. 5, (1973)11.

[4] B. B. Pokrovsky, B. Kaplunenko G. Influence of asymmetry cycle on the characteristics of the cyclic crack resistance for thermostable steels. Problems of strength. (1987), 11, 8–13. (В. В. Покровский, В. Г. Каплуненко, Влияние асимметрии цикла на характеристики циклической трещиностойкости Теплоустойчивых сталей. Проблемы Прочности. (1987) 11. 8~13.)

[5] Yu Yangui, Sun Yiming, MaYanghui and XuFeng. The Computing of intersectant relations for its Strength Problem on Damage and Fracture to Materials with short and long crack. International Scholarly Research Network ISRN Mechanical EngineeringVolume 2011, Article ID 876396.

[6] Yangui Yu，The Calculations of Evolving Rates Realized with Two of Type Variables in Whole Process for Elastic-Plastic Materials Behaviors under Unsymmetrical Cycle. Mechanical Engineering Research; Vol. 2, No. 2; ISSN 1927-0607(print) E-ISSN 1927-0615(Online). Canadian Center of Science and Education (2012) 77-87.

[7] Yu Yangui, Bi Baoxiang, MaYanghau, Xu Feng. Damage Calculations in Whole Evolving Process Actualized for the Materials Behaviors of Structure with Cracks to Use Software Technique. 12^{th} International Conference on Fracture Proceeding. 2009, July 12-19, Ottawa, (CD, Author Index Y, Yangui).

[8] S. Ya. Yaliema. Correction about Paris's equation and cyclic intensity character of crack. J. Strength Problem. Vol, 147，(1981) 9:20–28. (In Russian).

[9] Dugdale, D. s. J. Mech Physiss Solids 8 (1960),100–104.

[10] GB/T 19624-2004, Safety assessment for in-service pressure vessels containing defects (2005), China. 2005; 24–26 (in Chinese).