Calculation of bearing loads for fractured specimens by FRASTA simulation

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Abstract Based on the previously proposed simple bar hypothesis, the fracture surfaces can be assumed to be composed of independent rectangular bars. In this paper, by dividing the plastic deformation into single bars, the original lengths of these bars were deduced and then the global strains of these bars during the course of failure were calculated. According to the relationship between true stress and true strain of the material, the normal stress on the cross section of each bar was determined. Multiply the stress with the cross section area, the load acted on each single bar was obtained. Adding all loads on all bars together led to the total applied load of the specimen.

Keywords Fracture surface; FRASTA; Fatigue load

1. Introduction

To investigate the fractured surfaces always reveals a lot of useful information. For example, details of the processes that lead to failure can be determined from these surfaces, making it useful to investigate their morphology.

In order to investigate the reasons for a material's failure, it is necessary to know the temperature, environment and load imposed on it. Since these records are often unavailable, the reasons for failure must be deduced by analyzing fracture surfaces.

Dr. Kobayashi firstly proposed fracture-surface topography analysis (FRASTA) in 1987 [1]. FRASTA considers that as the crack extends, the material immediately beneath the newly formed fracture surfaces undergoes no further inelastic deformation. Based on this understanding, the process of fracture can be rebuilt from the conjugate fractured surfaces. Using the FRASTA technique, the details of the void nucleation and growth, the coalescence of the voids and cracks, and the crack propagation process can be clarified visually [2]. However, the FRASTA application has previously focused on smaller size scales and localized behaviors [3].

A novel method [4] of measuring the CTOA and determining the J integral using FRASTA was proposed in 2006. It extended the application of FRASTA to global fracture surfaces. Software [5] was developed for researching the global fracture surfaces based on this principle. The relationship between J integral and fracture surface average profile [6] and the relationship between J integral and COD [7] were proposed respectively based on FRASTA reconstruction.

The main goal of this paper is to propose a new method to calculate the specimen applied load during failure from the fracture surfaces. Based on the above researches especially the proposed simple bar hypothesis, the fracture surfaces will be divided into independent rectangular bars. The calculation of elongation, global strain, cross-section normal stress and so on of these bars will be deduced. By adding the applied force of each bar together, the total applied force of the specimen during the course of failure will be derived.

2. Method for calculating the applied loads of fractured specimens

The simple bar hypothesis was firstly proposed for determining the J integral [4]. Then it was used in the research of relationship between J integral and fracture surface average profile [6]; and relationship between J integral and COD [7] respectively. Results in these researches verified the validity of the proposed simple bar hypothesis. Based on the hypothesis, a new method for determining the applied loads of fractured specimens during the course of failure will be proposed and experiments will be performed to verify it.

It is well known that, by means of Mises yield criterion, for the Mode Ⅰ stress field, the boundary of plastic zone around crack tip for plain strain [8] can be expressed as Eq. (1)

$$r(\theta) = \frac{{K_{\mathrm{I}}}^2}{2\pi{\sigma_y}^2}\cos^2\frac{\theta}{2}\left[(1-2\nu)^2 + 3\sin^2\frac{\theta}{2}\right]. \quad (1)$$

where K_{I} is the stress intensity factor and ν is the Poisson's ratio. In plastic zone, the distribution of strain component that perpendicular to fracture surface is

$$\varepsilon(y) = \frac{K_{\mathrm{I}}}{2G(1+\nu')\sqrt{2\pi r}}\cos\frac{\theta}{2}\left[(1-\nu') + (1+\nu')\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]. \quad (2)$$

where $\nu' = \nu$ and $\nu' = \nu/(1-\nu)$ for plain stress and plain strain respectively, G is shear modulus.

Introduction of Eq. (1) into Eq. (2) leads to Eq. (3)

$$\varepsilon(y) = \frac{\sigma_y}{2G(1+\nu')\sqrt{(1-2\nu)^2 + 3\sin^2\frac{\theta}{2}}}\left[(1-\nu') + (1+\nu')\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right], \quad (3)$$

where σ_y is the yielding stress. It is clear that, on the boundary of plastic zone, for certain material, $\varepsilon(y)$ is only relative to θ. And for $\theta = \pi/2$, $\varepsilon(y)$ can be represented as Eq. (4),

$$\varepsilon(y)_{\frac{\pi}{2}} = \frac{\sigma_y}{2G(1+\nu')\sqrt{(1-2\nu)^2 + \frac{3}{2}}}\left(\frac{3}{2} - \frac{\nu'}{2}\right). \quad (4)$$

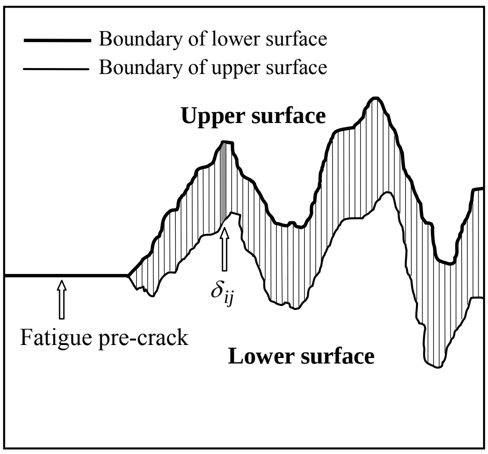

Figure 1. Division of plastic deformation on cross-sectional plot

This paper defines

$$\varepsilon_y = \varepsilon(y)_{\frac{\pi}{2}}. \tag{5}$$

It is obvious that ε_y is a constant. According to elastic-plastic fracture mechanics, at the crack tip, when the strain component along y axis direction reaches some certain value, crack extends. This certain value is defined as ε_f in this paper.

Fig. 1 displays a cross-sectional plot of a compact tension specimen. The hatched area is the overlap of the upper and lower fracture surfaces when driven to the initial position (state before test started) after being broken. It refers to the plastic deformation left on the specimen. In order to simplify the calculation, this paper supposes the plastic deformation is symmetry about the fracture surface as shown in Fig. 2. It means that the specimen fractures along x axis. Furthermore, this paper defines the area which is plastically deformed during the course of testing as plastic field. It is worth noting that plastic field is different from plastic deformation as shown in Fig. 2.

In this paper, according to the simple bar hypothesis, the fracture surfaces are assumed to be composed of independent rectangular bars. The sizes of these bars in the X and Y directions (the directions of crack extension and specimen thickness, respectively) are determined by the resolution of the laser microscope for the X and Y axes (represented by p and q, respectively). In this paper, they are both 50 μm. Thus, along the direction of crack extension, the amount of plastic deformation which is gray-scaled in Fig. 2 can be considered as the elongation of the bar. This paper supposes the original half length of the bar is l and the half elongation is Δl.

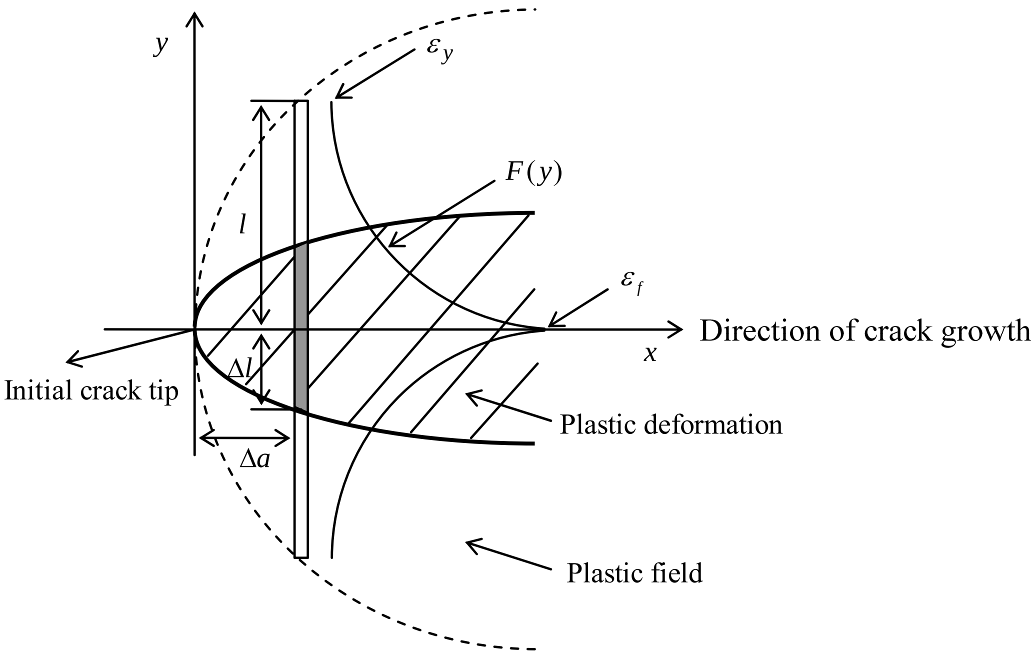

Figure 2. Principle of determining plastic field left on specimen

As introduced in [4], the strain distribution of a simple rectangular bar along the axis can be expressed as the curve shown in Fig. 2. The function of curve was supposed to can be written as Eq. (6),

$$F(y) = a\exp(by). \tag{6}$$

According to what discussed above,

$$F(0) = \varepsilon_f ,$$

$$F(l) = \varepsilon_y . \tag{7}$$

The substitution of Eq. (7) into Eq. (6) provides Eq. (8),

$$a = \varepsilon_f ,$$

$$b = \frac{1}{l} \ln \frac{\varepsilon_y}{\varepsilon_f} . \tag{8}$$

So, the elongation of the bar can be expressed as Eq. (9),

$$\Delta l = \int_0^l F(y)\,dy = \frac{\varepsilon_f - \varepsilon_y}{\ln \varepsilon_f - \ln \varepsilon_y} l . \tag{9}$$

Thus, the ratio of elongation of the bar can be expressed as Eq. (10),

$$P = \frac{\Delta l}{l} = \frac{\varepsilon_f - \varepsilon_y}{\ln \varepsilon_f - \ln \varepsilon_y} . \tag{10}$$

Because ε_f and ε_y are material constants, P is a constant, too. That is to say, the dimension of plastic field left on specimen is in proportion to the plastic deformation. At the same time, because Δl can be acquired from FRASTA, the plastic field left on specimen can be determined.

Return to the state as shown in Fig. 1, δ_{ij} represents the elongation of the bar at (x_i, y_j), where i and j represent the number of position along x and y axis respectively. The sizes of these bars along x and y direction (the direction of crack extension and specimen thickness respectively) are determined by the step lengths of laser microscope, which are represented by p and q respectively in this paper. Then, the original length l_{ij} of each bar should be

$$l_{ij} = \frac{\delta_{ij}}{P} , \tag{11}$$

the length of each bar after being broken is

$$l_{ij}' = \frac{\delta_{ij}}{P}(1+P) . \tag{12}$$

In order to calculate the load applied to the specimen during the course of fatigue, based on the above simple bar hypothesis, the fracture surfaces are assumed to be composed of independent rectangular bars. Once the normal stress on the cross section of each bar was obtained, the force applied on the bar can be determined by multiplying the normal stress with the cross section area. By this means, adding the force applied on each bar together, the load applied on the specimen can be obtained.

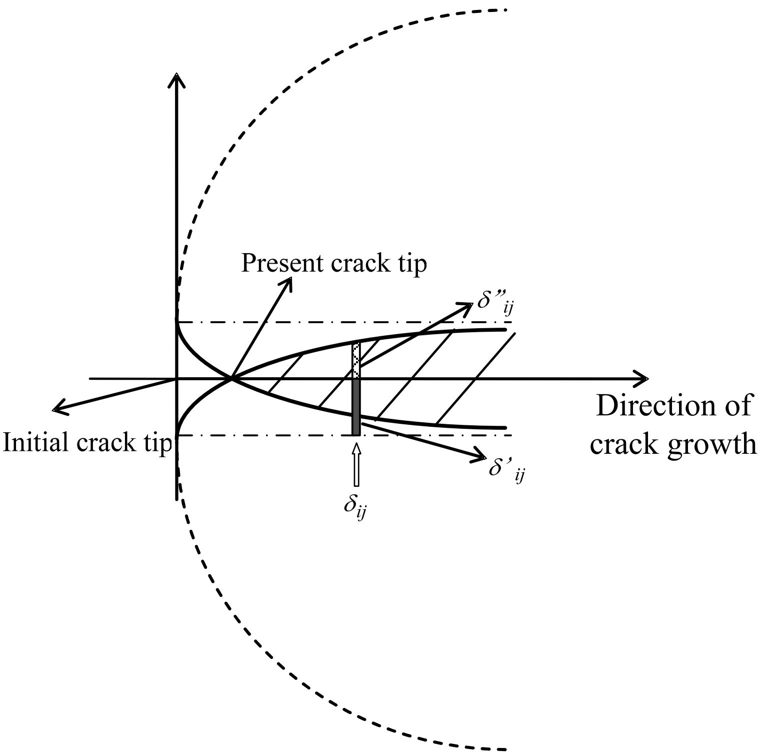

Figure 3. Two parts of the elongation of a single bar

According to this idea, we should firstly determine the global strain of each single bar during the course of failure. As shown in Fig. 3, using FRASTA to reconstruct the process of fracture, considering that the crack has extended to the "present crack tip", that is material behind the present crack tip (between the "initial crack tip" and the "present crack tip" in Fig. 3) has been fractured, thus forces acting on the bars composing this part had been released. While for material in front of the "present crack tip", it still bears load before failure. Add the loads acting on these bars together, the applied load of the specimen can then be determined. As shown in Fig. 3 and introduced above, suppose the final elongation of the bar is δ_{ij} and divide the elongation into two parts, δ'_{ij} and δ''_{ij}, where δ'_{ij} represents the elongation of the bar when the crack extends to the "present crack tip" and δ''_{ij} represents the residual elongation of the bar with the crack extending further from the "present crack tip" until final failure.

As introduced above, it is easy to get δ_{ij}, δ'_{ij} and δ''_{ij} by reconstructing the process of crack extension using FRASTA. According to Eq. (11), the initial length of the bar can then be calculated and the global strain of the bar when the crack reaches to the "present crack tip" can be calculated as Eq. (13),

$$\varepsilon_{ij} = \frac{\delta'_{ij}}{l_{ij}}. \tag{13}$$

Furthermore, the relationship between true stress σ and true strain ε can be expressed as Eq. (14),

$$\sigma = K\varepsilon^{n}, \tag{14}$$

where K is a constant and n is work hardening exponent.

Introduction of Eq. (13) into Eq. (14) leads to Eq. (15),

$$\sigma_{ij} = K\varepsilon_{ij}{}^{n}, \tag{15}$$

where σ_{ij} is the normal stress on the cross section of the single bar l_{ij}. Thus the applied load of the specimen can be calculated as Eq. (16),

$$F = \sum p \times q \times \sigma_{ij} = p \times q \times K \times \sum \varepsilon_{ij}{}^{n} \tag{16}$$

In order to simplify the calculation, use the average deformation of the fracture surfaces for instead, that is to add all deformations of the single bars with the same x coordinate together and divide it by the number of bars. By this means, Eq. (16) can be converted into Eq. (17),

$$F = p \times q \times K \times k \times \sum \varepsilon_{i}{}^{n} \tag{17}$$

where ε_i is the average strain at x_i and the k is the number of bars with the same x coordinate .

3. Conclusions

From the results presented in this paper, it is clear that by using FRASTA reconstruction, that is recording the elevation data of fracture surfaces and reconstructing the process of crack extension, it is possible to obtain the plastic deformation with crack extension. Based on the simple bar hypothesis and by dividing the plastic deformation into single bars, the original lengths of these bars can be determined thus the global strains of these bars during the course of failure can also be calculated. According to the relationship between true stress and true strain of the material, the normal stress on the cross section can thus be determined. Multiply the stress with the cross section area, the load acted on each single bar can be got. Adding all loads on all bars together leads to the total applied load of the specimen.

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