CMOD-Based J Integral Measurement for Surface Cracked Specimens

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Abstract This paper proposes an experimental approach based on the measured crack-mouth opening displacement (CMOD) to determine the mode I energy release rate for surface cracks in plate specimens subjected to four-point bending. The proposed experimental scheme measures the load versus CMOD relationships in a surface cracked specimen under four-point bending, and computes the relative rotation, θ, between the two crack planes based on the measured CMOD, and the bending moment (M) applied on the cracked cross section based on the measured load. The energy release rate, measured by the J-integral value, computes from the area under the M-θ curve, similar to the eta-approach originally proposed by Rice and co-workers for single-edge-notched bend, SE(B), specimens. This study examines the above approach for four plate specimens with fatigue pre-cracked, machined surface notches. The measured J value shows close agreement with the energy release rate computed using the domain integral approach.

Keywords surface crack, crack-mouth opening displacement, energy release rate, domain integral, compliance.

1. Introduction

The standard procedure to derive the fracture resistance curve, namely the J-R curve restricts to small-scale planar specimens with a straight, through-thickness crack front [1-2] under high-constraint, small-scale yielding conditions. The fracture resistance thus measured characterizes the capabilities of the material in resisting fracture failure in an idealized, plane-strain condition. In contrast, realistic cracks in structural details often entail a complex topology with a curved crack-front, sometimes resembling a semi-elliptical shape. The varying crack-extensions along the curved crack fronts impinge inevitably on the resistance against crack growth at individual crack-front locations, due to the stress redistribution caused by the non-uniform crack extensions along the curved crack front. The plane-strain fracture resistance obtained using the conventional through-thickness fracture specimens may not, therefore, represent the fracture resistance reserved at different crack-front locations in curved surface cracks. The fracture resistance measured from the through-thickness specimens may thus not provide an accurate description of the evolving fracture resistance as the surface crack grows in the integrity assessment for engineering structures, *e.g.*, the level 3C ductile tearing assessment. An experimental method to measure the J-integral values for surface crack specimens becomes much-needed to overcome the transferability of the J-R curve from the material level to the specimen/structural level.

This study proposes such an experimental approach to determine the energy release rate for surface cracked plate specimens. The calculation of the energy release rate utilizes the CMOD-based approach, frequently used in the conventional through-thickness fracture specimens in evaluating the energy release rate. The experimental procedure determines the gross energy release rate along the entire crack front using the area under the moment-rotation curve of the crack plane, similar to the η-approach originally proposed by Rice *et al*. [3]. The rotation of the crack plane derives from a numerically determined center of rotation. The dimensionless η-value is, on the other hand, determined by coupling the numerical strain energy (the area under the M-θ curve computed from finite element [FE] models with a stationary crack) and the domain-integral values. The energy release rate determined from the compliance method agrees closely with the energy release computed using the domain-integral approach implemented in WARP3D [4].

2. Proposed *J*-Measurement for Surface Cracks

The proposed approach to measure the energy release rate, quantified by the *J*-integrals, predicates the fundamental principle prescribing the variation of the strain energy with respect to the crack extension,

$$J = -\frac{1}{B}\frac{dU}{da}, \tag{1}$$

where B refers to thickness of the conventional planar fracture specimen with a through-thickness straight crack. The definition of the J value in Eq. (1) requires that the parameter B characterizes the length of the crack front, as the physical interpretation of the J refers to the energy released per extended crack area. Replacing B by the crack-front length for a curved surface crack, Eq. (1) becomes,

$$J_{avg} = -\frac{\partial U}{\partial A_{crack}}, \tag{2}$$

where A_{crack} denotes the crack area. The J-value determined from Eq. (2) thus represents the energy release rate averaged over the entire curved crack front in the surface crack specimens.

To facilitate the experimental measurement of the energy release rate, previous researchers [3, 5] have developed an η-approach for different fracture specimens with a through-thickness crack,

$$J = \frac{\eta U}{Bb}, \tag{3}$$

where η remains as a dimensionless parameter dependent on the specimen geometry, and equals 2.0 for SE(B) specimens [3]. The parameter b in Eq. (3) denotes the length of the remaining ligament in a fracture specimen with a through-thickness crack. The η parameter needs to be re-calculated for surface cracked specimens. Equation (3) provides a convenient calculation of the energy release rate based on some simple geometric parameters and the strain energy U, which often derives from the area under the load versus the load-line displacement. The denominator in Eq. (3) quantifies the remaining net area of the cracked cross section in the fracture specimen. When applied to specimens with a surface crack, Eq. (3) becomes,

$$J_{avg} = \frac{\eta U}{A_{net}}, \tag{4}$$

where A_{net} refers to the net intact area of the cracked cross section, or,

$$A_{net} = A_{total} - A_{crack}, \tag{5}$$

where A_{total} denotes the gross cross-sectional area of the specimen.

Equation (4) allows the determination of a single characteristic energy release rate for a surface crack in the experimental procedure. In contrast, the numerical domain integral approach often computes the varying energy release rates along the curved front for a surface crack. The energy release rate, averaged over the entire crack front, thus derives from the individual J-values along the crack front,

$$J_{avg} = \frac{\sum_{i}^{n} B_i J_i}{B_{total}}. \tag{6}$$

In calculating the average energy release rate along the crack front, Eq. (6) divides the curved crack front into multiple segments, each with a length B_i and a corresponding energy release rate J_i. B_{total}

in Eq. (6) denotes the total crack-front length. Equating Eqs. (4) and (6) allows the determination of the value of the dimensionless parameter η.

3. Experimental Validation

3.1. Specimen Configuration and Set-up

The experimental program includes four specimens made of high-strength steels extracted from a large-scale circular hollow section joint [6]. Figure 1 illustrates the geometric configuration of the steel plate specimens with a machined surface notch. The shape of the surface notch follows a semi-elliptical configuration with the crack aspect ratio (a/c) equal to 0.5, *i.e.*, the machined crack depth equals 10 mm.

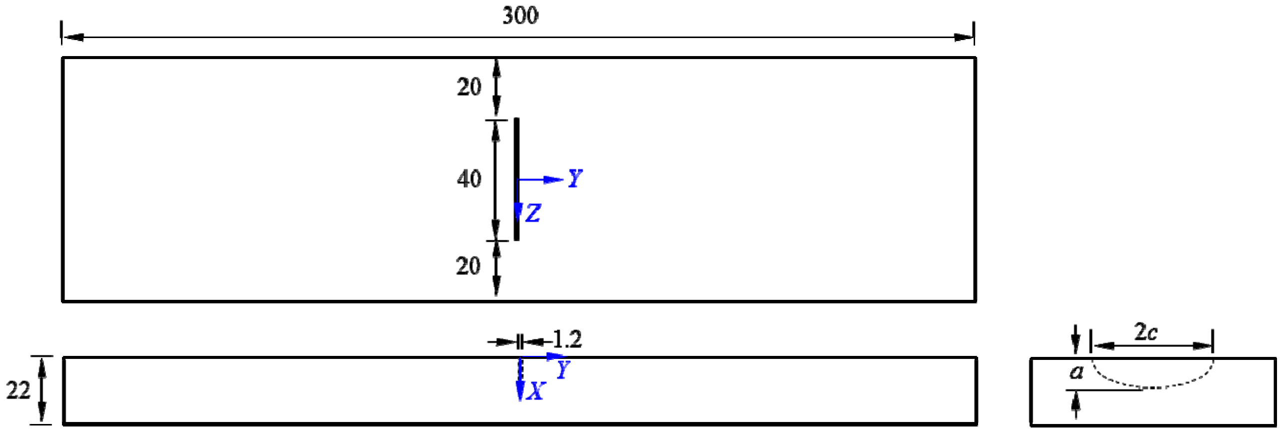

Figure 1. Configuration of the surface cracked plate specimens.

Figure 2 shows the uniaxial true stress-true strain curve obtained from the axial tension test following the ASTM E-8M [7]. The steel material has a specified grade of S690, with a measured yield strength equal to 810 MPa, and a tensile strength of 866 MPa. The Young's modulus of the material equals 206 GPa, with a Poisson's ratio of 0.3.

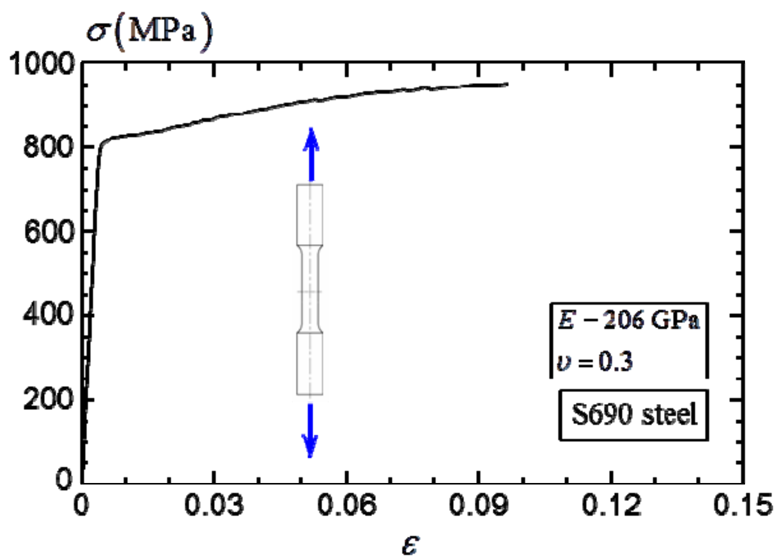

Figure 2. True stress-true strain curve of the S690 steel.

Figure 3 illustrates the four-point bend test set-up for the plate specimens. This four-point bend set-up, in contrast to the three-point bend set-up, prevents the plastic deformation caused by the indentation of the loading point to impinge on the plastically deformed crack tip under large deformations, which has frequently occurred in SE(B) specimens made of highly ductile materials.

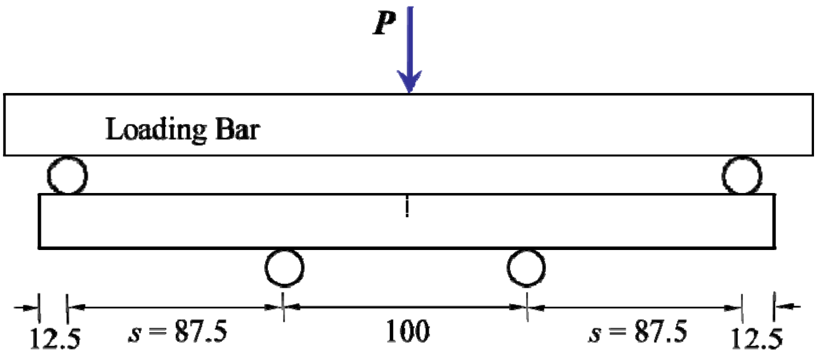

Figure 3. The four-point bend test set-up for the plate specimens.

3.2. Experimental Procedures

The experimental procedure separates into two stages: 1) a load-controlled fatigue pre-cracking cyclic load to generate a sharp crack-front along the machined notch; 2) a displacement-controlled monotonic loading coupled with multiple unloading-reloading cycles to monitor the compliance of the specimen, measured by the ratio of the measured CMOD range over the corresponding range of the applied load, throughout the procedure. The increasing compliance measured during the test indicates a growing crack during the test.

The experimental procedure monitors the specimen compliance during the fatigue pre-cracking stage, which terminates as the compliance increases by 20% to 30% of the initial compliance of the specimen. Table 1 lists the fatigue pre-cracked sizes for all four specimens.

Table 1. Fatigue pre-cracked sizes for the plate specimens

Specimen	Machined Crack Size		Fatigue Pre-crack	
	a (mm)	a/c	a (mm)	a/c
SP1	10	0.5	12.0	0.497
SP2			12.0	0.496
SP3			12.4	0.484
SP4			12.3	0.483

Figure 4 shows the load-CMOD measured during the experiment for the specimen SP1 shown in Table 1. The CMOD in Figure 4 refers to the crack mouth opening displacement measured at the center of the surface crack ($Z = 0$ in Figure 1).

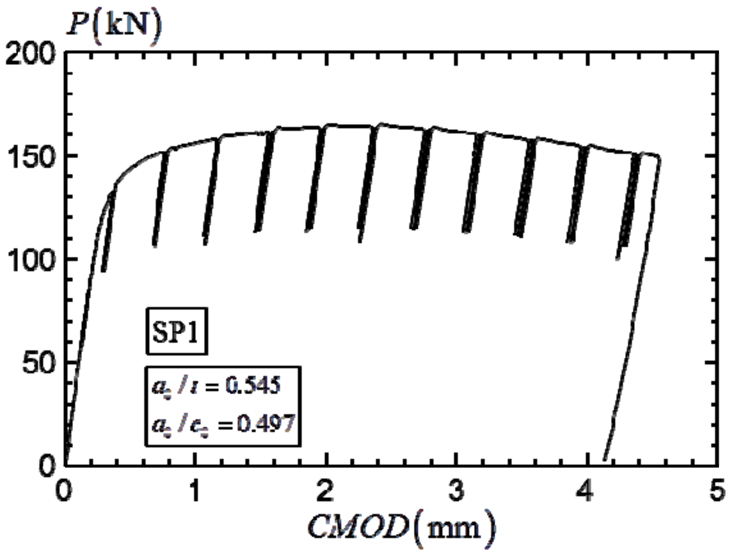

Figure 4. Definition of the crack-plane rotation angle θ.

3.3. Evaluation of Energy Release Rate

The calculation of the energy release rate in the plate specimens shown in Figures 1 and 3 depends on the evaluation of the strain energy U for the surface cracked specimens. The strain energy U, for the four-point bend specimens, follows,

$$U = 2\int_0^\theta M d\theta , \tag{7}$$

in which the angle θ defines the rotation of one of the two crack planes and M refers to the bending moment applied on the cracked section. Assuming a center of rotation at a distance of r $(W - a)$ away from the crack tip, the rotation of the crack plane thus derives from the measured crack-mouth opening displacement (CMOD),

$$\theta = \frac{CMOD}{2\left[a + r\left(W - a\right) + h\right]} , \tag{8}$$

where h denotes the thickness of the knife-edge aluminum plate glued to the surface of the plate specimens in order to mount the crack-opening displacement (COD) gauge, as shown in Figure 5.

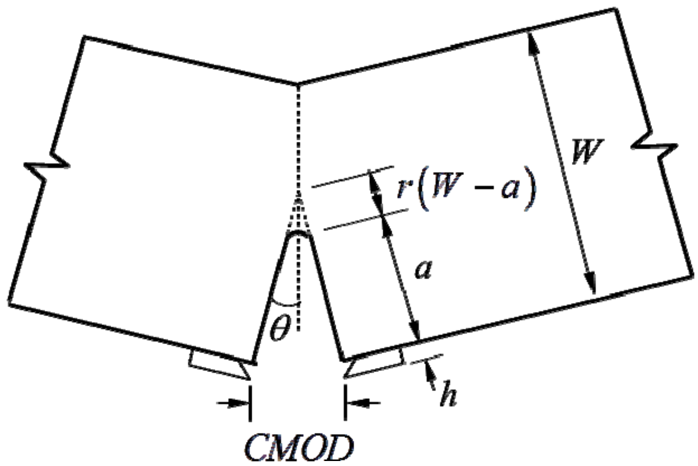

Figure 5. Definition of the crack-plane rotation angle θ.

The value of r in Eq. (8) equals 0.44 for SE(B) specimens, as outlined in standard material testing procedures [1]. This value of r, however, requires re-evaluation for the plate specimens subjected to the four-point bending shown in Figure 3. This study determines the r value through a detailed, large-deformation finite element analyses for plate specimens with a stationary crack. Figure 6 compares the deformed crack plane computed from the finite element analyses (represented by the discrete symbols) with the deformed shape calculated from a regressed rotation center with r = 0.18. The displacement u_Y in Figure 6 refers to the displacement in a direction perpendicular to the crack plane (see Figure 1 for the coordinate system *X*-*Y*-*Z*), while the *Z* value defines the crack-front position corresponding to which the displacements on the crack plane are measured. The results in Figure 6 indicates that the deformed crack plane aligns along a rotated plane with the rotation center calculated using r = 0.18.

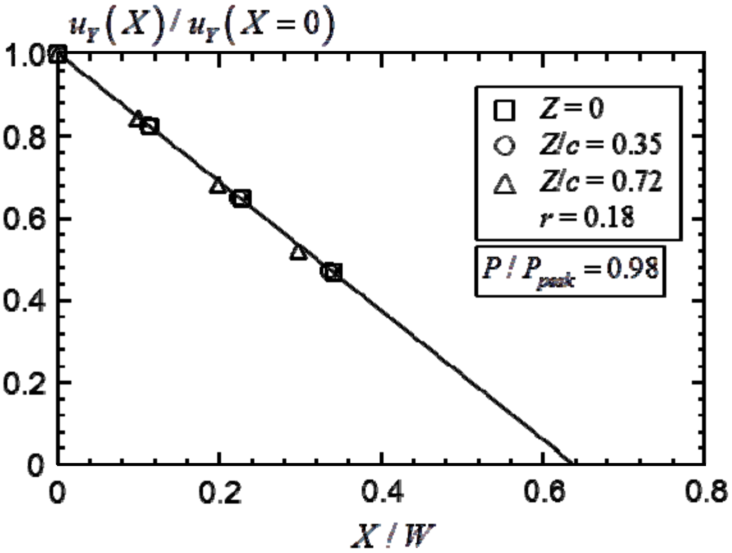

Figure 6. The r value for the plate specimens subjected to four-point bending.

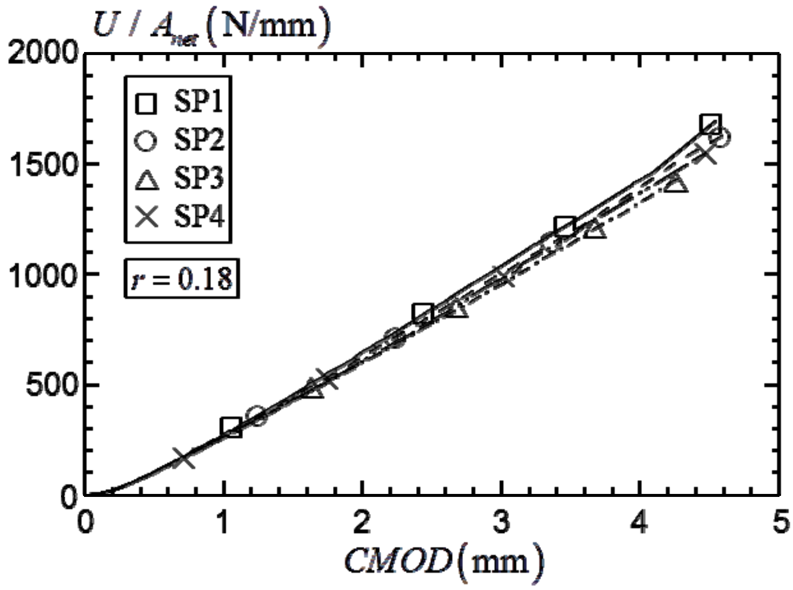

Figure 7. Evolution of the strain energy U for the four plate specimens.

Using an r value of 0.18, the strain energy U for the plate specimen can now derive from Eqs. (7)

and (8). Figure 7 presents the evolution of the strain energy U with respect to the crack mouth opening displacement measured at the center of the crack ($Z = 0$) for all four plate specimens.

The evaluation of the energy release rate requires the determination of the dimensionless η-value. This study determines the η-value using FE models with stationary cracks via equating Eqs. (4) and (6). The strain energy value U in Eq. (4) is computed using the strain energy derived from the area under the moment-rotation curve of the crack plane computed from the large-deformation finite element analyses, as shown in Eq. (7). Figure 8 illustrates the variation of the η-value as the applied load increases for the four plate specimens. The η-value remains approximately at the same magnitude at different load levels in all four specimens. The current study utilizes, therefore, a constant η-value of 1.1 for all plate specimens.

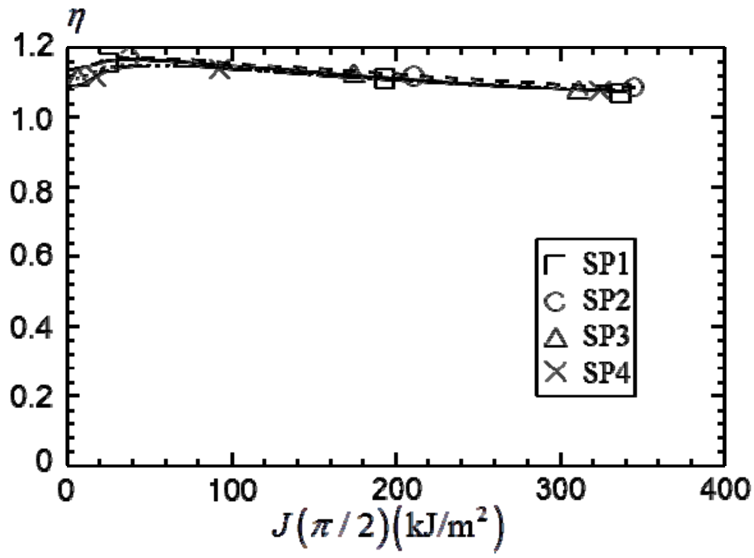

Figure 8. The dimensionless η-values for the plate specimens.

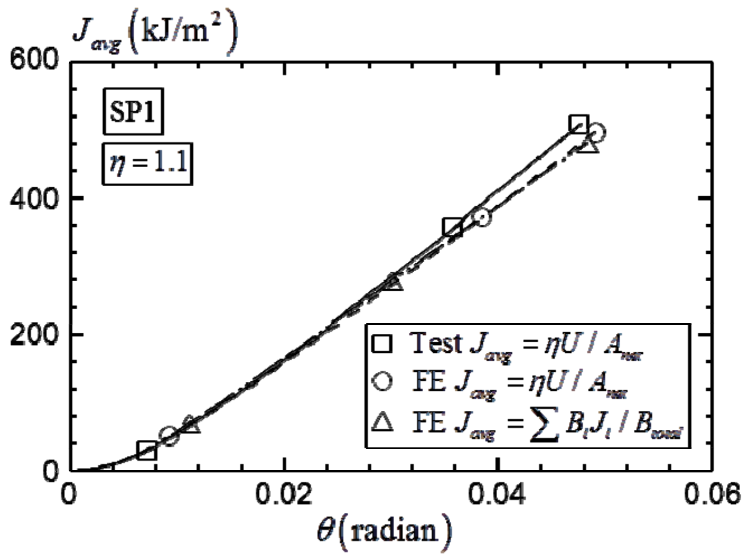

Figure 9. Comparison of the energy release rate for the surface cracked plate specimen SP1.

Figures 9 and 10 compare the energy release rate computed from the finite element model and those measured from the experiments for two selected specimens SP1 and SP4. The average J-values computed from the finite element analysis in Figures 9 and 10 correspond to the plate models with a stationary crack. The comparison in Figures 9 and 10 therefore limits the comparison for the experimental energy release rate to a small deformation level, at which the amount of ductile crack extension remains small. For both specimens, the numerically computed energy release rate agrees closely with those measured from the experiments.

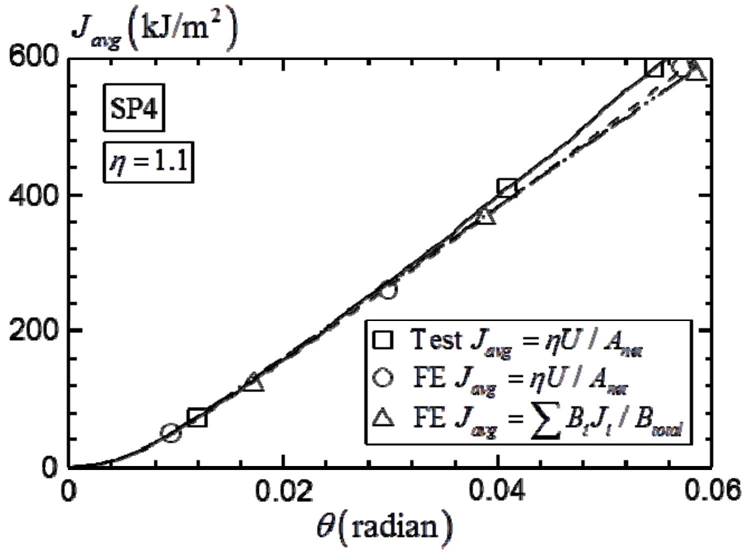

Figure 10. Comparison of the energy release rate for the surface cracked plate specimen SP4.

4. Summary and Conclusions

This study describes a compliance-based procedure to measure the energy release rate for surface cracks in plate specimens subjected to four-point bending. The average energy release rate along the crack front derives from the η-approach using the area under the M-θ curve for the cracked section. The center of rotation for the crack plane remains at about 18% of the remaining ligament measured from the crack tip at the deepest crack-front location, for the four surface cracked specimens considered in this study. This value, however, requires further validation when the η-approach is extended to other types of surface cracked specimens under different loading conditions. The energy release rate calculated using the η-approach based on the area under the numerical moment-rotation curves of the cracked section matches closely with the average energy release rate based on the domain-integral values computed in the large-deformation, elastic-plastic analyses. The energy release rate estimated using the η-approach from the experimental M-θ curves agrees well with that estimated using the numerical M-θ curve, before substantial crack extensions occur in the experimental specimen. Together with a compliance based method to determine the crack size in the surface cracked plate specimen, the approach proposed here provides a basis to determine the *J-R* curve for surface cracked specimens [8].

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