Modeling the propagation mechanism of two random micro cracks in rock Samples under uniform tensile loading

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Abstract

The breakage mechanism of brittle substances such as rocks under uniform tensile loading is considered. The random micro cracks are mostly produced in all of Polycrystalline substances resulting from stress concentration. This paper focuses on the interaction of two pre-existing micro cracks. The quality of the growing and coalescing of micro cracks in rock-like specimen are studied by using a Fortran Code $HDDMCR_{2D}$ which is based on a two dimensional displacement discontinuity method for the crack analysis (a boundary element computer code based on the linear elastic fracture mechanics (LEFM) theory). In order to improve the accuracy of the results and reduce the errors in calculating displacement discontinuity in the proposed model, the higher-order elements (quadratic elements) are used to discretize a sufficient number of boundary elements. Also to eliminate errors caused by stress and displacement singularity near the crack tip, three special elements has been used to discretize each crack end. In this research, the problems of wing cracks propagated at different angles from the original micro crack tips in an infinite specimen under uniform tension are considered.

Keywords micro crack interaction, crack initiation, LEFM, BEM, crack coalescence

1. Introduction

The presence of micro cracks in brittle substances may have higher effect on their macro mechanical behavior. It has been observed that the macro mechanical behavior of rocks is due to their micro mechanical structure [1]. The micro cracks initiate from their tips and extend at an angle with respect to the original micro crack plane [2]. In the path of development of micro cracks, the kinked and curved cracks may be produced and these types of cracks usually occur under shear or mixed mode loading [3]. The breakage mechanism of brittle substances with random micro cracks depends on the degree of their interaction and coalescence path that may leads to the formation of a macro crack [4].

In studying the breakage mechanism of brittle substances under uniaxial tensile stress, crack initiation and specimen breakage may happen very soon and the crack initiation is normally expected to occur in a direction perpendicular to the maximum tensile stress. In rocks, crack initiation in tension is preferred due to the lower toughness of rock substances in tension than in shear [5]. The mechanism of propagation and coalescence of micro cracks in forming of macro cracks or rupture of rock is still not a well understood subject and the problem of interaction between the micro cracks and macro cracks is always noted by the researchers [6]. A number of experimental investigations on the propagation and coalescence of micro cracks in rock-like specimens under uniaxial and biaxial compression have been published [7-14]. Some useful and modern numerical models such as FROCK code, damage model, Rock Failure Process Analysis (RFPA^{2D}) are used in this field [5, 15-19]. There has never been a random investigation on the effect of fracturing in brittle substances altering positions of random micro cracks which is under tension. In this research, a numerical model is proposed using a higher order displacement discontinuity code for micro crack analysis (HDDMCR^{2D} code). This model is designed on the basis of the linear elastic fracture mechanics (LEFM) principles so that it is able to simulate the micro crack interaction of specimens containing random micro cracks. In this paper, the numerical analysis of the growth of wing cracks from pre-existing micro cracks in rock-like specimens under uniform normal tensile stress is studied. In order to verify the validity of proposed numerical model, the experimental and numerical results of the wing crack initiation directions (given by Guo et.al. [20]) for different micro crack inclination angles of a center slant micro crack under uniform tension have been used. The crack propagation mechanism of two pre-existing micro cracks are investigated (for a specimen under uniform tension) in which the propagation direction of micro cracks is estimated by using the maximum tangential stress fracture criterion (σ-criterion) proposed by Erdogan & Sih [21].

2. The higher order displacement discontinuity method

In the higher order displacement discontinuity method, the boundaries are discretized into multiple segment elements. The formulations of three types of displacement discontinuity variations with: a constant variation of the displacement along the elements, a linear variation, and a quadratic variation were previously mentioned and used in the literature [22-24]. The quadratic element displacement discontinuity is fundamentally based on integrating the quadratic element shape functions over collinear, straight-line displacement discontinuity elements.

Fig. 1 shows the displacements distribution at quadratic collocation point ‘m’, which can be calculated as:

$$D_j(\zeta) = \sum \Omega_m(\zeta) D_j^m \ \mathit{for}\ m = 1, to 3,\ j = x, y \tag{2}$$

where D_j is the fundamental variable. It should be noted that two fundamental variables are calculated in each collocation point. Using $c_1 = c_2 = c_3$, it can be written:

$$\Omega_1(\varepsilon) = \zeta(\zeta - 2c_1)/8c_1^{\ 2},\ \Omega_2(\zeta) = -(\zeta^2 - 4c_1^{\ 2})/4c_1^{\ 2},\ \ \Omega_3(\zeta) = \zeta(\zeta + 2c_1)/8c_1^{\ 2} \tag{3}$$

which are the shape functions of the quadratic collocation point ‘m’. In the quadratic collocations, there are three collocation points for each element, for which the displacements are typically calculated. These collocations are located in the center of the elements (Fig. 1).

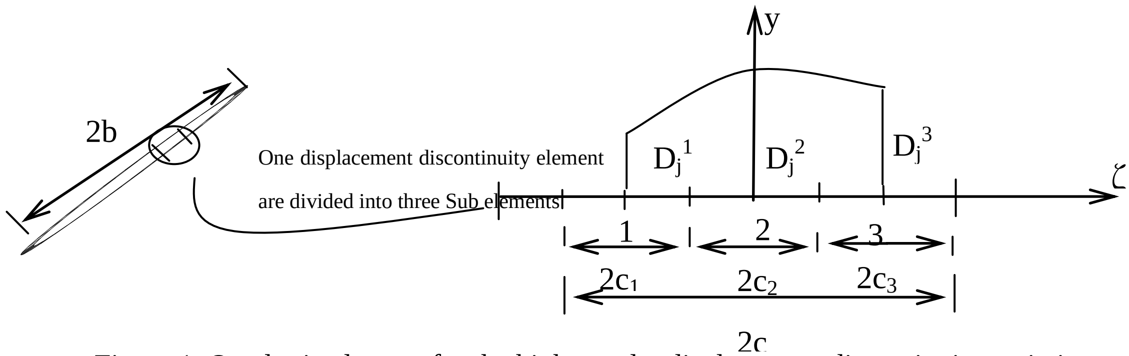

Figure 1. Quadratic element for the higher order displacement discontinuity variation

To eliminate the singularity of the displacements and stress calculation near the micro crack ends and increase the accuracy of order higher displacement discontinuity method around the original micro crack tip, a special treatment of the micro crack at the tip is necessary [24-27]. In this research three special crack tip elements at the end and initiation of each micro crack are used in the general higher order displacement discontinuity method. As shown in Fig. 2, using a special crack tip element with the length of 2c.The displacement discontinuity variations along this element can be written in the following form [24]: given as:

$$D_j(\zeta) = [\Omega_{T1}(\zeta)]D_j^1(c) + [\Omega_{T2}(\zeta)]D_j^2(c) + [\Omega_{T3}(\zeta)]D_j^3(c) \tag{4}$$

The crack tip element has a length $c = c_3 + c_2 + c_1$. Considering $c_1 = c_2 = c_3$, the shape functions $\Omega_{T1}(\zeta)$, $\Omega_{T2}(\zeta)$ and $\Omega_{T3}(\zeta)$ can be written as:

$$\Omega_{T1}(\zeta) = \frac{15\zeta^{\frac{1}{2}}}{8c_1^{\frac{1}{2}}} - \frac{\zeta^{\frac{3}{2}}}{c_1^{\frac{3}{2}}} + \frac{\zeta^{\frac{5}{2}}}{8c_1^{\frac{5}{2}}}, \qquad \Omega_{T2}(\zeta) = \frac{-5\zeta^{\frac{1}{2}}}{4\sqrt{3}c_1^{\frac{1}{2}}} + \frac{3\zeta^{\frac{3}{2}}}{2\sqrt{3}c_1^{\frac{3}{2}}} - \frac{\zeta^{\frac{5}{2}}}{4\sqrt{3}c_1^{\frac{5}{2}}} \quad \text{and}$$

$$\Omega_{T3}(\zeta) = \frac{3\zeta^{\frac{1}{2}}}{8\sqrt{5}c_1^{\frac{1}{2}}} - \frac{\zeta^{\frac{3}{2}}}{2\sqrt{5}c_1^{\frac{3}{2}}} + \frac{\zeta^{\frac{5}{2}}}{8\sqrt{5}c_1^{\frac{5}{2}}} \tag{5}$$

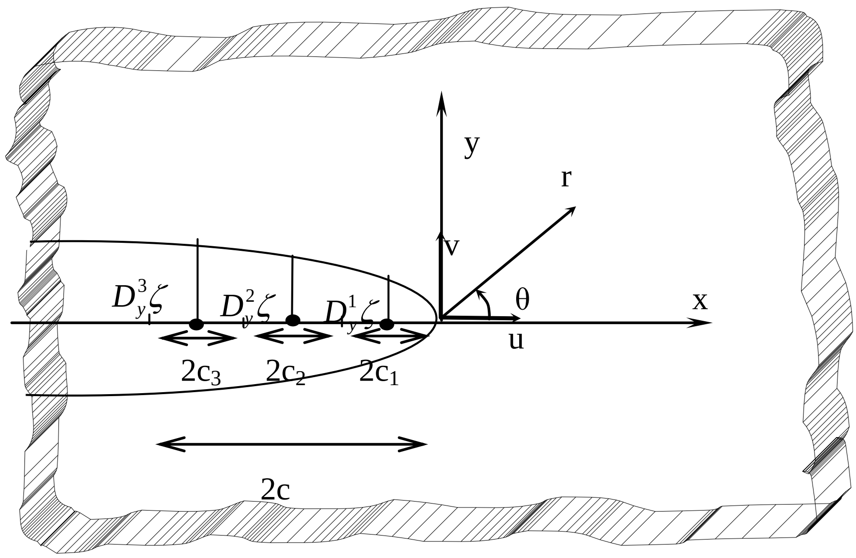

Figure 2. Quadratic element for the higher order displacement discontinuity variation

The mode I and Mode II stress intensity factors k_I and k_{II} can be estimated based on LEFM theory as the opening and sliding displacements [23]:

$$K_I = \frac{\rho}{4(1-\nu)}\left(\frac{2\pi}{c}\right)^{\frac{1}{2}} D_y(c), \quad \text{and} \quad K_{II} = \frac{\rho}{4(1-\nu)}\left(\frac{2\pi}{c}\right)^{\frac{1}{2}} D_x(c) \tag{6}$$

3. Verification of DDM with quadratic elements

A simple problem for verifying the numerical results and the proposed code is presented in this paper. This problem is the center slant micro crack in an infinite specimen shown in Fig. 3. The slant angle ψ, changes counter clock wise from the x (horizontal) axis, and the tensile stress σ^{∞} = 10 MPa is acting. Half of the micro crack length, b= 1 meter, modulus of elasticity E=10 GPa, Poisson's ratio $\nu = 0.2$, fracture toughness K_{IC}= 1.8MPa $m^{1/2}$ are assumed.

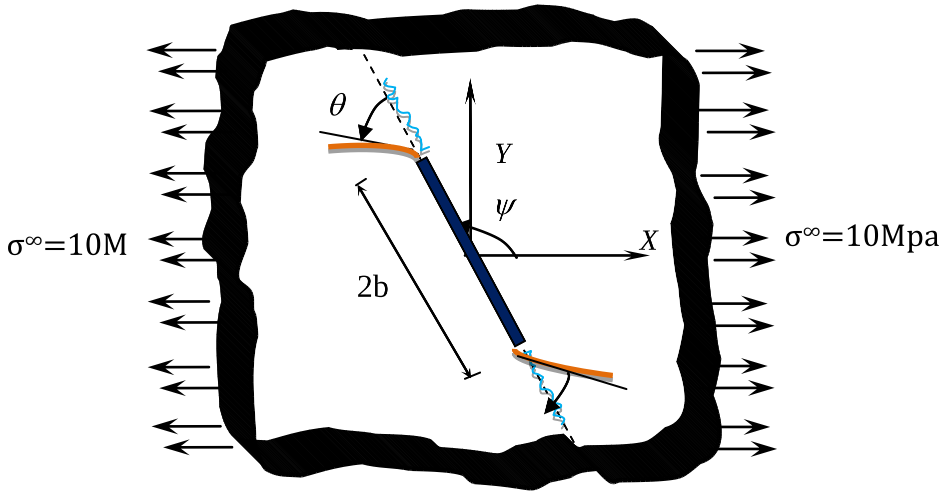

Figure 3. A center slant micro crack in an infinite body under uniform tension (parallel to the x axis)

The center slant micro crack problem has been solved by different researchers e.g. Guo et.al [20] to get a simple analysis, also these researchers used the constant element displacement discontinuity with a special crack tip element for angles 30, 40, 50,60,70,80 degrees. To evaluate the micro crack initiation angle θ_0, they used two initiation criteria: Maximum tangential tensile stress criterion (σ_θ-criterion), and Minimum strain energy density criterion (S-criterion), and compared their results with the results from other models. Fig. 4 compares the numerical results for the wing crack initiation directions θ_0 (obtained by $HDDMCR_{2D}$ code) and the results given by Guo et. al [20].

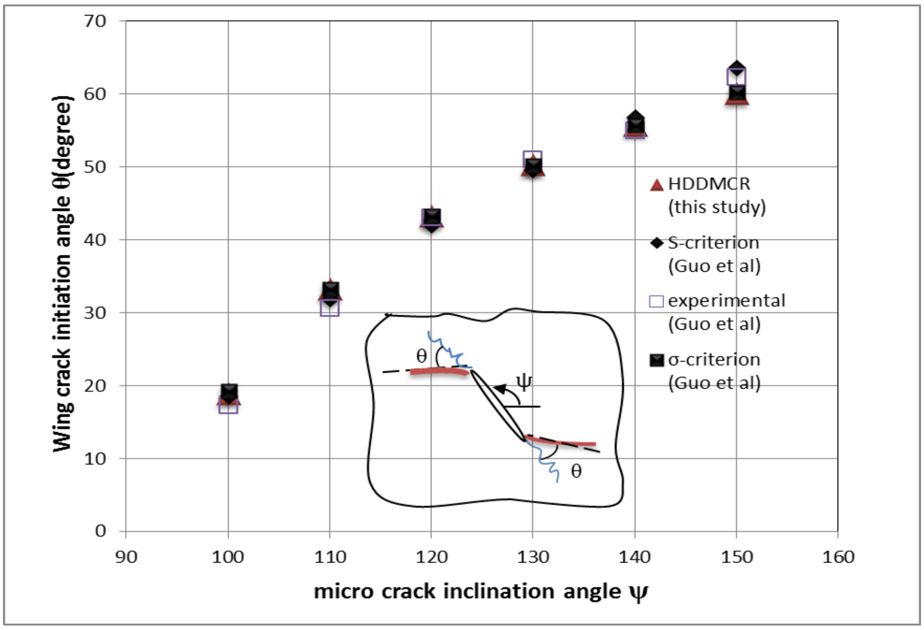

Figure 4. The micro crack initiation angle θ_0 as a function of micro crack inclination Angle ψ

4. The initiation and propagation of two non-parallel pre-existing micro cracks

In addition to the formation and growth of micro cracks in a rock mass, the reciprocal effects among the micro cracks may also be considered in the breakage process of the rock. The micro crack interactions can be modeled by the present numerical method which may allow micro cracks to interact reciprocally. It can be concluded that by increasing the number of micro cracks, the induced stresses among the original micro crack tips may have a significant effect on their initiation and some new micro cracks may also be produced. In fact, in rock specimens a series of micro cracks may remain closed during loading and propagation. In order to find out how the micro cracks connect under uniform tension (parallel to y axis), two micro cracks (of the same size as shown in Fig. 5), are considered. The body containing the micro cracks is an infinite plane, and the plane strain condition is assumed. It is under a uniform far field tensile stress, $\sigma^{\infty} = 10mpa$ with modulus of elasticity, $E=10\ GPa$, Poisson's ratio, $\nu=0.2$, fracture toughness, $K_{IC}=1.8MPa\ m^{1/2}$. In the present research, the effects of micro cracks randomness and spacing are considered.

The mode I and mode II stress intensity factors, K_I and K_{II}, near the original tips of two random micro cracks are estimated numerically. The non-dimensional mode I and mode II stress intensity factors, $\frac{k_I}{\sigma^{\infty}\sqrt{(\pi b)}}$ and $\frac{k_{II}}{\sigma^{\infty}\sqrt{(\pi b)}}$ are mainly used in the micro crack analyses. The negative sign of K_I value shows the micro crack closure, while the positive and negative signs in K_{II} only show the direction of relative shear between the two surfaces of micro crack.

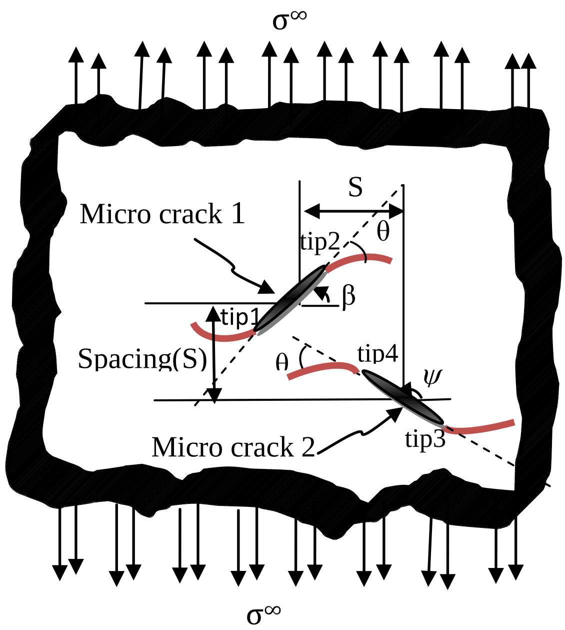

Figure 5. Two randomly slant micro cracks in an infinite specimen under uniform tension with the spacing, S=2b

4.1. Effects of inclination and randomness of micro cracks on SIF

Two random and completely non-parallel micro cracks are shown in Fig. 5. These micro cracks are known as micro crack 1 and micro crack 2. Their vertical and horizontal distances (center to center) are defined by S, and their inclination angles are denoted by β and ψ. The micro cracks are located in an infinite specimen under uniform tension (parallel to y axis). The normalized stress intensity factors $\frac{k_I}{\sigma^{\infty}\sqrt{(\pi b)}}$ and $\frac{k_{II}}{\sigma^{\infty}\sqrt{(\pi b)}}$ are shown graphically in Figs. 6 and 7. These normalized intensity stress factors are evaluated for four original tips of the two micro cracks (keeping a constant inclination angle, β=60°, for micro crack 1 but different angles, ψ= 150°, 140°,120°, 100°,80° and 60° for micro crack 2). For the original micro crack tips 3, 4, which have the most effect compared to other original tips, k_I value has decreased with an increase in the inclination angle of micro crack 2 (angle ψ) from 50° to 90°. In fact, k_I gets negative values when ψ is close to 90°. In tension field, the closing of micro crack is due to shear stress because shearing mode occurs sooner than opening mode in this field. The stress intensity factor k_I increases considerably as the inclination angle ψ increases from 90° to 150°. For the original micro crack tips 1 and 2, the effects of randomness are lower than those of other tips. The maximum value of k_I (~0.772) occurs at ψ=150° due to the complete non-orientation of both the micro cracks. The behavior of mode II SIFs of four original tips of micro cracks is notably different from that of mode I SIFs. For original tips 3, 4, $|k_{II}|$ value has increased in the inclination angle ψ from 50° to 130° and decreased from 130° to 150°, when the micro crack 1 and micro crack 2 are completely non-oriented. For original micro crack tips 1 and 2, there aren't any noticeable effects with a change in the micro crack inclination angle ψ. As it was shown in Figs. 6 and 7, the most changes were in original micro crack tips 3 and 4, when the original tips 3 and 4 are close to the other of original micro crack tips, both of modes I and mode II stress intensity factors, k_I and k_{II} are increased. In nearly vertical micro cracks, k_{11} plays a more important role than k_I. Fig. 8 shows the wing crack initiation angle of four tips versus the changes of inclination angle ψ. This figure shows that in tips 3, 4, wing crack initiation angle θ has increased in the inclination angle ψ from 60° to 90° and decreased from 90° to 150°. The maximum of wing crack initiation angle θ occurs at ψ= 90°. And also for tips 1 and 2, the changes of ψ are not noticeable due to non-orientation of micro cracks and it is assumed that there is zero interaction.

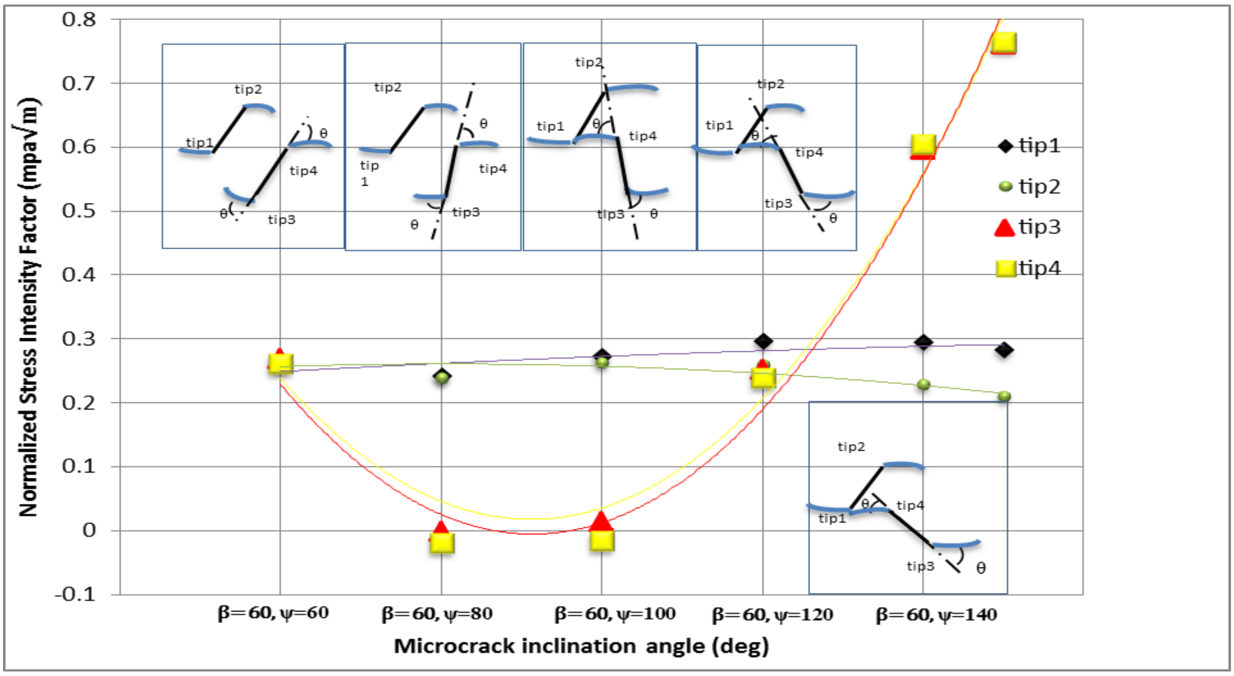

Figure 6. Treatment of mode I SIFs versus the locations of two micro cracks, under uniform tension (parallel to the y axis), for L/b=0.1 and 360+6 nodes for each micro crack

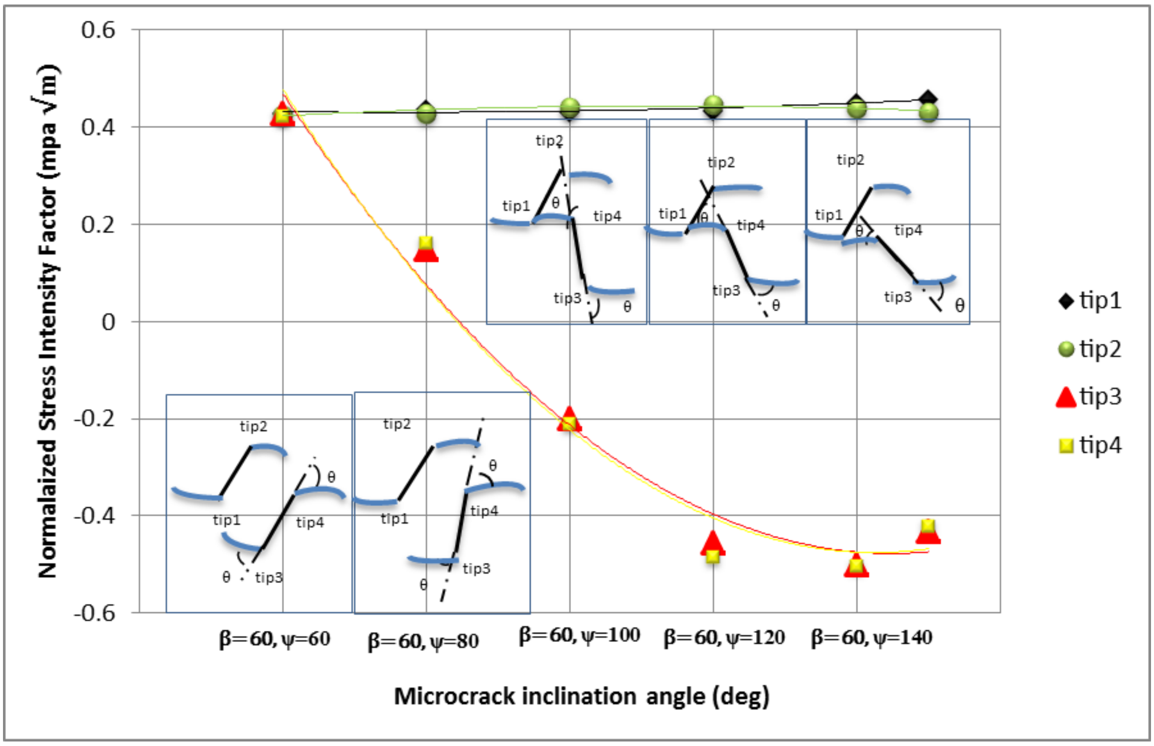

Figure 7. Treatment of mode II SIFs versus the locations of two micro cracks, under uniform tension (parallel to the y axis), for L/b=0.1 and 360+6 nodes for each micro crack

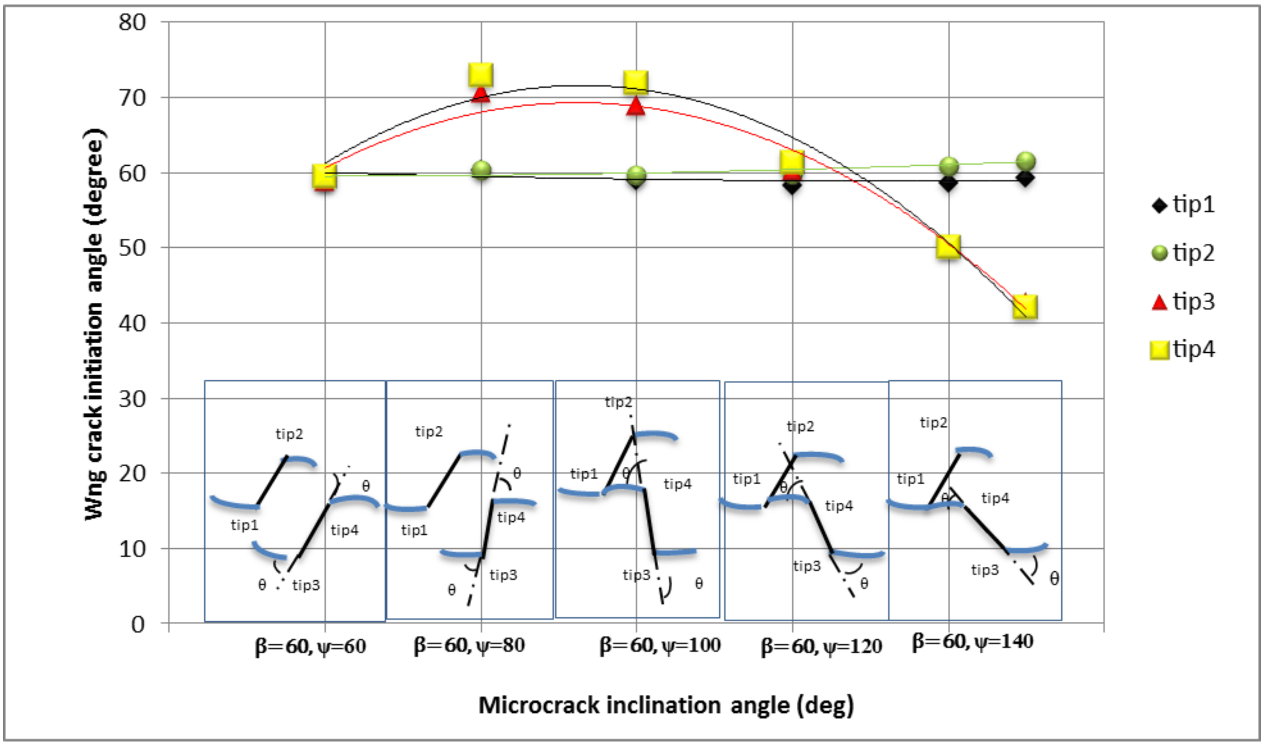

Figure 8. Wing crack initiation angle versus the locations of two micro cracks, under uniform tension parallel to the y axis, for L/b=0.1 and 360+3 nodes for each micro crack

5. Conclusions

A numerical model on the basis of computing the stress intensity factors and wing crack initiation angles for cracked substances under normal uniform tension is presented. In the present work, based on the Linear Elastic Fracture Mechanics (LEFM), the maximum tangential stress criterion or σ-criterion is implemented into code HDDMCR to investigate the interaction of micro cracks. In order to verify the validity of the proposed model, analytical solution of a typical sample problem e.g. a center slant micro crack under uniform tension is used, and its propagation mechanism is compared with numerical solution. In the present model, quadratic collocations with three special crack tip elements for each micro crack tip at the same time are implemented into code $HDDMCR_{2D}$. The numerical simulation is carried out considering infinite planes with two micro cracks under uniform tension. Comparing the parallel and non-parallel micro cracks, the effects of inclinations of two micro cracks show that these factors have a strong influence on the breaking path.

6. References

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