Elasto-Dynamic Behaviour of Interacting Inhomogeneities and Cracks

Shaker A. Meguid[1,*], **Xiaodong Wang**[2]

[1]Department of Mechanical and Industrial Engineering, University of Toronto, Toronto, Canada M5S 3G8
[2]Department of Mechanical Engineering, University of Alberta, Edmonton, Canada T6G 2G8
* Corresponding author: meguid@mie.utoronto.ca

Abstract The paper presents a theoretical treatment of the dynamic behavior of fibre reinforced composites containing matrix cracks and reinforcing inhomogeneities. A pseudo-incident wave method is used to treat the dynamic interaction between cracks and inhomogeneities. Using this method, the original interaction problem is reduced to the solution of coupled single crack/inhomogeneity subproblems, for which analytical solutions could be derived. The interaction effects are introduced via the superposition of the different subproblems. The steady state solution of the interacting crack problem is obtained using integral transform method and the solution of the inhomogeneity problem is determined using Fourier expansion. The dynamic stress intensity factors (SIFs) at the matrix crack are obtained and numerical examples are provided to show the effect of the frequency, geometry of microdefects and material properties upon the dynamic SIFs.

Keywords Dynamic interaction, Crack, Inhomogeneity, Composite

1. Introduction

A major issue in modeling the micromechanical behaviour of advanced composites is how to deal with the interaction between cracks and inhomogeneities, which governs the overall failure mechanism of the materials [1-4]. The quasi-static interaction problem in composite materials has received considerable attention but the dynamic interaction between cracks and inhomogeneities is still limited [5-9]. Compared with quasistatic problems, the formulation of dynamic problems is much more complicated and difficult to deal with and the experimental data are difficult to obtain. It should be mentioned that most advanced composite materials are currently being used or considered for use in situations involving dynamic loading. Numerical methods, such as finite element analysis or boundary element method, can be used for this type of dynamic analyses under certain conditions but has their own limitations when multiple defects are involved. Analytical study of interacting cracks under dynamic loads is still attracting researchers [10-12] because of its high reliability and accuracy in simulating the dynamic response of multiple defects in composite materials.

It is the objective of the present paper to review and present the usage of the pseudo-incident wave method for the analysis of steady state dynamic interaction problems. Based on this method, the original interaction problem is reduced into single crack and single inhomogeneity subproblems, which are coupled through the scattered waves. The single crack and single inhomogeneity problems are solved analytically using integral transform method and Fourier expansion, respectively. Following this introduction, the paper is divided into three more sections: the formulations, results and discussions and conclusions.

2. Formulation of the Problem

Consider now the dynamic interaction between arbitrary defects, which could be in forms of cracks or inhomogeneities, in an infinite elastic isotropic solid under steady state dynamic antiplane loading, as shown in Fig.1. The displacement field corresponding to a steady state dynamic loading can be generally expressed in terms of the frequency ω as

$$w^*(x,y,t) = w(x,y)e^{i\omega t} \tag{1}$$

For the sake of convenience, the time factor $e^{i\omega t}$ will be suppressed and only the magnitude $w(x,y)$ will be considered. The harmonic displacement field must satisfy the Helmholtz equation [13],

$$\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\omega^2}{c_T^2}w=0 \tag{2}$$

where $c_T=(\mu/\rho)^{1/2}$ is the shear wave speed of the medium. The non-vanishing shear stress components are

$$\tau_{yz}=\mu\frac{\partial w}{\partial y} \quad , \quad \tau_{xz}=\mu\frac{\partial w}{\partial x} \tag{3}$$

where μ is the shear modulus of the material.

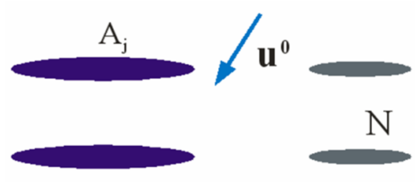

Figure 1. Interacting defects subjected to an incident wave

Instead of solving the original interaction problem, single defect problems will be considered. For any individual defect the interaction with other ones will be treated as an unknown wave, pseudo-incident wave. This wave represents the scattered waves from all other defects and will be determined by considering the consistency condition between defects.

2.1 Single crack problem

Consider the single defect problem first. For a single crack subjected to a dynamic antiplane loading, the boundary conditions along the crack surfaces are,

$$w(x,0)=0 \quad |x|\geq c \quad , \quad \tau_{yz}(x,0)=-\tau_1(x) \quad |x|< c \qquad sgn(y)=\begin{cases}1 & y>0\\ -1 & y<0\end{cases} \tag{4}$$

with τ_I being the shear stress caused by the incident wave. x is the axis along the crack surface and c is the half length of the crack.

By making use of Fourier transform, the general solution of the displacement and stress fields in the transformation domain can be expressed as

$$w^*(s,y)=-\frac{sgn(y)}{2\pi i}\int_{-c}^{c}f(u)\frac{1}{s}e^{isu-\alpha|y|}du \tag{5}$$

where y is an axis starts from the centre of the crack and is perpendicular to the crack surfaces, s is the Fourier transform parameter of x, $f(x)$ represents the deformation of the crack surfaces, defined by

$$f(x)=\frac{\partial w(x,0)}{\partial x} \tag{6}$$

and

$$\alpha=\sqrt{s^2-\frac{\omega^2}{c_T^2}} \quad with \quad Re(\alpha)\geq 0$$

The solution of the problem can be obtained by using Chebyshev polynomial expansion of $f(x)$ as,

$$f(x)=\sum_{j=0}^{\infty}\frac{c_j}{\sqrt{1-\frac{x^2}{c^2}}}T_j(\frac{x}{c}) \tag{7}$$

where T_j are Chebyshev polynomials of the first kind and c_j are unknown constants. By satisfying the boundary condition at selected collocation points along the crack surfaces, the parameters c_j can be determined in terms of the boundary stress as,

$$[S]\{A\} = \{f\} \tag{8}$$

where *[S]* is a known matrix, $\{A\}=\{c_1, c_2, \dots\}^T$ and *{f}* is a matrix containing the boundary stresses at the collocation points along the crack surfaces. From this solution, the stress and displacement field caused by this crack can be calculated in terms of *{A}*.

2.2 Interaction problem

The solution of other single defect problems can also be determined and the solution can be expressed in the similar format as shown in Eq. (8). When multiple defects are involved, for defect A_j, as shown in Fig.2(a), all scattered waves from other defects will become an incident wave, i.e. the pseudo-incidenr wave (u_j^p). Therefore, defect A_j is subjected to both the original incident wave and the pseudo-incident wave and results in a scattered wave, as shown by Fig.2(b).

(a) (b)

Figure 2 Illustration of pseudo-incident waves
(a) scattering from other defects, (b) total incident wave for a defect

Based on the relation between defects discussed above, for defect A_j the solution can be expressed as

$$[S_j]\{A_j\} = \{f_j\} + \{f_j^p\} \tag{9}$$

where $[S_j]$ is the matrix given by (8) for A_j, and the two terms on the right hand side represent the original incident wave and the pseudo-incident wave.

If Eq. (9) is applied to all the defects and the pseudo-incident waves are represented in terms of the scattered waves, the governing equation for the interaction problem can be determined,

$$\left\{\begin{bmatrix}[S_1] & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & [S_n]\end{bmatrix} + \begin{bmatrix}[Q_1]\\ \cdots\\ [Q_n]\end{bmatrix}\right\}\begin{Bmatrix}\{A_1\}\\ \cdots\\ \{A_n\}\end{Bmatrix} = \begin{Bmatrix}\{f_1\}\\ \cdots\\ \{f_n\}\end{Bmatrix} \tag{10}$$

where $[Q_j]$ are determined by the scattered waves of the defects, $\{f_j\}$ are the original incident wave at different defects. It should be mentioned that both [*S*] and [*Q*] matrices are obtained from the analytical solution of single defects. By solving this linear equation, the solution of the interaction problem can be determined.

3. Results and Discussion

The solution presented in the previous section can provide reliable and accurate prediction of the stress field caused by the dynamic interaction. The method can be used to treat interaction between different defects [8,10,11]. Although only the single crack solution is presented in the previous section, solutions of other single defects can be similarly and easily assembled into Eq. (10). In this section, typical examples are presented to illustrate the dynamic interaction between different defects. The numerical simulation is conducted by solving Eq. (10) and the convergence of the solution has been carefully evaluated. Specifically, numerical results are presented to illustrate the dynamic interaction between a main crack and a second crack or an inhomogeneity. The incident antiplane wave is perpendicular to the crack surface.

To evaluate the accuracy of the current method, Fig. 3 shows the static interaction between a circular inhomogeneity and a collinear crack subjected to an initial stress intensity factor (K_0). Comparing with the closed form solution (lines) [14] excellent agreement is observed.

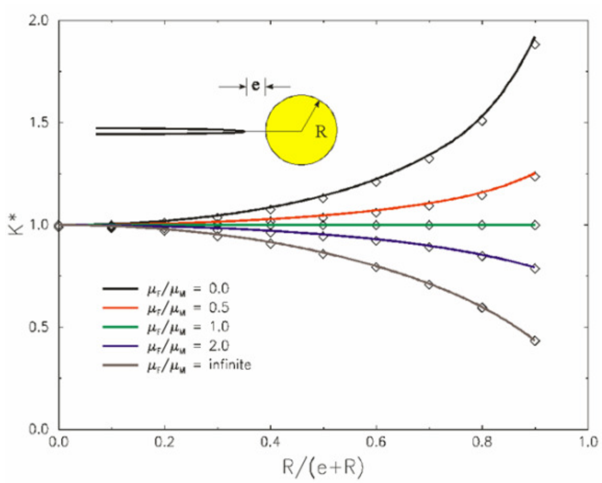

Figure 3. Static interaction

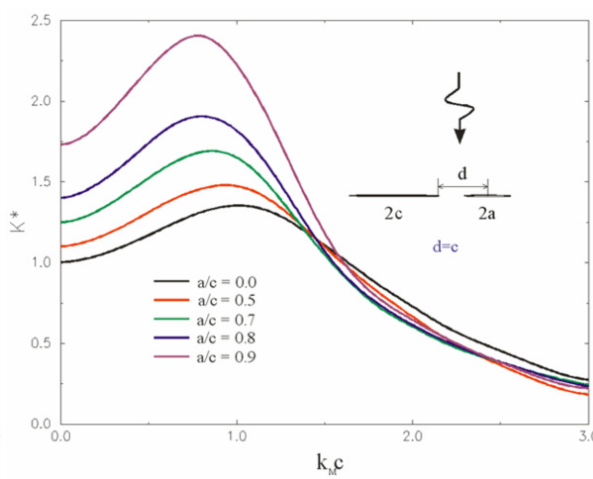

Figure 4. Interacting cracks

Fig.4 shows the effect of a collinear crack of length *2a* on the normalized stress intensity factor crack of a main crack of length *2c*. *a/c=0* corresponds to the case of a single crack. Comparing these curves indicates that significant interaction between cracks exists for low frequencies. But the interaction effect is significantly reduced at higher frequencies.

Fig.5 shows the result of interaction between a crack and an inhomogeneity. The variation of the normalized stress intensity factor for different crack-inhomogeneity configurations is presented. Unlike the collinear crack case where only amplification effects are observed, when the inhomogeneity, with a higher stiffness, is ahead of crack, the stress field is shielded. i.e. the dynamic stress intensity factor at the main crack attains a value lower than the dynamic single crack solution. Fig.6 presents the result of a similar inhomogeneity with a partially debonded interface near the crack tip, as shown. With the increase of the size of the debonded interface, the stress intensity factor at the crack tip increases and indicates an amplification effect. For high loading frequencies, the interaction effect becomes insignificant because the distance between the crack and the inhomogeneity is much larger than the wave length.

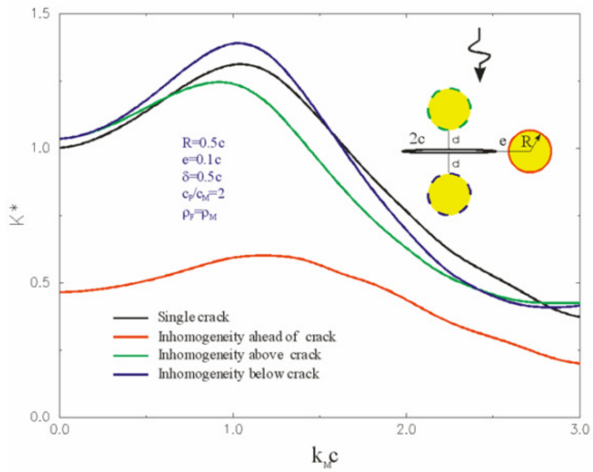

Figure 5. Crack and inhomogeneity

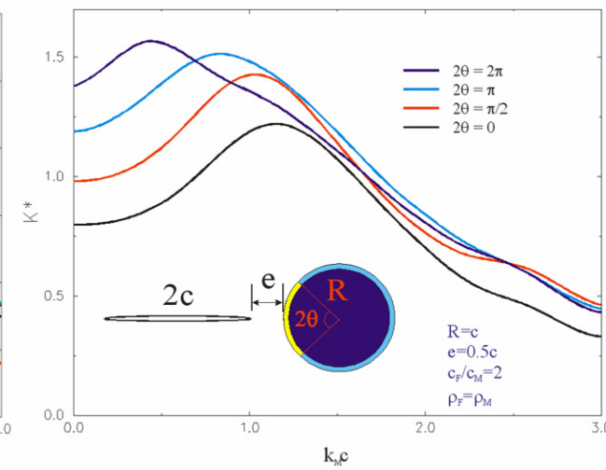

Figure 6. Crack and debonded inhomogenity

4. Conclusions

A pseudo-incident wave method is developed to predict the dynamic interaction effects between cracks and inhomogeneities. This approach enabled us to reduce the original interaction problem into the solution of coupled single crack/inhomogeneity subproblems, for which analytical solutions could be easily obtained which are coupled through the scattered waves. The steady state solution of the crack problem is obtained using integral transform method and the solution of the inhomogeneity problem is determined using Fourier expansion. The dynamic stress intensity factors (SIFs) at the matrix crack are obtained and numerical examples are provided to show the effect of the frequency, geometry of microdefects and material properties upon the dynamic SIFs. Both shielding and amplification effects are experienced and discussed.

Acknowledgements

This research is supported by the Natural Sciences and Engineering Research Council of Canada.

References

[1] R. G. Hoagland, G. T. Hahn and A. R. Rosenfield, Influence of microstructure on the fracture propagation in rock. Rock Mech. **5**, (1973), 77-106.
[2] N. Claussen, J. Steeb, R. F. Pabst, Effect of inducted micro-cracking on the fracture toughness of ceramics. Bull. Am. Ceram. Soc. **56**, (1977), 559-562.
[3] A. G. Evans, K. T. Faber, Crack-growth resistance of micro cracking brittle materials. J. Am. Ceram. Soc., **67**, (1984), 255-260.
[4] M. Ruhle, A. G. Evans, R. M. McMeeking, P. G. Charalambides, J. W. Hutchinson, Microcrack toughening in alumina/Zirconia. Acta Metall. **35**, (1987), 1701-1710.
[5] D.K.L. Tsang, S.O. Oyadiji, A.Y.T. Leung, Multiple penny-shaped cracks interaction in a finite body and their effect on stress intensity factor. Eng. Frac. Mech. **70**, (2003), 2199-2214.
[6] X. Yan, C. Miao, A numerical method for a void-crack interaction under cyclic loads, Acta Mech **223**, (2012), 1015-1029.
[7] D. Gross, Ch. Zhang, Diffraction of SH waves by a system of cracks: solution by an integral equation method. Int. J. Solids Structures. **24**, (1988), 41-49.
[8] S. A. Meguid, X. D. Wang, On the dynamic interaction between a microdefect and a main crack. Proc. R. Soc., Lond. **A409**, (1995), 449-464.

[9] Ch. Zhang, Elastodynamic analysis of a periodic array of mode III cracks in transversely isotropic solids. J. Appl. Mech. **59**, (1992), 366-371.
[10] S.A. Meguid, X.D. Wang, On the dynamic behaviour of interacting microdefects in advanced composite materials, Int. J. Impact Eng. **19**, (1997), 503-514.
[11] X. D. Wang, S. A. Meguid, Diffraction of SH-wave by interacting matrix crack and an inhomogeneity. *ASME J. App. Mech.* **64**, (1997), 568-575.
[12] W.T. Ang, L. Athanasius, Dynamic interaction of multiple arbitrarily oriented planar cracks in a piezoelectric space: a semi-analytic solution. Eur. J. Mech. A-Solid. **30**, (2011), 608-618.
[13] J.D. Achenbach, Wave propagation in elastic solids. North-Holland Publishing Company, Amsterdam (1973).
[14] E. Turska-Klebek, M. Sokolowski, On the influence of defects upon stress concentration at the crack. Arch. Mech. **36**, (1984), 121-126.