# General laws of multiple fracture at static, cyclic and dynamic loading

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**Abstract** Multiple fracture patterns of specimens from low - and medium carbon steel tested in conditions of static, dynamic and cyclic loading were studied. Cumulative distributions of microcracks by length under static and cyclic loading, distributions of the number of fragments by their mass under dynamic loading and amplitude distributions of acoustic emission signals were plotted. The parameters of these distributions at different stages of loading were estimated. It was studied the influence of specimen thickness, the distance to the fracture surface, the grain size and the failure mechanism on these parameters. The general laws characterizing multiple fractures at different loading conditions were found. They include, in particular, a change in function describing the distributions and a reduction of the exponents of these functions before fracture. The physical meaning of the estimated distribution parameters, the possibility of their use for the fracture prediction and material selection are discussed.

**Keywords** Multiple fracture, Damage evolution, General fracture laws

## 1. Introduction

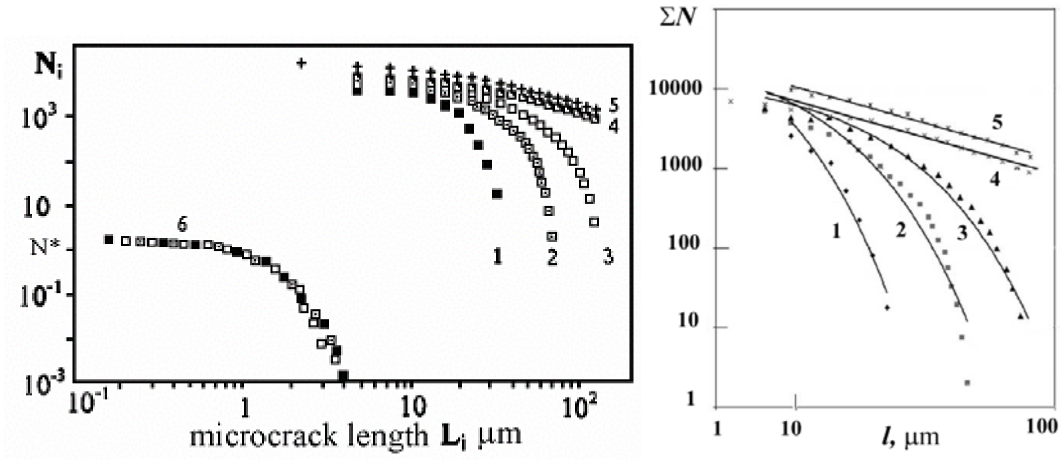
The fundamental problem of materials science is associated with the need to relate macro characteristics of mechanical behavior of material with parameters of structure and its response to the loading conditions. This response is expressed in the formation of the localized plastic zone with a certain geometry and accumulation of internal damage, characterized by the size and density of microcracks. The evaluation of these parameters by the method of replicas removed from the surface of the polished specimens tested under different types of loading showed that there are some general regularities of the damage accumulation or the multiple fracture at the stage of macrocrack initiation, which will be discussed below.

## 2. Results of the study

### 2.1 Two stage of defect growth

Multiple fracture under conditions of irradiation and thermal fatigue [1], was studied using the histograms of the number - size distribution of voids in the aluminum irradiated using different neutron doses [2] and histograms of the distribution of thermal fatigue microcracks in the 06Cr18Ni11steel tested at different number of cycles [3].
Analysis of the cumulative number - size distributions of radiation defects with size of several angstroms in the aluminum [1] showed that these curves corresponding to different neutron doses can be reduced to a single distribution curve by normalization. This demonstrates the self-similarity of the process of accumulation defects at this early stage of their growth [4].
The thermal fatigue microcracks are longer than the radiation voids by several orders of magnitude. Figure 1 shows that the first self-similar stage of defect accumulation is described by an exponential law (the curves 1 – 3, Fig.1, a, b) and the second stage, by a power law (the curves 4 - 5). The exponents in the exponential laws decrease with the number of cycles and are 0.38, 0.17, 0.09 ($R^2$>0.92), respectively. The distribution curves 4 - 5 corresponding to stage of coalescence of voids are well described by a power law with the exponent 0.91 ($R^2$=0.98) (Fig.1, b, solid lines).
Thus, there are two stages of accumulation of defects: the first stage of growth of isolated voids

characterized by exponential distribution curves and the stage of accelerated growth due to void coalescence which is described by the power - law function.



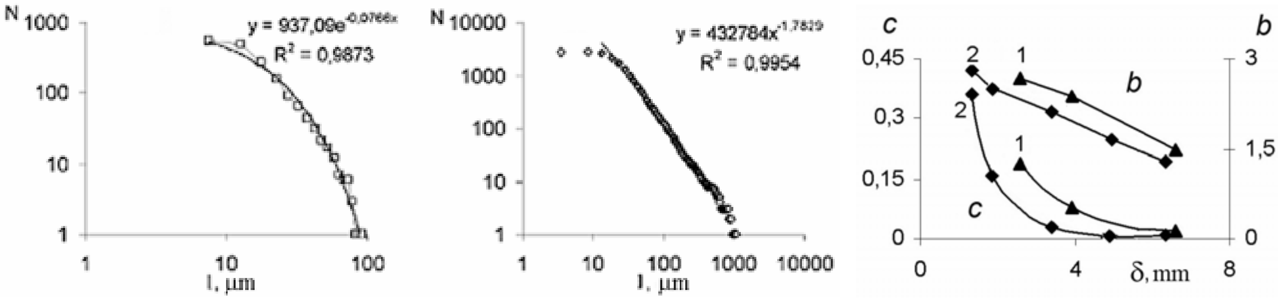

a b

Figure 1. The cumulative number - size distributions of thermal fatigue microcracks in steel X6CrNi181 plotted using initial data [3] on microcrack growth at various numbers of cycles *N*: 2500 (curve 1), 4500 (2), 6500 (3), 11000 (4), 15000 (5). The curve 6 was obtained by normalizing the curves 1 - 3 with respect to the coordinates of their knee points. Solids lines in (b) correspond to the exponential (1-3) and power law (4, 5) relations

We assume that both the initial stages may obey the hypothesis of self-similarity, even though the characters of the laws describing these two stages of damage accumulation are different. It is important to note that the exponents in the exponential functions characterizing the first stage decrease with increasing a number of cycles.

## 2.2. Distributions of microcracks by their length at static loading

Similar results were obtained by means of analyzing the multiple fracture patterns in plastic zone of notched specimens from low carbon steel [5, 6]. The cumulative length - frequency distributions of microcracks in the plastic zones are shown in Fig. 2, which allows us to observe evolution of the distribution curves over different loading stages. It shows that as the tensile load increases, the change in these curves is similar to the change observed in thermal fatigue.




a b c

Figure 2. The cumulative distributions of microcracks in notched specimens from low carbon steel at different tension stages (a, b) (the specimen thickness is 6 mm): (a) – δ =2.8, $P/P_{gy}$=1,7; (b) –δ = 5.2, $P/P_{gy}$ = 2.1, and dependencies (c) of the b - and c - values in the power-law and exponential cumulative number-length distributions of microcracks on the displacement of specimens (1) 16 and (2) 4 mm thick [4]

At the stage of the main crack formation, the exponential law

$$N \sim A \exp(-cl) \quad (1),$$

which describes multiple fracture patterns at the initial loading stage (Figure 2 a) is replaced by the power law

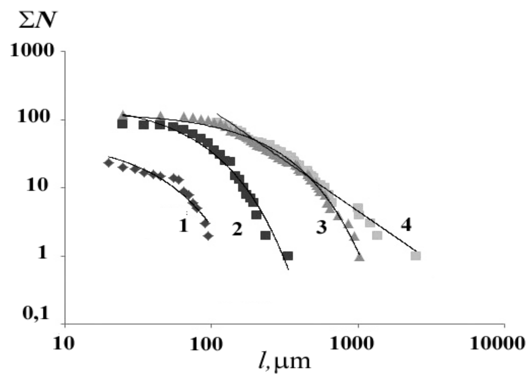
$$N \sim B\, l^{-b} \quad (2)$$

(Fig. 2 b). Both laws are characterized by the respective c and b exponents, whose absolute values decrease with load (Figure 2 c) and the specimen thickness.

An analysis of the amplitude distributions of the acoustic emission signals evaluated during tension has revealed the regularities similar to those, which were observed during tension; these are the change of the type of cumulative functions describing the number-amplitude distributions of signals and the reduction in the exponents of these functions before fracture.

### 2.3. Distributions of microcracks by their length at cyclic loading

The analysis of the distribution curves of fatigue microcracks in mild steel leads to the conclusion that the change of function describing these curves occurs with increasing a number of cycles (Fig. 3 a) and decreasing a distance from the specimen fracture surface (Fig. 3 b). As in the case of thermal fatigue, the change is accompanied by a reduction of the exponents of these functions.



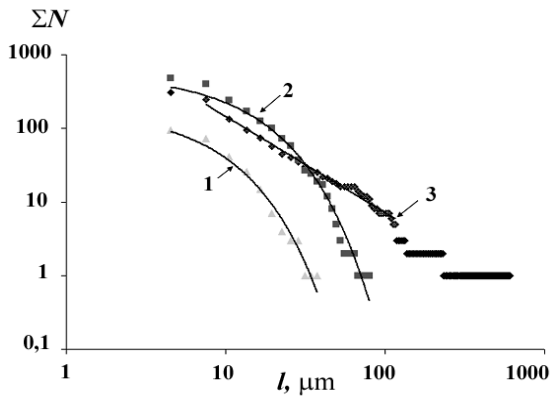



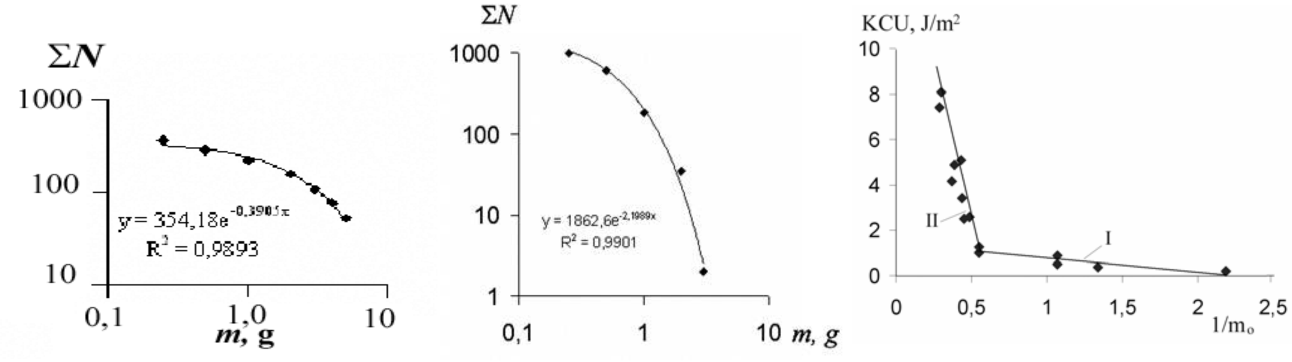

Figure 3. The cumulative distributions of fatigue microcracks in mild steel plotted on the data of Suh et al [7] at different relative number of cycles (a): $n/n_f$= 0,17 (1), 0,43 (2), 0,87 (3), 0,95 (4) and on data [5, 6] for microcracks located at a distance of 5.39 (1) 2.31 (2) and 0.79 mm from the fracture surface (b). Solid lines obey the exponential (a-curves 1-3, and b -1, 2) and power- law functions (a- curve 4, b- curve 3)

### 2.4. Distributions of fragments by their mass at dynamic loading

The similar conclusion was also made in studying the dynamic fracture [8, 9] (Fig. 4). The data presented in the Figure 4 show, firstly, that the number - mass distributions of fragments of shells made of brittle and ductile steels may be described by a simple exponential function:

$$\sum N \sim N_O \exp(-m/m_o) \quad (3),$$

and, as is seen from Fig. 4 b, the fragment distributions for the ductile material approach the power law $N$ - $m$ dependence. Secondly, the exponents of these distributions ($1/m_o$) are determined by the mechanical properties of the fragment material and, in particular, by impact toughness of material in initial state (Fig. 4 c).




a b c

Figure 4. Cumulative number-mass distributions of shell fragments from brittle (a) and ductile (b) carbon steels, and dependence of impact toughness of the brittle (I) and ductile (II) steels on the exponents in the exponential relations describing distributions of fragments (c)

### 2.5. Concentration criterion of defects

At different stages of tension and fatigue, the average length of the microcracks $L_{av}$, their density $n$, fraction of the damages ω, and the values of concentration criterion $k$ suggested in [10] and estimated by the relation

$$k = n^{-1/2} / L_{av} \tag{4}$$

were evaluated. It allowed to find that the transition from the exponential to the power - law relation with development of damage accumulation process is accompanied by reduction of the concentration criterion related to the coalescence of microcracks in the plastic deformation zone.

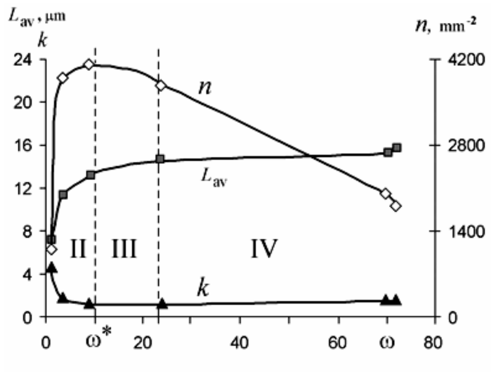



Fig. 5. Dependences of the average length ($L_{av}$), density of microcracks ($n$) and the concentration criterion ($k$) on the relative fraction of damages (ω) at the different loading stages of damage development (II, III, IV) (at the stage I plastic zone forms)

Four stages of the damage accumulation were found. At the first (I) stage, in the notch tip of specimen, slip bands appear and plastic zone is formed (before approaching the yield strength); at the second stage (II), the formation and the accumulation of the microcracks occur. Interaction of microcracks at the third stage (III) leads to their coalescence and initiation of macrocrack. At the fourth stage (IV), the main crack appears in a secondary plastic zone in its tip; the area of the damaged surface grows as a result of the opening of microcracks. The development of the main crack leads to the complete fracture. Changes of the damage parameters at the observed stages of multiple fracture are shown in Fig. 5. It follows from the graphs that the critical situation appears at reaching the area of damages (ω*) close to 10%, when the microcrack opening increases, density of defects decreases as a result of their coalescence, and concentration criterion falls to the constant value (k~1.5). The value of ω*=10% is close to the threshold of percolation for many systems; therefore, changes at the third stage precede the critical event (specimen fracture). With reaching ω*, the exponential relation, which describes the cumulative distributions of microcracks and acoustic emission signals, becomes close to the power relation, and exponents in these relations reduce.

### 2.6. Activation energy of fracture process

An analysis of the established regularities of changes in exponents of the functions describing the statistical distributions allows us to connect them with a change in another important parameter of kinetic process, namely, in the activation energy of fracture.

According to Arrhenius – Zhurkov equation of [11], the lifetime ($t$) of solids is the function of the activation energy of fracture process $U(\sigma)$, the absolute temperature $T$, the energy of a thermal motion $R$ and obeys the exponential relation:

$$t = t_0 \exp\left[\frac{U_0 - U(\sigma)}{RT}\right] \tag{5}$$

This relation may be obtained from a correlation $\lg t = \lg t_0 + b(\sigma)/T$, where $b(\sigma)$ is the slope of dependence $\lg t - 1/T$, $b(\sigma) = b = \Delta(\lg t)/\Delta(1/T)$,

or $t = t_0 \exp(\beta/T)$, where $\beta = 2.3b$.

Multiplying the numerator and denominator of the exponential function index by the Boltzmann constant and denoting $k\beta = U$ leads to (Eq. 5).

Thus, the activation energy is evaluated on the slope coefficient $b$ of creep curves plotted in semi logarithmic coordinates [12]. The stress dependence of the activation energy of fracture of polymethylmethacrylate, which was plotted using these data [13], showed that the activation energy decreases with increasing the stress and the stress dependence of activation energy corresponds to the exponential relation with the exponent equal to 0.02 and $R^2$=0.98. Similar decrease of the slope coefficient with increasing the load or specimen displacement is presented in Fig. 2 c.

The activation energy of solids is traditionally related (by the analogy with that of chemical reactions) to the energy given for the overcoming of inter-atomic interaction by thermal fluctuations. Although the fracture process in the whole is determined by the strength of atomic bonds in the region of fracture localization, this approval calls for the special confirmation. Indeed, the slope coefficient of strength dependencies obtained for laboratory specimens is the macroscopic parameter revealing the kinetics of initiation and accumulation of microcracks and growth of a macrocrack. These processes are connected to changes in the fracture process zone size and damage accumulation in this zone. Therefore, the decrease in the activation energy with increasing stress may be a consequence of considered above regularities of damage accumulation in the fracture process zone that are due to the interaction of defects resulting in a decrease in the slope coefficients of statistical distributions. This assumption is confirmed by an analysis performed in [6, 13], which showed an interrelationship between the activation energy and exponents in power law equations characterizing the fracture development, namely, in both the Paris relation and the equation based on the phase transition theory.

## 3. Summary

The patterns of multiple fractures in carbon steel specimens at different stages of tension, cyclic and dynamic loading were studied; characteristics of damage accumulation were estimated and the cumulative distributions of microcracks and shell fragments were plotted. Analysis of damage accumulation process with the use of mechanical and physical methods allows us to establish some general laws characterizing this process on different stages. They include, in particular, the following.

- Distributions of microcracks under static and cyclic loading and amplitude distributions of acoustic emission signals measured during static loading are described by an exponential function at the initial stage of fracture and a power –law one at the final stage of fracture.
- Cumulative number-mass distributions of the shell fragments obey the exponential relations with the slope coefficients, which decrease for the brittle materials.
- There is a quantitative relationships of the exponents of functions approximating the distributions with material properties and loading parameters.
- Parameters of the cumulative distributions of microcracks and amplitude distributions of acoustic emission signals, as well as the concentration criterion are reduced at the stage of pre-fracture and can serve as diagnostic signs of the approaching fracture.

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