A new macroscopic model based on non-local interactions to predict damage and failure in quasibrittle materials

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Abstract The purpose of this paper is to propose a new macroscopic approach to describe the evolving non-local interactions during damage and failure in quasi-brittle materials. A new integral-type non-local model is proposed where the weight function is directly built from these interactions. The structure is considered as an assembly of inclusions, which are successively elastically dilated in order to characterize the transfer of information inside the material. By this way, the new macroscale weight function takes into account intrinsically the interactions evolution during the material failure similarly as a mesoscale model does. This new model is first validated on simple 1D cases and its performances are compared with the performances of other models proposed in the literature. It is shown that the new model is able to describe the continuous/discrete transition during the dynamic failure of a rod. It is also shown that the new model is able to describe boundary effect during a spalling test. Finally, the model is used to predict damage and failure during 3 points bending fracture tests on notched and unnotched concrete beams.

Keywords Non-local model, interactions, damage, quasi-brittle materials

1. Introduction

Classical failure constitutive models involve strain softening due to progressive cracking and a regularization technique for avoiding spurious strain and damage localization. Different approaches have been promoted in the literature such as integral-type non-local models (e.g. [1]), gradient damage formulations (e.g. [2]), cohesive cracks models (e.g. [3] with classical finite elements and e.g. [4] with extended finite elements), or strong discontinuity approaches (e.g. [5]). Such macroscale failure models have been applied on a wide range of problems, including the description of damage and failure in strain softening quasi-brittle materials [1], softening plasticity [6–8], creep [9] or composite degradation [10]. They may exhibit, however, some inconsistencies such as (*i*) incorrect crack initiation, ahead of the crack tip; (*ii*) propagating damage fronts after failure due to non-local averaging, (*iii*) incorrect shielding effect with non-zero non-local interactions across a crack surface; (*iv*) deficiencies at capturing spalling properly in dynamics, with spalls of zero thickness when the expected spall size is below the internal length of the model (see e.g. [11–14]). Moreover changing geometry, e.g. from tensile to bending loads or from unnotched to notched specimens, results generally in the loss of predictive capabilities of the macro-scale non-local models [15, 16]. On the contrary, it has been shown recently [15, 17] that meso-scale models gave good prospect in the prediction of failure and size effect for notched and unnotched concrete beams. Indeed meso-scale results have been compared to a new experimental database [16] consisting in 3 point bending failure tests for similar notched and unnotched concrete specimens of four different sizes but made from the same formulation. Not only the different peak loads for all geometries are recovered but also the failure softening phase is well predicted which is a more challenging issue. It means that the meso-scale models intrinsically contain relevant information leading to a good description of the size effect, the boundary effect and the whole failure process.

At the macro-scale, the prediction of failure in quasi-brittle materials needs enhancement of existing non-local damage models and the way the non-locality is taken into account in the macro-scale models has to be redefined. Non-locality finds its origin in the interaction between material points

undergoing damage in the course of failure [18, 19]. There are several mechanisms, which should be considered when looking at the non-locality due to the interaction between damaged points: (*i*) an interaction exists if there is damage, which produces this interaction. Assuming that damage corresponds to the growth of micro-cracks, this interaction grows with the size of the defect; (*ii*) shielding effects are also expected: the interaction between two points located apart from a crack should not exist; (*iii*) on free existing or evolving boundaries, and along the normal to these boundaries, non-local interactions should vanish as demonstrated in [20]. The internal length in the non-local model is the parameter inside the weight function that encompasses the non-locality and there is a consensus that this quantity may not be constant, but should depend on the geometry of the specimen or on the state of damage. Therefore enhanced non-local models accounting for a variation of the internal length have been proposed recently [13, 20, 21]. Proposals discussed in [13, 20] are considered on academic one-dimensional problems. Their implementation and extension to 2D or 3D problems are really not trivial as they involve the computation of path integrals, which are tedious in a finite element setting. The stress-based model in [21] is more tractable in 2D/3D computations but the evolution of non-locality is rather empirical.
The purpose of this paper is to discuss a new approach to non-local interactions during failure in quasi-brittle materials and to upscale the relevant information present from the meso-scale to the macro-scale. Therefore, the paper focuses on the estimation of the non-local weight function directly from interactions. The material is modeled as an assembly of inclusions and the elastic interactions upon dilation of each inclusion are computed in a similar ways to a classical Eshelby's problem [22]. A new interaction-based weight function is then built from these interactions. This new interaction-based non-local model is validated on simple 1D problems and its performances are compared with the classical integral-type nonlocal model.

2. A new interaction-based non-local model

2.1. Non-locality in integral-type macro-scale models

In classical non-local models, such as the integral-type [1], the internal length is the parameter inside the weight function that encompasses the non-locality. Associated with a classical Gaussian weight function, it set how and how far the interactions produce inside the materials. However, the main drawback of the formulation is that this parameter is constant whatever the geometry and the failure process. For instance, close to a boundary, the part of the nonlocal averaging domain that protrudes outside the boundary is classically chopped off [1]. Improved models can be found in the literature, with a different averaging process close to the boundary of the solid [12,23] or with a varying internal length in the course of damage [13, 20, 21]. However, even if the internal length variations are based on micro-mechanical concepts, such as the crack growth interaction effect or the transfer of information through a damaged area, the final choice of the weigh function and thus the evolution of non-locality are rather empirical.

2.2. Non-locality in meso-scale models

In meso-scale models, the non-locality is intrinsically included by representing the meso-structure of the materials (e.g. granular, matrix and interfaces in concrete). Therefore, the non-locality does not behave the same close to a boundary, close to a damaged area, at initiation or during the failure process. It has been shown recently that such models are able to capture challenging issues of quasi-brittle materials failure such as predicting the peak loads and even the whole softening load-displacement responses of notched and unnotched beams in three-point bending [15, 17]. In the following, we aim at building a new interaction-based non-local weight function, which will evolve intrinsically when damage occurs inside the materials.

2.3. A new interaction-based non-local weight function

The purpose of the paper is to discuss a new approach to non-local interactions during failure in quasi-brittle materials and to upscale the relevant information present from the meso-scale to the macro-scale. Therefore, we aim at estimating the non-local weight function directly from interactions.

Before we get to the weight function to be inserted in a non-local integral model, let us first consider elastic interactions. In order to compute the effect of point ξ on point x, we look at the strain induced at point x due to the dilation of ε^* of a circular inclusion of radius a centered at point ξ (see Fig. 1).

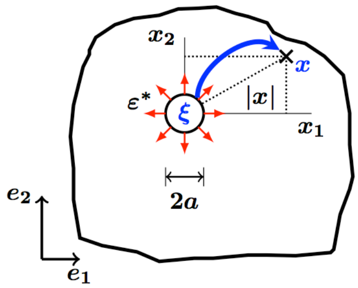


Figure 1. Non-local contribution seen by a point x when a perturbation is produced in ξ.

Assuming now that the induced strain at point x has been computed, numerically for instance. The growth of damage is often defined from energy considerations and we shall look at a norm of this strain, denoted as A, instead of the strain tensor itself:

$$A(x,\xi,\varepsilon^*,a) = \sqrt{\sum_{i=1}^{3} \left|\varepsilon_i(x)\right|^2} \tag{1}$$

where $\varepsilon_i(x)$ is the ith principal strain. Note that we could have chosen the true elastic energy instead of a norm of the strain tensor. It would not have changed much the following development. Then, the interaction is represented by this norm transmitted from the dilation in the inclusion centered at ξ to x. It depends on the geometry of the solid, on the inclusion size a, and on the material elastic properties inside and outside the inclusion. Formally, the norm A transmitted to x by the dilation ε^* writes also:

$$A(x,\xi,\varepsilon^*,a) = \left\|\varepsilon^*\right\| A^*(x,\xi,a) \tag{2}$$

where A^* represents the interaction produced at x due to ξ for a unit dilation.

2.4. Final formulation

2.4.1. Non-local averaging

We assume now that it is this interaction A^*, which governs the weight function involved in non-local averaging. This non-local averaging writes:

$$\bar{\varepsilon}_{eq}(x) = \frac{1}{\Omega_r}\int_\Omega \psi(x,\xi)\varepsilon_{eq}(\xi)d\xi \quad \text{with} \quad \Omega_r = \int_\Omega \psi(x,\xi)d\xi \tag{3}$$

where $\bar{\varepsilon}_{eq}$ is the non-local strain and ε_{eq} is the effective strain defined by Mazars [24] as:

$$\varepsilon_{eq} = \sqrt{\sum_{i=1}^{3} \left\langle \varepsilon_i \right\rangle_+^2} \tag{4}$$

where $\langle \bullet \rangle_+$ is the positive part function, Ω is the volume of the structure, Ω_r is a characteristic volume introduced in such a way that the non-local operator does not affect an uniform distribution of equivalent strain far away from the boundary when no damage occurs in the structure. The analogy between the interactions defined above and the weight function ψ suggests:

$$\psi(x,\xi) \equiv A^*(x,\xi,a) = \frac{A(x,\xi,\varepsilon^*,a)}{\left\|\varepsilon^*\right\|} \equiv \frac{\sqrt{\sum_{i=1}^{3}\left|\varepsilon_i(x)\right|^2}}{\left\|\varepsilon^*\right\|} \quad \text{with} \quad \Omega_r = \int_\Omega A_0^*(x,\xi,a)d\xi \tag{5}$$

where $A_0^*(x,\xi,a)$ is the interaction function reconstructed when no damage occurs in the structure (typically at the beginning of the computation).

Practically, the computation of the interactions (function $A*$) is performed using a finite element setup, which is identical to that of the mechanical problem to be solved, with the same mesh. The finite elements which belong to each inclusion centered on a given integration point are subjected to a thermal expansion ($\varepsilon* = \alpha\Delta TI$) where α is the thermal expansion coefficient, I is the identity tensor, and T is the temperature. If the structure has n inclusions (integration points), n elastic computations are performed to build the weight function at each loading step. Since the construction process of the interaction-based weight function is cinematically driven by the successive thermal expansions, all boundary conditions are clamped during the reconstruction process in order to avoid the perturbation of the kinematics on the boundary.

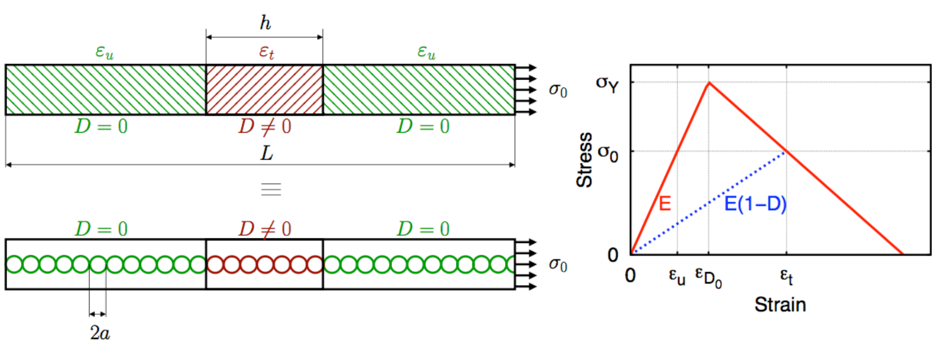
The single model parameter which remains to be determined is the inclusion size a. This inclusion size is the internal length involved in the formulation. It ought to be related to the average size of the heterogeneities in the underlying heterogeneous material to be modeled.

2.4.2. Constitutive model

Damage is considered to be isotropic. Temperature and time-dependent effects are neglected. Damage is a function of the amount of extension in the material, defined locally by the equivalent strain (see Eq. 4). The evolution of damage is a function of the non-local equivalent strain and it is governed by the Kuhn-Tucker loading-unloading condition (see [15] or [16] for details).

3. Validation and performances

3.1. Clamped bar in tension



(L =10 cm, σ_Y =3 MPa, E =30 GPa, ε_{D0} =10^{-4}, σ_0 =1.8 MPa, D =70 %)

Figure 2. Simple problem of a clamped bar in tension.

We are going to illustrate the influence of the inclusion size a on the weight function and we shall look at the evolution of the non-local contributions nearby the boundary of a damaged zone. For this purpose, we look at a one-dimensional problem of a clamped bar subjected to tension. The bar has a damaged zone in the middle, the rest being undamaged. The distribution of damage is fixed a priori. The strain distribution inside the bar corresponds to the onset of evolution of damage from this initial state. The strains ε_u and ε_t are obtained from a bilinear softening constitutive law (see Fig. 2).

Although the bar is one-dimensional, the interactions are computed following a 2D, plane stress description. The weight functions are computed from a discrete set of circular inclusions located on the neutral axis of the bar (see Fig. 2). Their size is much smaller than the bar depth in order to avoid interactions with upper and lower boundaries. The finite element meshes consist in triangular elements with 1 integration point and the meshes are built in such a way that there are always 4 elements in the inclusion diameter. Each inclusion is dilated successively in order to reconstruct the weight functions. The weight functions are normalized afterwards so that their integral over the bar is equal to 1 (through the functional Ω_r in Eq. 5).

Fig. 3 presents the influence of the size of the inclusion on the weight function in the case where damage is equal to zero.

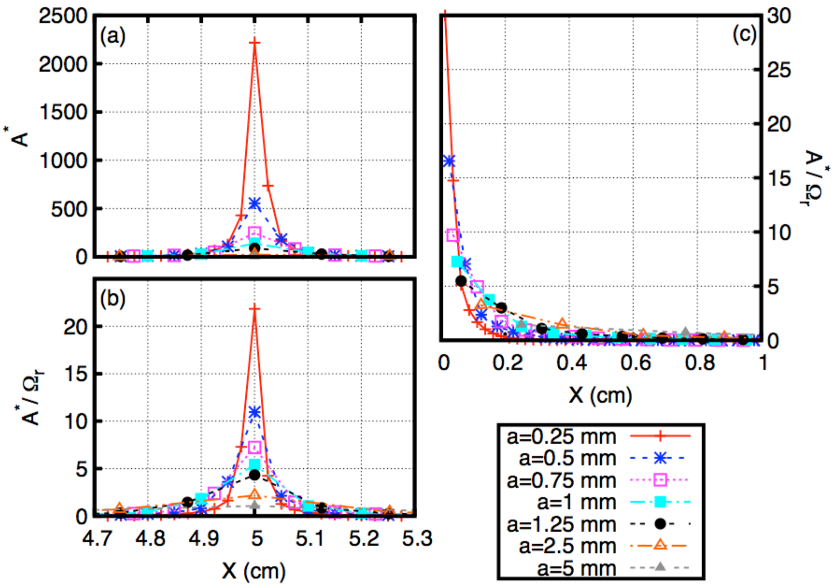


Figure 3. Influence of the inclusion size on the weight function (Reproduced from [25]):
(a) computed far from the boundaries; (b) normalized and computed far from the boundaries;
(c) normalized and computed near the boundary.

Far away from the center of the inclusion (Fig. 3.a), the weight function does not depend on the inclusion size and decreases as $1/X^2$ following the Eshelby's theory (see e.g. [22]). Fig. 3.b presents the same influence of the size of the inclusion but on the normalized weight function. If the inclusion size tends to zero, the computation of the interactions reduces to the construction of Green functions in which it is well known that no internal length is involved. One can demonstrate from the construction of the normalized weight function that it becomes a Dirac delta function and the constitutive model becomes local. Fig. 3.c shows the same calculation nearby the boundary of the solid. The weight function is centered in the inclusion, which sits right next to the boundary. Again, upon decreasing of the radius of the inclusions, the weight function converges toward a Dirac Delta function. According to the results due to [12], [20] and [23], it is expected that at the boundary of the solid the response becomes local. In mesoscale models, there is a wall effect on the boundaries and large inclusions may not be fitted in a boundary layer smaller than their radius. Nearby a boundary, the inclusion size is constrained by the distance to the free surface, as it cannot protrude outside the solid. This feature can be easily introduced in the present model: at points located close to the boundary, the inclusion size is decreased so that it cannot protrude outside the solid. In a boundary layer of thickness l, inclusions of diameter l shall be considered only when $l < a$. Thus interactions tend to vanish as we consider points closer and closer to the boundary of the solid.

Fig. 4.b presents the normalized non-local contributions when the non local strain is computed at the center of the inclusion located close to the damage band in the region which unloads on the left side. The damage band contains 7 inclusions (*a* = *1.25mm, h* = *8.75mm*). A comparison with the gauss-type weight function used in the classical non-local damage model is also provided (Fig. 4.a). With the Gauss weight function and because the strain inside the damage band is larger than outside the damage band, the non-local contribution from points lying inside the band is much larger than those of points lying outside the band. This will trigger the propagation of the damage band, which should expand in the course of the calculation eventually. Fig. 4.b shows that a shielding effect is observed with the new formulation. The non-local weights outside the damage band are the most important. As a matter of fact, the weight at points lying inside the band is decreasing with increasing damage. There is a shielding effect due to damage, which derives directly from the method used for the calculation of interactions. In the extreme case of a fully damaged band, the dilation of an inclusion sitting inside the band will not be transmitted to the stiffer zone outside the band.

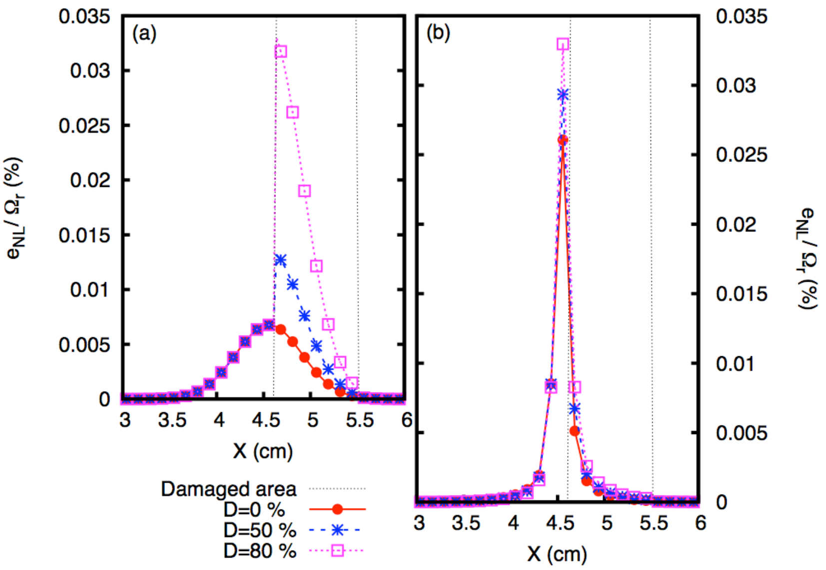


Figure 4. Response close to a damaged area (Reproduced from [25]):
(a) original formulation; (b) interaction-based formulation.

There is, however, a limitation to the shielding effect when the radius of the inclusions is larger than the width of the damage band. In this case, the interaction induced by the dilation of the inclusion will extend across the band. It is expected then that a point lying on one side of the band will feel the interaction from points lying on the other side. We recover here the case of an inclusion located near a boundary, a fully formed crack being two free boundaries facing each other. In order to avoid this problem, we impose that the radius of the circular inclusion reduces as damage grows and we adopt the following rule, which encompasses the situation where an inclusion is centered at a point nearby a damage zone or near a boundary of the solid:

$$a(x) = \min(a_0\sqrt{1-D(x)}, d(x)) \quad (6)$$

where *a(x)* represents the radius of the sphere containing the integration points where the thermal expansion is imposed to reconstruct the interaction-based weigh function, a_0 is a model parameter related to the maximum aggregate size, D is the local damage, d is the minimal distance from any boundary of the structure.

3.2. Dynamic failure of a rod

This example is used to test the relevance of the proposed model and its capabilities to describe progressive failure and complete failure. A bar is submitted at both extremities to constant strain waves, which propagate toward the center in the linear elastic regime (see Fig. 5 and Table 1). When the two waves meet at the center, the strain amplitude is doubled, the material enters the softening regime suddenly, and failure occurs. In all computations, the time step is chosen to be equal to the critical time step.

Table 1. Characteristics of the rod dynamic failure test

Parameter	L	v	E	ρ	l_c/a_0	ε_{D0}	α_t	A_t	B_t	α_c
Unit	*cm*	cms^{-1}	*MPa*	kgm^{-3}	*cm*					
Value	30	0.7	1	1	4	1	1	1	2	0

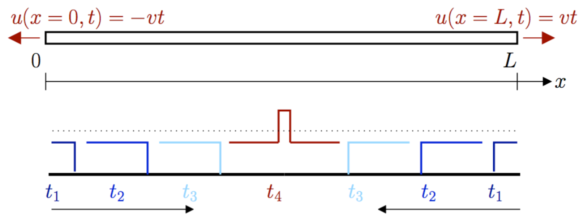


Figure 5. Dynamic failure of a rod: test description and time evolution of the strain amplitude repartition along the rod.

In the course of damage, the crack opening displacement (COD) can be estimated using the method proposed by [26] and compared to an ideal crack opening profile obtained from a strong discontinuity analysis (single crack). The comparison, e.g., the distance between the two profiles, indicates how close the strain and damage distributions are from those corresponding to a single crack surrounded by a fracture process zone. Details may be found in [13] based on [26].

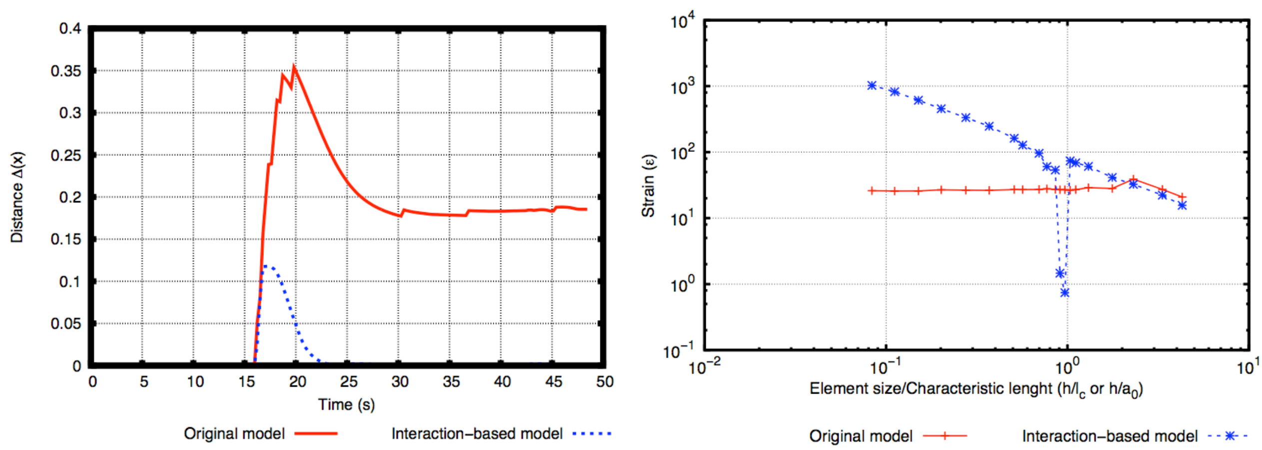


Figure 6. Dynamic failure of a rod (Reproduced from [25]): (left) distance between the computed COD and an ideal opening profile obtained for a strong discontinuity versus time; (right) strain in the cracked element versus adimensional element size at complete failure.

Figure 6 shows that the failure process is better described with the interaction-based model since the distance between its crack opening profile and the one corresponding to a strong discontinuity COD tends rapidly to zero. At complete failure, the crack opening computed according to the same technique should be independent of the element size. In a simple 1D setting, for instance, and assuming that the crack opening is smeared over the finite element that contains the discontinuous displacement at complete failure, the crack opening is equal to the strain distribution times the element size. Therefore, after complete failure, the strain in the cracked element should evolve in inverse proportion of the element size (for constant strain element). Figure 6 shows that the complete failure is better described with the interaction-based model since the strain versus adimensional element size curve follows a linear trend in a logarithmic plot. Moreover the slope is coherent with the CMOD estimated at complete failure. Note that in the original model, the element size is dimensioned by the internal length l_c whereas in the new model, it is dimensioned by the characteristic length a_0 . For the integration-based model, a peak discontinuity is observed when the element size is approximately equal to the characteristics length a_0. It means that several elements are needed inside the inclusion where the perturbation is produced to well reconstruct the interaction-based weight function.

3.3. Spalling test

A second 1D example is used to test the response of the new model close to a boundary. This 1D example consists of a spalling test presented by [12] based on a split Hopkinson pressure bar test primarily developed by [27] for material dynamic behavior characterization, but often adapted for dynamic fracture testing [28, 29]. A striker bar generates a square compressive wave that then propagates along the bar in the linear elastic regime. When this compressive wave reaches the free extremity of the bar, it is converted into a tensile wave and added to the incoming compressive wave (see Fig. 7 and Table 2). The resulting wave stays equal to zero until the tensile one reaches a distance from the boundary equal to half the initial signal length. Failure is initiated at this point if the amplitude is greater than the tensile strength, generating a spall at a controlled distance from the boundary that depends on the initial compressive signal duration. For all numerical studies, the time step is chosen to be equal to the critical time step of the corresponding element size.

Figure 7 shows that the spalling failure is better described with the interaction-based model since the spall location is predicted inside the bar whereas the damage is maximum on the boundary with the original model.

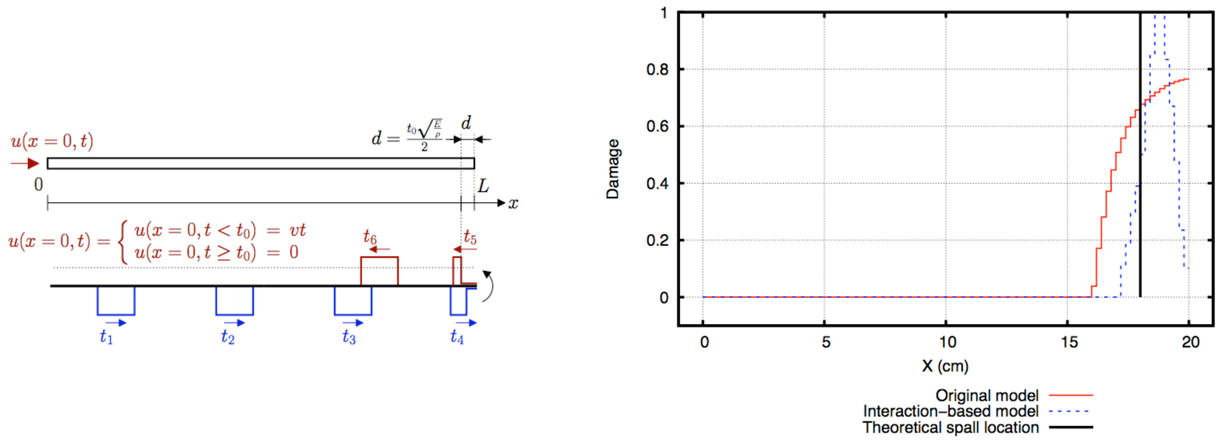


Figure 7. Spalling test: test description, time evolution of the strain amplitude repartition along the rod (left) and damage repartition along the bar after failure (right).

Table 2. Characteristics of the spalling test

Parameter	L	t_0	v	E	ρ	l_c/a_0	ε_{D0}	α_t	A_t	B_t	α_c
Unit	cm	s	cms^{-1}	MPa	kgm^{-3}	cm					
Value	20	4	1,5	1	1	4	1	1	1	2	0

4. Concluding remarks

A new interaction-based non-local formulation has been proposed. In this formulation, the material is modeled as an assembly of inclusions and the elastic interactions upon dilation of each inclusion are computed in a similar ways to a classical Eshelby's problem. A new interaction-based weight function is then built from these interactions. This new interaction-based non-local model has been first validated on simple 1D problems and its performances have been compared with the classical integral-type non-local model.

Different results have been presented in the paper:
(i) In the course of damage, the crack opening displacement has been estimated and the comparisons show that the failure process is better described with the new formulation. Indeed the crack opening profile is very close to an ideal opening profile obtained for a strong discontinuity.
(ii) At complete failure, the crack opening should be independent of the element size. Therefore, after complete failure, the strain in the cracked element should evolve in inverse proportion of the element size, assuming that the crack opening is smeared over the finite element that contains the discontinuous displacement. It has been shown that for the new formulation, the strain versus element size curve follows a linear trend in a logarithmic plot. Moreover the slope is coherent with the CMOD estimated at complete failure.
(iii) Close to a boundary, it has been shown that the spalling failure is better described with the interaction-based model since the spall location is predicted inside the bar whereas the damage is maximum on the boundary with the original model.

Finally, it has been shown that this new interaction-based formulation fulfill several deficiencies of the classical integral-type non-local model and the formulation has to be implemented in 2D in order to test its performance on more challenging issues of quasi-brittle materials failure such as reproducing the peak loads and even the whole softening load-displacement responses of notched and un-notched beams in three-point experimental bending tests [16].

Acknowledgements
Financial support from ERC advanced grant Failflow (27769) is gratefully acknowledged.

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