Criticality of Damage-Failure Transitions under Dynamic and Fatigue Loading

Oleg Naimark

Institute of Continuous Media Mechanics UB RAS, 614013 Perm, Russia
Corresponding author: naimark@icmm.ru

Abstract Statistical theory of evolution of typical mesoscopic defects (microcracks, microshears) revealed specific type of criticality–structural-scaling transitions and allowed the development of phenomenology of damage-failure transition in solids with defects. The key results of statistically based phenomenology are the establishment of characteristic multiscale collective modes of defects responsible for damage-failure transitions under dynamic and fatigue loading. High resolution experiments and structural (SWFM and AFM) study in terms of scaling invariance supported the linkage of the evolution of these modes with material responses in large range of load intensity (dynamic crack propagation, fragmentation statistics, crack path under high cycle (HCF) and very high cycle (VHCF) fatigue)

Keywords multiscale damage kinetics, collective modes of defects

1. Introduction

The problem of fracture treated as a critical phenomenon represents one of the key problems of fundamental and applied physics of materials science. Experimental studies of material responses in a large range of loading rates show that the behavior of solids is intimately linked with the evolution of typical mesoscopic defects (microcracks, microshears). This characterizes generically solids under dynamic and fatigue loading, when the internal times of the evolution of ensemble of defects for different structural levels are approaching the characteristic loading times. Statistical theory of typical mesoscopic defects (microcracks, microshears) revealed specific type of criticality – the structural-scaling transitions and allowed the development of phenomenology of damage-failure transition based on the definition of non-equilibrium free energy of solid with defects. Multiscale aspects of damage-failure transition are analyzed to consider recent experiments on dynamic crack propagation, fragmentation statistics, scaling laws of fatigue.

1. Structural-scaling transitions in solid with defects

Statistical theory of the evolution of typical mesoscopic defects (microcracks, microshears) allowed us to establish specific type of critical phenomena in solid with defects – structural-scaling transitions and to propose the phenomenology of damage-failure transition [1]. The key results of the statistical theory and statistically based phenomenology are the establishment of two order parameters responsible for the structure evolution – the defect density tensor p_{ik} and the structural scaling parameter $\delta = (R/r_0)^3$, which represents the ratio of the spacing between defects and characteristic size of defects. Non-equilibrium free energy F represents generalization of the Ginzburg-Landau expansion in terms of mentioned order parameters – the defect density tensor (defect induced deformation $p = p_{zz}$ in uni-axial case) and structural scaling parameter δ:

$$F = \tfrac{1}{2} A(\delta,\delta_*)p^2 - \tfrac{1}{4} Bp^4 - \tfrac{1}{6} C(\delta,\delta_c)p^6 - D\sigma p + \chi(\nabla_l p)^2, \quad (1)$$

where $\sigma = \sigma_{zz}$ is the stress, χ is the non-locality parameter, A, B, C, D are the material parameters, δ_* and δ_c are characteristic values of structural-scaling parameter (bifurcation points) that define the areas of typical nonlinear material responses on the defect growth (quasi-brittle, ductile and fine-grain state) in corresponding δ–ranges: $\delta < \delta_c \approx 1$, $\delta_c < \delta < \delta_*$, $\delta > \delta_* \approx 1.3$. The damage kinetics is determined by the kinetic equations for the defect density p and scaling parameter δ

$$\dot{p} = -\Gamma_p \frac{\Delta F}{\Delta p}, \quad \dot{\delta} = -\Gamma_\delta \frac{\partial F}{\partial \delta}, \quad (2)$$

where Γ_p, Γ_δ are the kinetic coefficients, $\Delta(...)/\Delta t$ is the variation derivative. Kinetic equations Eq.2 and the equation for the total deformation $\varepsilon = \hat{C}\sigma + p$ ($\hat{C}$ is the component of the elastic compliance tensor) represent the constitutive equations of materials with mesodefects. Material responses on the loading realize as the generation of characteristic collective modes – the solitary waves in the range of $\delta_c < \delta < \delta_*$ and the “blow-up” dissipative structure in the range $\delta < \delta_c \approx 1$. The generation of these collective modes under the loading provides the change of the system symmetry and initiates specific mechanisms of the momentum transfer (plastic relaxation) and damage-failure transition on the scales of damage localization with the blow-up kinetics.

The damage-failure scenario includes the “blow-up” kinetics of damage localization as the precursor of crack nucleation according to the self-similar solution:

$$p = g(t) f(\xi), \xi = x/L_H, \quad g(t) = G(1 - t/\tau_c)^{-m}, \quad (3)$$

where τ_c is the so-called "peak time" ($p \to \infty$ at $t \to \tau_c$ for the self-similar profile $f(\xi)$ localized on the scale L_H, $G > 0, m > 0$ are the parameters of non-linearity, which characterise the free energy release rate for $\delta < \delta_c$. The self-similar solution Eq.3 describes the blow-up damage kinetics for $t \to \tau_c$ on the set of spatial scales $L_H = kL_c$, $k = 1,2,...K$, where L_c and L_H corresponds to the so-called “simple” and “complex” blow-up dissipative structures. Generation of the complex blow-up dissipative structures appears when the distance L_S between simple structures approaches to the scale L_c. Similar scenario of the “scaling transition” proceeds for the blow-up structures of different complexity to involve in the process of the final stage of damage localization the larger scales of material.

The description of damage kinetics as the structural-scaling transition allowed the consideration of solid with defects as a dynamical system with spatial degrees of freedom (corresponding to the set of blow-up dissipative structures of different complexity). Stochastic behavior in this case can be linked with the dynamics of the critical state with the features of flicker noise, or $1/f$ - statistics. The systems reveal the so-called self-organized criticality (SOC) with universal behavior that is typical for the late state evolution of dynamic systems when the correlation will appear on all length of scales. The self-similar nature of mentioned collective modes associated with damage localization zones has the great importance in the case of dynamic loading, when the “excitation” of these modes can lead to the subjection of relaxation and failure to the dynamics of these modes. The examples for this situation are the transition from the steady-state to the branching regimes of crack propagation, qualitative change of the fragmentation statistics with the increase of the energy density imposed into the material.

2. Nonlinear crack dynamics. Crack branching

The understanding of self-similar scenario of damage-failure transition stimulated our experimental study of crack dynamics for the explanation of mechanisms of transition from the steady-state to the branching regime, fragmentation statistics and failure wave phenomenon [2]. The stress field in the area of crack tip in the preloaded (by external stress σ) PMMA plate and the diagram “crack velocity V versus applied stress σ” are presented in Fig.1 according to the data of high speed framing with the usage REMIX REM 10-8 camera (time lag between pictures $10\,\mu s$). Three characteristic regimes of crack dynamics were established in the different ranges of crack velocity: steady-state $V < V_C$, branching $V > V_C$ and fragmenting $V > V_B$, when the multiply branches of main crack have the autonomous behavior (Fig.1, 2). Steady-state regime of crack dynamics is the consequence of the subjection of damage kinetics to the self-similar solution of the stress distribution at the crack tip (mechanically speaking to the stress intensity factor). Bifurcation point V_C ($V_C \approx 0.4 V_R$ where V_R is the Rayleigh wave speed) corresponds to the transition to the regime, when the “second attractor” (with the symmetry properties related to the number of the blow-up dissipative structures) disturbs the steady-state regime due to the excitation of numerous new failure hotspots (the daughter cracks having the image of mirror zones on the fracture surface). The change of the symmetry properties of nonlinear system were studied under the recording of dynamic stress signal (polarization of laser beam) at the front of propagating crack in the point deviated on 4 mm from the main crack path. The corresponding phase portraits $\dot{\sigma} \sim \sigma$ for steady-state and branching regimes of crack dynamics are presented in Fig. 3 and confirmed the existence of two “attractors”, which subject the crack dynamics. The first attractor is related to the intermediate asymptotic solution for the stress distribution at the crack tip. The second attractor has degrees of freedom corresponding to the set of blow-up dissipative structures of different complexity that can be responsible for different scenario of fragmentation statistics and failure under dynamic and shock wave loading.

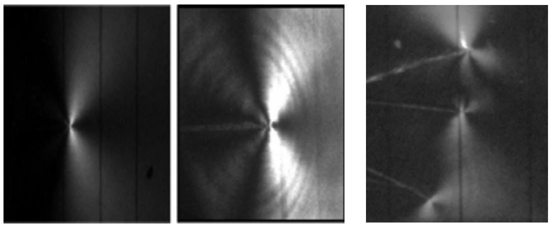


Figure 1: Stress pattern for steady-state $V < V_C$, branching $V > V_C$ and fragmenting $V > V_B$ scenario of crack dynamics.

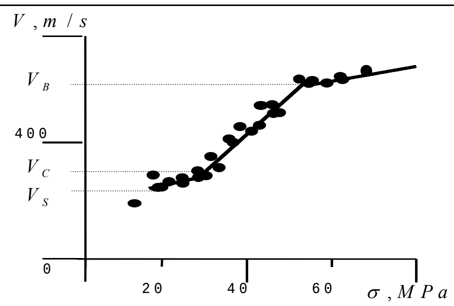
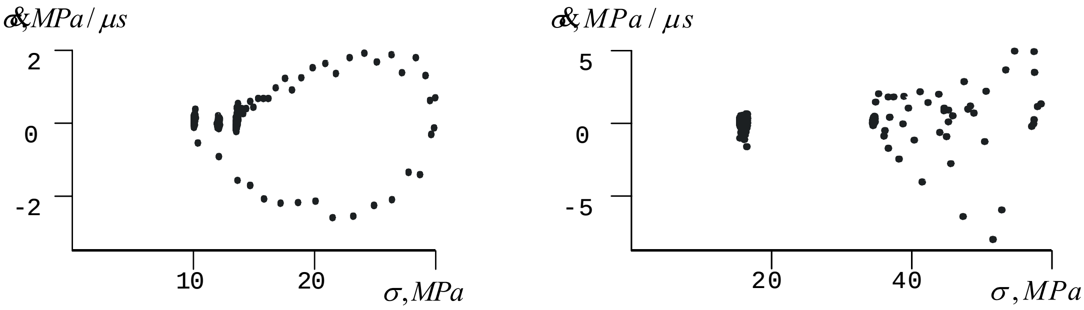


Figure 2: Crack velocity V versus stress σ.



a b

Figure 3: Stress phase portraits: $\dot{\sigma} \sim \sigma$: a - $V = 200\, m/s$, b - $V = 615\, m/s$

3. $1/f$ -fragmentation statistics

Fundamental properties of failure are central in determining the temporal scenario of fragmentation statistics and fragment size distribution. Fragment size distributions can range from the relatively tight exponential functions to power-law relations spanning a number of decades in fragment size. Onset of fracture asymptotes to a range of length scales in which fragmentation is self-similar requiring that failure temporal sequences and the fragment size distributions exhibit a power-law dependence [3]. The linkage of scenario of crack propagation and symmetry properties of dynamic system “solid with defects” allowed us to propose the interpretation of temporal and spatial fragmentation statistics depending on the energy density imposed. A large number of the fragmentation statistics were proposed: log-normal, power-law, exponential, combination of exponential and power laws. These theories have focused on the prediction of mean fragment size through energy and momentum balance principles, and on statistical issues of fragment size distribution. The energy density $E < E_C$ (E_C corresponds to the critical velocity V_C of the steady state – branching transition) provides the stress intensity controlled failure scenario. The transient densities $E_B > E > E_C$ $(V_C < V < V_B)$ lead to the exponential fragmentation statistics that is sensitive to both self-similar solutions: the self-similar stress distribution at the crack tip and collective blow-up modes of damage localization. "In-situ" fracture-luminescence sequences in dynamically loaded fused quartz rods and fragmentation statistics in recovered samples were analyzed. The power law of fracture luminescence sequences and the fragment size distribution was observed [4,5] under the increase of the energy density $E > E_B$ (Fig.4) when the multiscale damage localization occurs according to the set of blow-up dissipative structures that have the image of the stochastic "cloud" on the phase portrait (Fig. 3b). It was shown that the fragmentation time exceeds at least in two orders of magnitude the characteristic acoustic time on the sample length (100mm). The fracture luminescence signals represent the wave envelopes with the amplitudes revealing non-monotonic decay.

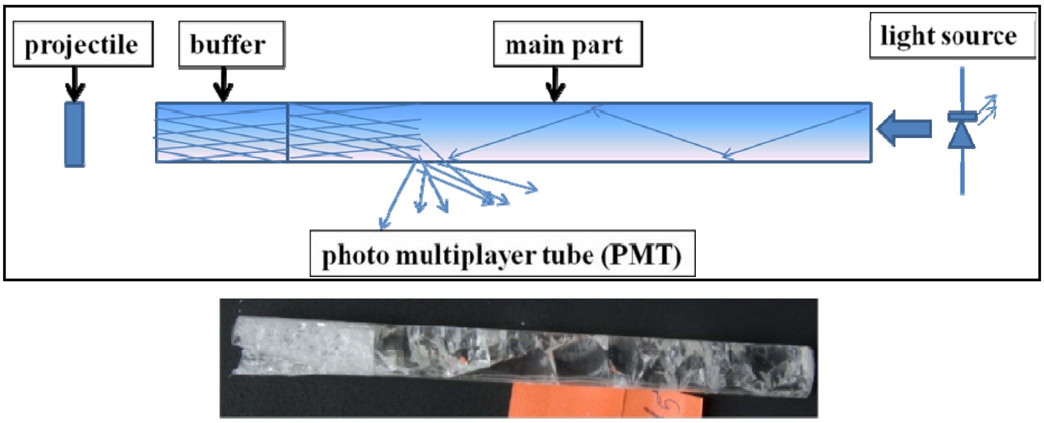


Figure 4. Fracture luminescence and fragmentation experiments

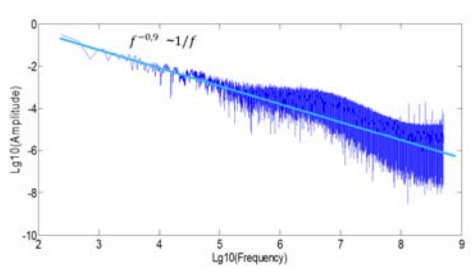


Figure 5. *1/f* fracture luminescence spectrum in dynamically loaded fused quartz rod

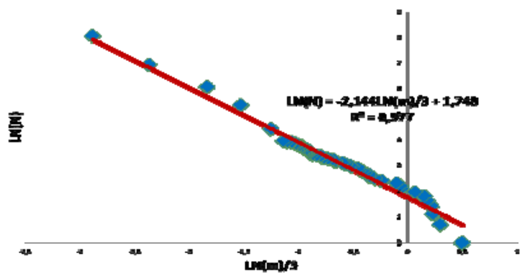


Figure 6. Fragment size cumulative distribution

Analysis of «in-situ» fracture luminescence sequences and fragmentation size statistics in dynamically loaded fused quartz established the power laws for the temporal (*1/f* spectrum) and spatial failure scenario that are characteristic for the multiscale collective modes (blow-up dissipative structures) associated with collective behavior of the set of damage localization hotspots.

4. Resonance excitation of damage localization

The solution (3) allowed us to link the self-similar features of damage localization kinetics and generation of blow-up dissipative structures. It was established the correspondence of failure hotspots nucleation having the image of mirror zones in experiments with numerous spall failure in shocked cylindrical rods of PMMA and ultraporcelain [5,6]. The multiple mirror zones with an equal size were excited on different spall cross sections in the shocked rod when the stress wave amplitude exceeded some critical value corresponding to the transition to the so-called “dynamic branch” under spalling (Fig. 7). Theoretically predicted low limit of damage localization scale L_C shows the existence of critical energy density, which provides the limit size of fragmented structure close to L_C and the degeneration of the power low statistics into the mono-disperse distribution.

Such fragmentation dynamics can be linked to the failure wave phenomenon [8]. The important feature of failure wave phenomenon is that the velocity of failure wave doesn’t depend on the velocity of propagation of the single crack. The stored elastic energy in material is the main factor, which provides the ability of brittle solid to the generation of failure wave. The failure waves represent the specific dissipative structures (the "blow-up" dissipative structures) in the microshear ensemble that could be excited due to the shock wave pass [9]. Experimental study of failure wave generation and propagation was realized for the symmetric Taylor test on fused-quartz rods [10]. Fig. 8a shows the processing of a high-speed photography (upper picture) for the flyer rod traveling at 534 m/s at impact.

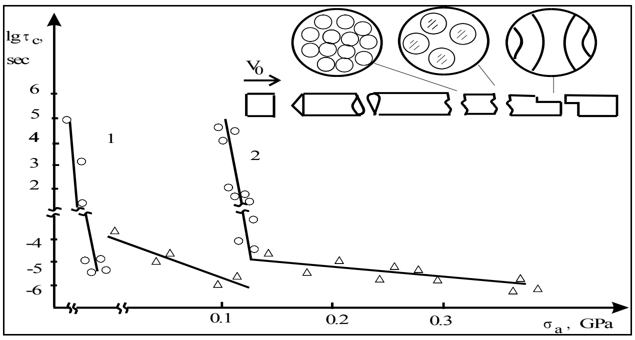


Figure 7: Fracture time t_c for shocked rod of PMMA (1) and ultraporcelain (2) versus stress amplitude σ_a.

Insert: surface pattern with mirror zones in different spall cross sections [6].

Three dark zones correspond to the image of impact surface (green triangle), failure wave (red square) and (blue diamond) the shock wave. The initial slope for the failure wave gives the front velocity $V_{fw} \approx 1.57\, km/s$ that is close to traditionally measured in the plate impact test [10]. However, the experiment revealed the increase of failure front velocity up to the value

$V_{fw} \approx 4\,km/s$. Approaching of failure wave front velocity to the shock front velocity supports theoretically based result concerning the failure wave nature as "delayed failure" with the limit of "delay time" corresponding to the "peak time" in the self-similar solution [10].

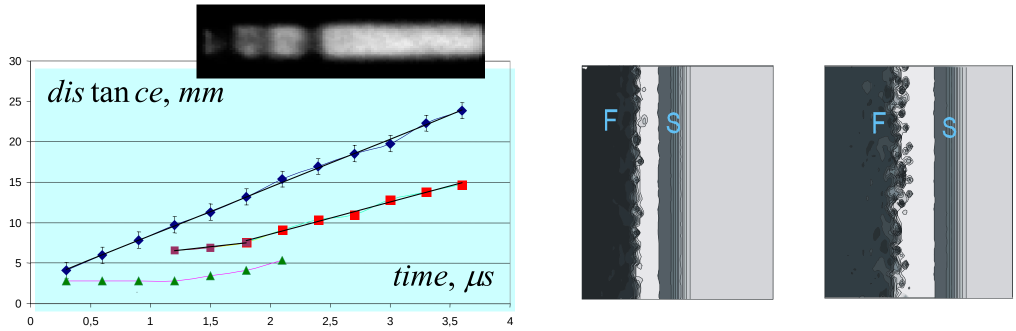


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Figure 8. a – The Taylor test data; b - Simulation of shock wave (S) and failure wave (F) propagation for different time [10].

Numerical simulation (Fig.8b) of damage kinetics describes the self-similar "blow-up" dynamics of damage-failure transition, supports the assumption concerning the failure wave mechanism as delayed failure with the delay time of the development of "peak regime" of "blow-up" dissipative structure. Time of the delay τ_D represents generally the sum of the induction time τ_I, that is the time of the formation of damage spatial distribution close to the self-similar profile, and the "peak time" τ_C, that is the time of "blow-up" damage kinetics. Steady-state regime of failure wave front propagation can be linked to the successive „resonance" activation of "blow-up" dissipative structures with characteristic "delay time" that is close to the "peak time" τ_C.

5. Scaling transitions and fatigue crack kinetics

Damage kinetics according to Eq. 2 reveals the specific system behavior in the ranges of scaling parameter $\delta_C < \delta < \delta_*$ and $\delta < \delta_C$ when the defect density tensor p_{ik}, influences on the correlation properties of defects on different spatial scales. The existence of two ranges of δ characterizes qualitative difference in relaxation mechanisms, which provide different dissipation ways: the orientation ordering and the formation of PSBs structures and the blow-up dissipative structures as the final stage of damage localization. The scenario of defect evolution in the range $\delta_C < \delta < \delta_*$ leads to the anomaly of the relaxation properties and, as the consequence, the energy absorbing. The high level of the structural relaxation time in the orientation metastability area in respect of the loading time $\tau_l \approx (\dot{\varepsilon})^{-1}$ coupled with the self-similar features of structure rearrangement in the scaling transition regime explains the anomaly of the energy absorbing under cycle load starting from some characteristic level of strain. This stain corresponds to the saturation stress σ_S providing the start of the scaling transition and the saturation effect as the anomaly of energy absorbing. The microscopic mode of fatigue crack growth is strongly affected by the multiscale slip arrangement, applied stress level and the extent of near tip plasticity. In ductile solids, cyclic crack growth is observed as a process of intense localized deformation in slip bands near the crack tip which leads to the creating of new crack surfaces by shear decohesion. The important feature of cyclic loading conditions, when the onset of crack growth from pre-existing defects can occur, at stress intensity values that are well below the quasi-static fracture toughness. This observation was used as a physical basis for the Paris model, when small scale yielding assumption allowed the formulation of the crack kinetics as

$$\frac{da}{dN} = C\,\Delta K^m, \tag{4}$$

in the term of the stress intensity factor range defined as $\Delta K = K_{\max} - K_{\min}$, where $K_{\max}$ and $K_{\min}$ respectively are the maximum and minimum stress intensity factors, C and m are empirical constants which a functions of material properties and microstructure. This formula predicted the Paris exponent of $m \approx 4$ in agreement with experiments for most metals [11]. Since the crack growth kinetics is linked to the temporal ability of material to the energy absorbing at the crack tip area the understanding of the saturation nature can be the key factor for the explanation of the 4th power universality. It was shown that the saturation nature can be considered as a consequence of the anomaly energy absorbing in the course of structural-scaling transition in dislocation system with the creation of PSBs and long-range interaction of dislocation substructures. Taking into account that the p - kinetics is given by the 4th order difference in the power in the metastability area the qualitative dependence of the damage rate reads $\dot{p} \sim A\sigma_S^4$, where σ_S is the saturation stress in the metastability area for $\delta_C < \delta < \delta_*$. This result supports the phenomenological law proposed by Paris [11] for the HCF crack growth kinetics. It is interested to note that this channel is very powerful in the sense of the energy absorbing. For instance, similar to the Paris law, the 4th power law $\dot{\varepsilon} \approx A\sigma_A^4$ of the linkage of plastic strain rate $\dot{\varepsilon}_p$ on the stress amplitude σ_A was established at the steady-state plastic wave front for several materials experienced shock loading [9, 12].

6. Defect induced scaling and scaling laws for fatigue crack path

With the purpose of understanding of crack behavior near the fatigue crack threshold the replica technique was used to study the correlation between the scaling properties of defect induced roughness and scaling laws of fatigue crack. 3D New View high resolution data of defect induced roughness in the crack process zone (Fig.9a) under fatigue crack path revealed the existence of two characteristic scales: the scale of the process zone L_{Pz} and the correlation length l_{sc} that is the scale when the correlated behavior of defect induced roughness has started [13].

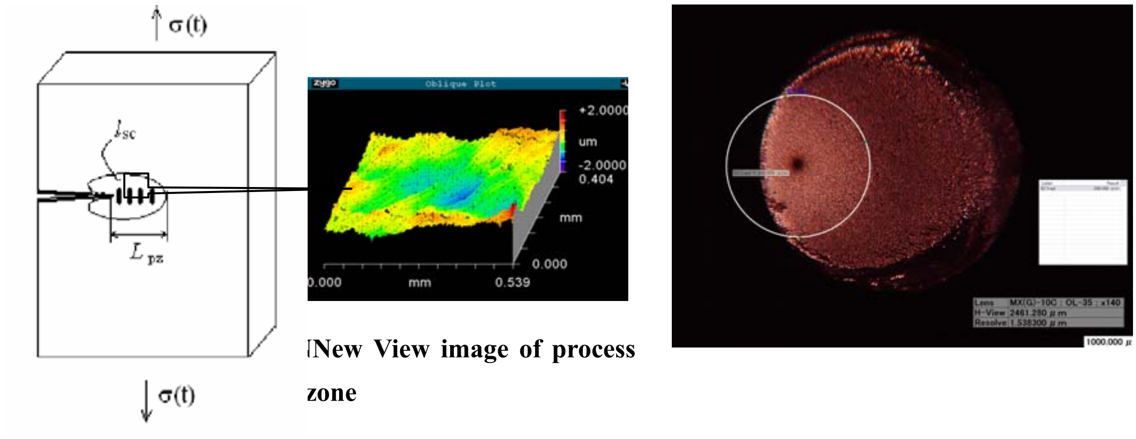

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Figure 9: a –roughness scaling in the process zone; b – "fish eye" area of damage localization under VHCF (fine grain titanium)

Assuming incomplete self-similarity in the variables $\Delta K / \left(E\sqrt{l_{sc}}\right) \to 0$, $l_{sc}/L_{Pz} \to 0$ and the definition of the effective stress intensity factor $\Delta K_{eff} = \Delta K \left(l_{sc}/L_{PZ}\right)^{\beta/\alpha}$ the kinetics of fatigue crack path reads

$$dl/dN = l_{sc}\left(\Delta K_{eff} / \left(E\sqrt{l_{sc}}\right)\right)^{\alpha}, \tag{5}$$

where the exponent α and β are the parameters related to the intermediate asymptotic dependence of crack rate on the stress intensity range. There is important case of the fatigue crack advance near-fatigue threshold when the scale of the process zone has the limit $L_{Pz} \to l_{sc}$, the effective stress intensity factor range ΔK_{eff} is approaching to the value ΔK and Eq.5 has the form $dl/dN = l_{sc}\left(\Delta K / \left(E\sqrt{l_{sc}}\right)\right)^{\alpha}$ This form is similar to the equation proposed by Hertzberg [14] for $l_{sc} \to b$, where b is the Burgers vector. Generally, in the limit of small scales $l_{sc} \approx b$ the application of stress intensity factor conception is problematic and numerous multiscale mesodefects interaction and corresponding scaling laws must be introduced [15]. This situation is characteristic for the fatigue of fine grain materials and especially for Very High Cycle Fatigue, when the stage of crack nucleation is dominated in the life time. The image of fracture surface of fine grain titanium (grain size ~300nm) in the condition of VHCF (number of cycles ~10^{10}) is shown in Fig.9b and revealed the "fish eye" image and the large "process zone" occupying fracture surface area in the bulk of specimen.

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