FRACTURE TOUGHNESS CRITERIA OF SMALL-SIZE SPECIMENS WITH ULTRAFINE-GRAINED STRUCTURE

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Abstract A new method for determining crack toughness of materials is described based on test data of small-size chevron-notched specimens in terms of commercial titanium VT1-0 and titanium alloy VT6 with ultrafine-grained (UFG) structure, obtained by methods of severe plastic deformation (SPD). A problem of separating a part, connected with variations in specimen ductility under crack propagation, of the total displacement of load application point, is solved. Equations to calculate specific fracture energy are obtained. The calculated values of stress intensity factor K_{Ic} are in good agreement with known test data of standard specimens.

Keywords ultrafine-grained (UFG) structure, strain localization, fracture specific fracture energy

1. Introduction

Standard crack toughness tests of materials are generally conducted using the bulk specimens not less than 10 mm in thickness. Although in many cases it is more convenient to use specimens of essentially smaller thickness for this purpose. These specimens do not require a large amount of material and high-power testing machines. Due to this, there is an issue in assessing crack toughness of ultrafine-grained (UFG) and nanostructured materials. The production of these materials in bulk specimens is connected with a series of technical problems. When testing the fracture toughness (crack toughness) of small-size specimens, the chevron-notched specimens are generally used [1-5]. Moreover, specimens with configuration of this type are not required to be fatigue pre-cracked.

In the given paper, a new method for determining crack toughness of materials is described based on test data of small-size chevron-notched specimens in terms of commercial titanium VT1-0 and titanium alloy VT6 with ultrafine-grained (UFG) structure, obtained by methods of severe plastic deformation (SPD).

In course of this study, important computational works, connected with using of chevron-notched specimens, were performed:

- Calculation of the Young 's modulus of the material *E*;
- Definition of specific fracture energy (crack-driving force [6]) under crack propagation G_{s} .

2. Determination of Young's modulus when testing chevron-notched specimens

The data on Young's modulus value for the materials with UFG structure is limited. It is known that SPD strongly affects the E value [7, 8]. A method of Young's modulus determination by test data of the chevron-notched specimens is described below.

A scheme of the chevron-notched specimen is presented in Fig. 1. This configuration of the specimen can be considered as a double-cantilever design.

A single cantilever was presented as a beam of elementary cantilevers (mini-cantilevers) of infinitesimal thickness dx. As seen from Fig. 1, length of the elementary cantilever at a distance of x from the specimen axis is equal to $l(x) = l_0 + x \cdot \operatorname{ctg}(\alpha/2)$, where l_0 is the minimum distance from the load application point to the chevron notch boundary, α is the angle at the end of the chevron notch (Fig. 1). The known formula from the elasticity theory is valid for each cantilever in the beam [9-11]:



Fig. 1. Scheme of the chevron-notched specimen.

where *b* is the cantilever thickness, *dP* is the load that provides cantilever deflection in width of *dx* by the value of λ' . Displacement of load application points λ for a double-cantilever design exceeds λ' twice, i.e. $\lambda = 2\lambda'$. Accordingly, from the equation (1) we derive the dependence of elementary load *dP*, applied to the mini-cantilever's end on the variable *x*:

$$dP(x) = \frac{E\lambda}{8} \left(\frac{b}{l(x)}\right)^3 dx.$$
 (2)

Integration of elementary forces (2) affecting each mini-cantilever along the full width of the specimen *a*, determines the actual load *P*, to which the displacement of load application points by the value of λ corresponds:

$$P = \frac{E\lambda b^3}{8} \int_{-\frac{a}{2}}^{\frac{a}{2}} \left(l_0 + x \cdot \operatorname{ctg}\left(\frac{\alpha}{2}\right) \right)^{-3} dx = \frac{E\lambda a}{4} \left(\frac{b}{l_0}\right)^3 \left[4 + \frac{a}{l_0} \operatorname{ctg}\left(\frac{\alpha}{2}\right) \right] \left[2 + \frac{a}{l_0} \operatorname{ctg}\left(\frac{\alpha}{2}\right) \right]^{-2},$$

where a is the specimen width (Fig. 1).

Hence we derive the working formula to determine the Young's modulus:

$$E = \frac{8M}{a} \left(\frac{l_0}{b}\right)^3 \left[2 + \frac{a}{l_0} \operatorname{ctg}\left(\frac{\alpha}{2}\right)\right]^2 \left[4 + \frac{a}{l_0} \operatorname{ctg}\left(\frac{\alpha}{2}\right)\right]^{-1}.$$
(3)

The value of $M = P/\lambda$ characterizes the specimen rigidity at the initial stage of elastic loading. According to the formula (3), the Young's modulus calculations for commercial titanium VT1-0 and titanium alloy VT6 were conducted. For the VT6-alloy with coarse-grained (CG) structure (grain size *d* ranges from 7 to 10 μ m), the value of *E* equal to 110±8 GPa was obtained. For the same material with UFG structure, the value of *E* is equal to 114±8 GPa. These values agree with reference data [7]. The Young's moduli of commercial titanium VT1-0 in CG and UFG states appeared equal to 111±8 GPa and 113±8 GPa, respectively, that slightly differs from the value of *E* equal to 110 GPa for commercial titanium by reference data [12, 13]. Thus, the conducted calculations with use of experimental data have shown the following:

- equation (3) can be used for approximate Young's modulus calculation for materials by test data of the chevron-notched specimens;

- grain structure refinement by SPD methods does not lead to essential change in elastic behavior of the studied specimens.

3. Definition of specific fracture energy under crack propagation G_s

Energy approach is reasonable when determining condition of unstable crack propagation. The gist of energy fracture criterion can be defined as follows: crack growth takes place if system can release energy to start crack propagation at elementary distance dl. Energy necessary for crack growth appears entirely due to elastic strain energy occurring in bulk of the material under the action of external applied force.

Let us consider a double-cantilever beam specimen with a narrow straight-through notch (Fig. 2) to begin with. Distance from load application points to the notch boundary is a crack in length of l_0 . It was shown in the papers [6, 14] for this case that a necessary condition for crack propagation obeys the equation

$$G = \frac{P^2 d\eta}{dS},\tag{4}$$

where *P* is the load applied to the specimen, $dS = 2a \, dl$ is the doubled area swept by the crack when propagating to the short distance dl (Fig. 2), $\eta = \lambda_e/P$ is the specimen ductility (value reverse to rigidity $M = \lambda_e/P$). The value of *G* determines elastic energy release rate under crack propagation. Further we shall call the characteristics of *G* a specific fracture energy.



Fig. 2. Straight-through notched specimen.

According to [6], displacement of load application points λ_e for the specimen in width of *a* with crack length *l* is provided by load:

$$P = \frac{E\lambda_{\rm e}a}{8} \left(\frac{b}{l}\right)^3.$$
 (5)

Ductility of such specimen is equal to:

$$\eta = \frac{\lambda_{\rm e}}{P} = \frac{8}{Ea} \left(\frac{l}{b}\right)^{\rm s}.$$

Considering that dS is equal to 2a dl, we shall find the derivative $d\eta/dS$ in the equation (4):

$$\frac{d\eta}{dS} = \frac{d\eta}{2adl} = \frac{12l^2}{Ea^2b^3}.$$

Substituting the given expression into (4), we shall obtain:

$$G = P^{2} \frac{d\eta}{dS} = \frac{12P^{2}l^{2}}{Ea^{2}b^{3}}.$$
 (6)

Equation (6) determines specific fracture energy along the crack length l and external load value P, wherein the crack starts to propagate.

Substituting expression (5) into this equation, we shall derive an equation for G_s , which allow us to calculate specific fracture energy based on crack length *l* and the value of λ_e :

$$G = \frac{3E}{16} \frac{\lambda_{\rm e}^2 b^3}{l^4}.$$
 (7)

It is seen that in the given presentation the value of G does not depend on the specimen width a.

Let us apply these considerations to the chevron-notched specimen. Assume that in the process of loading of the given specimen, the material lost discontinuity in segment of Δl (Fig. 3). Crack front is presented as a straight line. It is easy to find from geometrical constructions that length of this line is equal to $x = 2\Delta l \cdot tg(\alpha/2)$. An equation (7) can be applied to the middle part of the specimen in width of x, wherein crack length 1 makes $l_0 + \Delta l$. Using the equation (7), a specific fracture energy G can be found, if increment Δl is known. Moreover, it is necessary to know displacement of force application point λ_e , caused by enhancement in specimen ductility when increasing the crack length by Δl .



Fig. 3. Determination of the specific fracture energy $G_{\rm s}$.

It should be noted that the experimentally measured value of λ (Fig. 3), in addition λ_e , includes the contribution due to plastic deformation of the material at the mouth of the crack, and in the volume of sample as a whole.

The value of λ_e can be determined if its dependence from the external force *P* is found. To determine *P*, let us present a specimen with a crack as a set of double-cantilever beams: with straight-through notch in width of *x* and with chevron notch in width of *a* - *x* (Fig. 4). We shall find forces *P*₁ and *P*₂ for each part, determining equal displacement λ_e of these force application points. Using equation (5), it is easy to derive expression for the force *P*₁ affecting the specimen in width of $x = 2\Delta l \cdot tg(\alpha/2)$, which provides displacement of load application points *P*₁ to the specified value of λ_e :

$$P_{1} = \frac{E\lambda_{e}\Delta l}{4} \operatorname{tg}\left(\frac{\alpha}{2}\right) \left(\frac{b}{l}\right)^{3}.$$
(8)

Fig. 4. Presentation of the double-cantilever beam specimen.

When applying equation (3), with regard for the width of the chevron-notched specimen equal to a-x, we shall derive an expression for the force P_2 , which provides displacement of load application points to the same value of λ_e :

$$P_{2} = \frac{E\lambda_{e}a}{8} \left[1 - \frac{2\Delta l}{a} \operatorname{tg}\left(\frac{\alpha}{2}\right) \right] \left(\frac{b}{l}\right)^{3} \frac{l}{l_{0}} \left[4 + \frac{a}{l_{0}} \operatorname{ctg}\left(\frac{\alpha}{2}\right) + \frac{2\Delta l}{l_{0}} \right] \left[2 + \frac{a}{l_{0}} \operatorname{ctg}\left(\frac{\alpha}{2}\right) \right]^{-2}, \quad (9)$$

where $l = l_0 + \Delta l$.

From equations (8) and (9), an expression for λe is determined:

$$\lambda_{\rm e} = \frac{8Pl^3}{Eab^3} \left[\frac{2\Delta l}{a} \operatorname{tg}\left(\frac{\alpha}{2}\right) + \left(1 - \frac{2\Delta l}{a} \operatorname{tg}\left(\frac{\alpha}{2}\right)\right) \frac{l}{l_0} \left[4 + \frac{a}{l_0} \operatorname{ctg}\left(\frac{\alpha}{2}\right) + \frac{2\Delta l}{l_0} \right] \left[2 + \frac{a}{l_0} \operatorname{ctg}\left(\frac{\alpha}{2}\right) \right]^{-2} \right]^{-1}, \quad (10)$$

where $P = P_1 + P_2$.

Equations (7) and (10) were used to calculate fracture energy determining the necessary condition for spontaneous crack propagation in studied materials.

Figure 5 presents typical loading diagrams for titanium alloy VT6 and commercial titanium VT1-0 with UFG structure obtained under testing of small-size chevron-notched specimens. Both diagrams correspond to loading rate $v = 2,0 \text{ }\mu\text{m/s}$. Specimens in length of 18 mm were made of bars in section of 6x6 mm².

Calculations have shown that the value of G_s is maximal at the peak of loading and therefore, can serve as a crack resistance criterion of studied materials at specified geometrical parameters and loading conditions of the specimen.

Stress intensity factor is generally used as a crack resistance criterion in engineering fracture mechanics for a cleavage crack:

$$K_{\rm Ic} = \sqrt{\frac{EG_{\rm s}}{1 - v^2}},\tag{11}$$





Fig. 5. Loading diagrams of the VT6-alloy (a) and commercial titanium VT1-0 (b) with UFG structure.

Values of crack resistance characteristics for the studied materials are presented in Tab. 1. It is seen that specific fracture energies of the VT6-alloy in CG and UFG states differ significantly.

Dimension of the G_s characteristics is energy per unit area. However, this value is not a surface energy of the material. The last one is several orders of magnitude smaller than fracture energy G_s . Thus, surface energy of titanium is equal to 1.7 J/m² [12], at the same time, energy fracture value of commercial titanium, according to our calculations, is $G_s = 27.82 \text{ kJ/m}^2$. Agreement in values of G_s and surface energy will be observed only in case of totally brittle fracture. A colossal difference is caused by plastic strain processes intensively developing in metals and alloys that lead to essential change in shape and strain-stress state locally at the crack tip.

Material	λ_p/λ_e	$G_{\rm s}$, kJ/m ²	K_{Ic} , MPa/m ^{1/2}	E, GPa
VT1-0 UFG	0.11	27.82	56.37	113
VT1-0 CG				111
VT6 UFG	0.11	31.48	63.2	114
VT6 CG	0.37	53.00	90.8	110

Table 1. Mechanical characteristics of commercial titanium VT1-0 and VT6

According to equation (10), the values of λ_e are equal to 1.07 and 1.914 mm for VT6 and VT1-0, respectively. These values appeared smaller than experimentally measured displacement values of load application points λ , i.e. λ is equal to 1.21 mm for VT6 and λ makes 2.12 mm for VT1-0. Difference between measured and calculated values is caused by additional contribution of λ_p plastic strain into displacement of load application points, i.e. λ_p is equal to 0.14 mm for VT6 and λ_p is equal

to 0.28 mm for VT1-0. The relation λ_p/λ_e , apparently, can serve as mechanical characteristics determining a relative contribution of the specimen plastic distortion into displacement caused by change in specimen ductility.

4. Conclusion

A new method for determining crack toughness of materials is described based on test data of small-size chevron-notched specimens in terms of commercial titanium VT1-0 and titanium alloy VT6 with ultrafine-grained (UFG) structure, obtained by methods of severe plastic deformation (SPD). A problem of separating a part, connected with variations in specimen ductility under crack propagation, of the total displacement of load application point, is solved. A series of important computational problems connected with testing of chevron-notched specimens is solved in the study. Analytical expressions are obtained to calculate the Young's modulus of the material and to determine specific fracture energy.

The calculated values of the Young's modulus E and stress intensity factor K_{Ic} agree with known test data of standard specimens made of commercial titanium VT1-0 and titanium alloy VT6.

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References

- 1. L.M. Barker, Theory for determining K_{Ic} from small, non-LEFM specimens, supported by experiments on aluminum. Int. J. of Fracture, 16, No. 6 (1979) 515-536.
- 2. C.T. Wang, R.M. Pillar, Short-rod elastic-plastic fracture toughness test using miniature specimens. J. Mater. Sci. 24 (1989) 2391-2400.
- 3. T.J. Grant, L. Weber, A. Mortensen, Plasticity in Chevron-notch fracture toughness testing. Engineering Fracture Mechanics, 67 (2000) 263-276.
- 4. B.A. Drozdovsky, T.V. Polischuk, V.P. Volkov, Chevron notch as a method of thickness decrease of the specimen at determining of K_{Ic}. Zavodskaya Laboratoriya, 6 (1987) 74-76.
- R.R. Rakhimkulov, Matching of fracture toughness values of K_{1c}, obtained in chevron-notched specimens and based on standard technique for the steel St3sp. Neftegazovoe Delo, 2 (2010) 1-10. <u>http://www.ogbus.ru/authors/Rakhimkulov/Rakhimkulov_1.pdf</u>
- 6. D. Broek, Fundamental concepts of fracture mechanics, Vysshaya shkola, Moskow, 1980.
- G.A. Salischev, R.M. Galeev, S.P. Malysheva et al., Change in elasticity modulus at annealing of submicrocrystalline titanium, The Physics of Metals and Metallography (FMM), V. 85. No. 3 (1998) 178-181.
- 8. V.I. Betekhtin, O.R. Kolobov, M.V. Narykova et al., Mechanical properties, density and defect structure of submicrocrystalline titanium VT1-0, obtained after severe plastic deformation at screw and lengthwise rolling. Journal of Technical Physics, 81, Iss. 11 (2011) 58-63.
- 9. M.D. Podscrebko, Resistance of materials, Vyseishaya shkola, Minsk, 2007.
- 10. S.P. Timoshenko, J. Gud'er. The theory of elasticity, Nauka, Moskow, 1975.
- 11. GOST 16483.9-73. Drevesina. Methods of elasticity modulus determination under static bending, Moskow, 1973.

- 12. I.S. Grigor'ev, B.Z. Meilikhova (Eds), Physical quantities, Handbook, Energoatomizdat, Moscow, 1991.
- 13. A.A. Il'in, B.A. Kolachev, I.S. Pol'kin, Titanium alloys. Composition, structure, properties: Handbook, VILS MATI, Moskow, 2009.
- 14. R.W. Hernzberg, Deformation and Fracture Mechanics of Engineering materials, John Wiley and Sons, Toronto, New York, 1983.