Size Effect Analysis of Recycled Concrete Fracture based on Micromechanics Using the Base Force Element Method

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Abstract In this paper, the base force element method (BFEM) on damage mechanics is used to analyze the fracture properties of recycled aggregate concrete (RAC) on meso-level. The recycled aggregate concrete is taken as five-phase composites consisting of natural coarse aggregate, new mortar, new interfacial transition zone(ITZ) , old mortar and old ITZ on meso-level. The random aggregate model is used to simulate the meso-structure of recycled aggregate concrete. The size effects of mechanics properties of RAC under uniaxial tension loading are simulated using the BFEM on damage mechanics. The simulation results agree with the test results. This analysis method is the new way for investigating fracture mechanism and numerical simulation of mechanical properties for recycled aggregate concrete.

Keywords Base force element method (BFEM), Micromechanics, Fracture property, Size effect, Recycled aggregate concrete (RAC)

1. Introduction

Concrete is considered as heterogeneous composites which mechanical performance is much related with the microstructure of material. The mechanical performance is usually obtained using experimental method. However, testing usually consumes a large amount of manpower and material resources, and test results are usually more discrete. In order to overcome this defect, the concept of numerical concrete was presented by Wittanmm et al (1984) based on micro-mechanics. Subsequently, some scholars did some creative works in this field, and made a number of models. Among them, the two important models are the lattice model and the random aggregate model. For example, Schlangen et al (1992, 1997) applied the lattice model to simulate the failure mechanism of concrete. Liu et al (1996) adopted the random aggregate model to simulate cracking process of concrete using FEM. Peng et al (2001) adopted the random aggregate model to simulate the mechanics properties of rolled compacted concrete on meso-level using FEM. Du et al (2008) simulated the failure mechanism of beam under impact loading and triangular cyclic loading by using displacement-controlled FEM, stress-strain curves and dynamic bending strengths of specimens.

Recycled concrete material which is used as a green building material has attracted more and more researchers with the shortage of resources and an increasing number of construction wastes. They have carried out series of experiments and some conclusions have been reached. An overview of study on recycled aggregate concrete has been given by Xiao et al (2012). However, because of the complexity of recycled coarse aggregates, conclusions made by different researchers are usually not very accordant, even opposite sometimes. To remove effects of experimental conditions, some numerical researches on meso-level was considered. For example, numerical simulation on stress-strain curve of recycled concrete was taken by Xiao et al (2009) with Lattice Model under uniaxial compression. A method on meso-mechanics analysis was proposed by Peng et al (2011) using FEM for recycled aggregate concrete based on random aggregate model. However, the Numerical researches on the damage mechanism for recycled concrete material have just begun.

In recent years, a new type of finite element method - the Base Force Element Method (BFEM) has been developed by Peng et al (2006-2012) based on the concept of the base forces by Gao (2003). In this paper, the BFEM on potential energy principle is used to analyze recycled aggregate concrete (RAC) on meso-level. The size effects of mechanics properties of recycled aggregate concrete in uniaxial tension test are simulated using the BFEM. The simulation result agrees with the test result. This research method is the new way for investigating fracture mechanism and numerical simulation of mechanics properties of recycled aggregate concrete.

2. Basic Equation

Consider a two-dimensional domain of solid medium, let $\boldsymbol{x}^{\alpha}(\alpha = 1,2)$ denote the Lagrangian coordinate system, where $\boldsymbol{P}$ and $\boldsymbol{Q}$ the position vectors of a material point before and after deformation, respectively. Two triads for original and current configurations can be defined as:

$$\boldsymbol{P}_{\alpha} = \frac{\partial \boldsymbol{P}}{\partial x^{\alpha}}, \qquad \boldsymbol{Q}_{\alpha} = \frac{\partial \boldsymbol{Q}}{\partial x^{\alpha}}, \tag{1}$$

Let $\boldsymbol{u}$ denotes the displacement of a point, then

$$\boldsymbol{u} = \boldsymbol{Q} - \boldsymbol{P} \tag{2}$$

The gradient of displacement $\boldsymbol{u}_{\alpha}$ can be written as:

$$\boldsymbol{u}_{\alpha} = \frac{\partial \boldsymbol{u}}{\partial x^{\alpha}} = \boldsymbol{Q}_{\alpha} - \boldsymbol{P}_{\alpha} \tag{3}$$

Then, the Green strain $\boldsymbol{\varepsilon}$ can be written as

$$\boldsymbol{\varepsilon} = \frac{1}{2}(\boldsymbol{u}_i \otimes \boldsymbol{P}^i + \boldsymbol{P}^i \otimes \boldsymbol{u}_i) \tag{4}$$

In order to describe the stress state at a point $\boldsymbol{Q}$, a parallelogram with the edges $\mathrm{d}x^1\boldsymbol{Q}_1, \mathrm{d}x^2\boldsymbol{Q}_2$ is shown in Figure 1. Define,

$$\boldsymbol{T}^{\alpha} = \frac{\mathrm{d}T^{\alpha}}{\mathrm{d}x^{\alpha+1}}, \qquad \mathrm{d}x^{\alpha} \to 0 \tag{5}$$

where 3=1 for indexes. Quantities $\boldsymbol{T}^{\alpha}(\alpha = 1,2)$ are called the base forces at point $\boldsymbol{Q}$ in the two-dimensional coordinate system x^{α}.

According to the definitions of various stress tensors, the relation between the base forces and various stress tensors can be given. The Cauchy stress is

$$\boldsymbol{\sigma} = \frac{1}{A_Q}\boldsymbol{T}^{\alpha} \otimes \boldsymbol{Q}_{\alpha}. \tag{6}$$

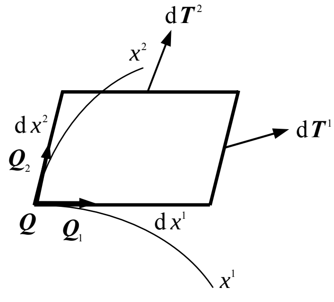


Figure 1. Base Forces on a plane element

Further, the base forces are given as follows

$$\boldsymbol{T}^{\alpha} = \rho A_Q \frac{\partial W}{\partial \boldsymbol{u}_{\alpha}} = \rho_0 A_P \frac{\partial W}{\partial \boldsymbol{u}_{\alpha}} \tag{7}$$

in which W is the strain energy density, ρ_0 is the mass density before deformation.

Equation (7) expresses the $\boldsymbol{T}^{\alpha}$ by strain energy directly. Thus, $\boldsymbol{u}_{\alpha}$ is just the conjugate variable of $\boldsymbol{T}^{\alpha}$. It can be seen that the mechanics problem can be completely established by means of $\boldsymbol{T}^{\alpha}$ and $\boldsymbol{u}_{\alpha}$.

3. Model of BFEM with Triangular Element

We will derive explicit expressions for stiffness matrices of a triangular element now, base on the concept of "base forces". Consider a triangular element with boundary S as shown in Figure 2.

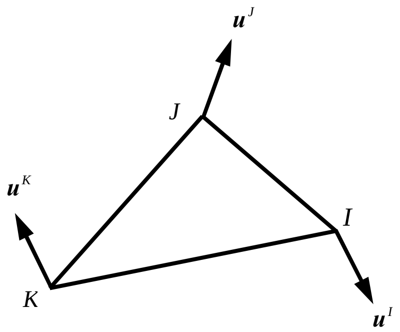


Figure 2. A triangular element

For the small displacement case, the real strain $\boldsymbol{\varepsilon}$ can be replaced by $\bar{\boldsymbol{\varepsilon}}$. We can obtain the average stress in element as

$$\bar{\boldsymbol{\varepsilon}} = \frac{1}{A}\int_{A} \boldsymbol{\varepsilon}\, \mathrm{d}\, A \tag{8}$$

in which A is the area of element.

Substituting Equation (4) into Equation (8), we have

$$\bar{\boldsymbol{\varepsilon}} = \frac{1}{2A}\int_{A} \left(\boldsymbol{u}_{\alpha} \otimes \boldsymbol{P}^{\alpha} + \boldsymbol{P}^{\alpha} \otimes \boldsymbol{u}_{\alpha}\right) \mathrm{d}\, A \tag{9}$$

Using Green' theorem, Equation (9) becomes

$$\bar{\boldsymbol{\varepsilon}} = \frac{1}{2A}\int_{S} \left(\boldsymbol{u} \otimes \boldsymbol{n} + \boldsymbol{n} \otimes \boldsymbol{u}\right) \mathrm{d}\, s \tag{10}$$

where $\boldsymbol{n}$ is the current normal of boundary S.

When the element is small enough, Equation (10) can be written as

$$\bar{\boldsymbol{\varepsilon}} = \frac{1}{2A}\sum_{i=1}^{3} L_i \left(\boldsymbol{u}_i \otimes \boldsymbol{n}_i + \boldsymbol{n}_i \otimes \boldsymbol{u}_i\right) \tag{11}$$

where L_i is the length of edge $i(i = 1,2,3)$, $\boldsymbol{n}_i$ denotes the external normal of edge $i(i = 1,2,3)$, $\boldsymbol{u}_i$ is the displacement of geometric center of edge $i(i = 1,2,3)$.

Further, we assume that any edge of the triangular element in the deformation process keeps its edges straight lines. Then, we can obtain the following expression for $\boldsymbol{u}_i$:

$$\boldsymbol{u}_i = \frac{1}{2}\left(\boldsymbol{u}_I + \boldsymbol{u}_J\right) \tag{12}$$

where $\boldsymbol{u}_I$ and $\boldsymbol{u}_J$ denote the displacements of both ends of edge $i(i = 1,2,3)$, respectively.

Substituting Equation (12) into Equation (11) yields

$$\bar{\boldsymbol{\varepsilon}} = \frac{1}{2A}\left(\boldsymbol{u}_I \otimes \boldsymbol{m}^I + \boldsymbol{m}^I \otimes \boldsymbol{u}_I\right) \tag{13}$$

The summation rule is implied in the above equation, and $\boldsymbol{m}^I$ is

$$\boldsymbol{m}^I = \frac{1}{2}\left(L_{IJ}\boldsymbol{n}^{IJ} + L_{IK}\boldsymbol{n}^{IK}\right) \tag{14}$$

where L_{IJ} and L_{IK} are the length of edges IJ and IK, $\boldsymbol{n}^{IJ}$ and $\boldsymbol{n}^{IK}$ denote the external normal of edges IJ and IK, respectively.

Then, for an isotropic material, the strain energy in the element is reduced to

$$W_D = \frac{AE}{2(1+\nu)}\left[\frac{\nu}{1-2\nu}\left(\bar{\boldsymbol{\varepsilon}} : \boldsymbol{U}\right)^2 + \bar{\boldsymbol{\varepsilon}} : \bar{\boldsymbol{\varepsilon}}\right] \tag{15}$$

in which E is Young's modulus, ν is Poisson's ratio.

Substituting (13) into (15) we have

$$W_D = \frac{E}{4A(1+\nu)}\left[\frac{2\nu}{1-2\nu}\left(\boldsymbol{u}_I \cdot \boldsymbol{m}^I\right)^2 + \left(\boldsymbol{u}_I \cdot \boldsymbol{u}_J\right)m^{IJ} + \left(\boldsymbol{u}_I \cdot \boldsymbol{m}^J\right)\left(\boldsymbol{u}_J \cdot \boldsymbol{m}^I\right)\right] \tag{16}$$

where

$$m^{IJ} = \boldsymbol{m}^I \cdot \boldsymbol{m}^J \tag{17}$$

From Equation (16), we can obtain the force acting on this element at node I

$$\boldsymbol{f}^I = \frac{\partial W_D}{\partial \boldsymbol{u}^I} = \boldsymbol{K}^{IJ} \cdot \boldsymbol{u}_J \tag{18}$$

where

$$\boldsymbol{K}^{IJ} = \frac{E}{2A(1+\nu)}\left[\frac{2\nu}{1-2\nu}\boldsymbol{m}^I \otimes \boldsymbol{m}^J + m^{IJ}\boldsymbol{U} + \boldsymbol{m}^J \otimes \boldsymbol{m}^I\right] \tag{19}$$

Here, $\boldsymbol{K}^{IJ}$ is a second-order tensor that is called the stiffness matrix.

The characteristics of the stiffness matrix $\boldsymbol{K}^{IJ}$ compared with the traditional FEM are as:

(1) This expression of stiffness matrix $\boldsymbol{K}^{IJ}$ can easily be extended to apply for arbitrary polygonal elements problem in two dimensions or arbitrary polyhedral element problem in three dimensions.

(2) The expression of the stiffness matrix $\boldsymbol{K}^{IJ}$ is a precise expression, and it is not necessary to introduce the Gauss' integral for calculating the stiffness coefficient at a point.

(3) This expression of $\boldsymbol{K}^{IJ}$ can be used for calculating the stiffness of various elements with a unified method.

(4) This expression of stiffness matrix $\boldsymbol{K}^{IJ}$ can be used in any coordinate system.

(5) The method of constructing the stiffness matrix does not regulate the introduction of interpolation.

The model of the base force element method will be used to analysis the damage problem for recycled aggregate concrete and be used to analysis the relationships of meso-structure and macroscopic mechanical performance of recycled concrete.

4. Random Aggregate Model for RAC

Based on the Fuller grading curve, Walraven J.C et al (1981) put the three dimensional grading curve into the probability of any point which located in the sectional plane of specimens, and its expression as follow:

$$P_c(D<D_0)=P_k\left(1.065\left(\frac{D_0}{D_{\max}}\right)^{\frac{1}{2}}-0.053\left(\frac{D_0}{D_{\max}}\right)^4-0.012\left(\frac{D_0}{D_{\max}}\right)^6\right.$$
$$\left.-0.0045\left(\frac{D_0}{D_{\max}}\right)^8-0.0025\left(\frac{D_0}{D_{\max}}\right)^{10}\right) \tag{20}$$

where P_k is the volume percentage of aggregate volume among the specimens, in general $P_k = 0.75$, D_0 is the diameter of sieve pore, $D_{\max}$ is the maximum aggregate size.

According to (20), the numbers of coarse aggregate particles with various sizes can be obtained. By Monte Carlo method, random to create the centroid coordinates of all kinds of coarse aggregate particles, namely to generate random aggregate model.

According to the projection method, we dissect the specimens of RAC with different phases of materials. Then, the phase of recycled coarse aggregate, the phase of new hardened cement, the phase of old hardened cement, and the phase of new and old interfacial transition zone (ITZ) can be judged by a computer code as Figure 3:

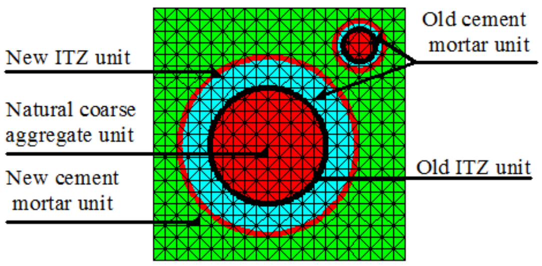


Figure 3. Attribute recognition figure

5. Damage Model of Materials

Components of RAC such as recycled coarse aggregate, new mortar, old mortar, new interfacial transition zone (New ITZ) and old interfacial transition zone (Old ITZ) are basically quasi-brittle material, whose failure patterns are mainly brittle failure.

In this paper, according to the characteristics of recycled concrete on meso-structure, the damage degradation of recycled concrete is described by the bilinear damage model, and the failure principal is the criterion of maximum tensile strain. Damage constitutive model is defined as $\widetilde{E}=E(1-D)$ as shown in Figure 4, where the damage factor D can be expressed as fallow:

$$D=\begin{cases}0 & \varepsilon<\varepsilon_0\\ 1-\dfrac{\eta-\lambda}{\eta-1}\dfrac{\varepsilon_0}{\varepsilon}+\dfrac{1-\lambda}{\eta-1} & \varepsilon_0<\varepsilon\le\varepsilon_r\\ 1-\lambda\dfrac{\varepsilon_0}{\varepsilon} & \varepsilon_r<\varepsilon\le\varepsilon_u\\ 1 & \varepsilon>\varepsilon_u\end{cases} \tag{21}$$

where f_t is the tensile strength of material, the residual tensile strength is defined as $f_{tr}=\lambda f_t$, the residual strength coefficient λ ranges from 0 to 1, the residual strain is $\varepsilon_r=\eta\varepsilon_0$, η is the residual strain coefficient, the ultimate strain is defined as $\varepsilon_u=\xi\varepsilon_0$, where ξ is ultimate strain coefficient, ε is principal tensile strain of element.

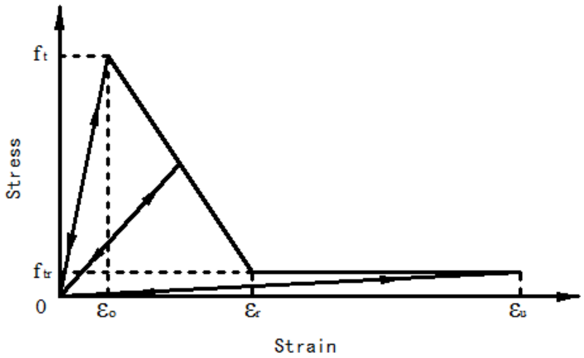


Figure 4. Bilinear damage model

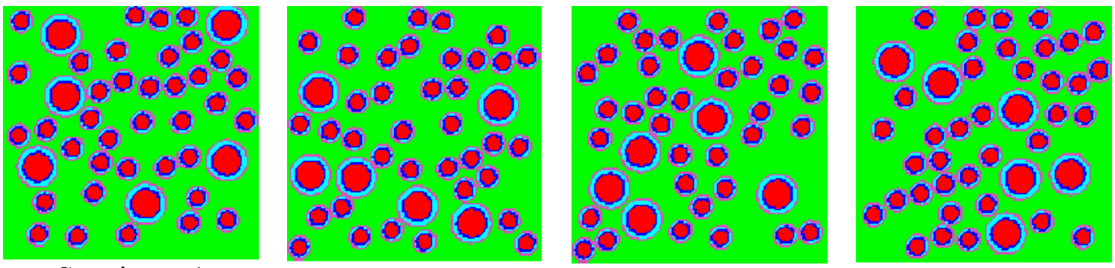
6. Numerical Example

According to the test results got from the experiment, material parameters of recycled aggregate are selected, which is completely coherent with model material used in the experiment. Material parameters of numerical simulation are shown in table 1. The recycled concrete specimens were loaded by displacement steps.

Table 1. Material parameters of uniaxial tensile tests

Materials	Elastic modulus/GPa	Position ratio	Tensile strength/MPa	λ	η	ξ
Natural coarse aggregate	50	0.16	10	0.1	5	10
Old ITZ	25	0.2	2	0.1	3	10
Old cement mortar	25	0.22	2.5	0.1	4	10
New ITZ	30	0.2	2	0.1	3	10
New cement mortar	30	0.22	3	0.1	4	10

For the size 100mm×100mm×100mm of tension specimen, the numbers of coarse aggregate particles can be obtained according to Equation (20). By Monte Carlo method, random to create the centroid coordinates of all kinds of coarse aggregate particles, namely to generate random aggregate model as Figure 5:



Specimen 1 Specimen 2 Specimen 3 Specimen 4

Figure 5. Random aggregate model with 100mm×100mm×100mm

The uniaxial tensile stress -strain curve of recycled concrete is getting as shown in Figure 6. The tensile strengths of the four specimens were 2.64Mpa, 2.69MPa, 2.67MPa and 2.69MPa. The uniaxial tensile strength average of the specimen group is 2.67MPa. The result BFEM on meso-damage analysis for RAC is consistent with the test results.

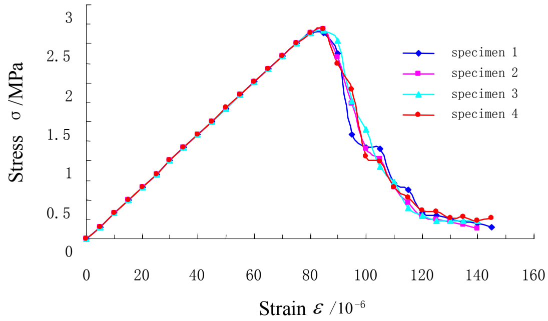
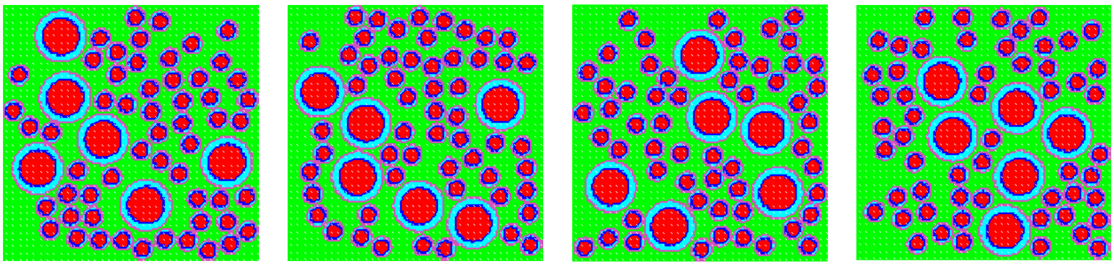


Figure 6. Stress-strain curve of RAC with 100mm×100mm×100mm

For the size 150mm×150mm×150mm of tension specimen, the numbers of coarse aggregate particles can be obtained according to Equation (20). By Monte Carlo method, random to create the centroid coordinates of all kinds of coarse aggregate particles, namely to generate random aggregate model as Figure 7:



Specimen 1 Specimen 2 Specimen 3 Specimen 4

Figure 7. Random aggregate model with 150mm×150mm×150mm

The uniaxial tensile stress -strain curve of recycled concrete is getting as shown in Figure 8. The tensile strengths of the four specimens were 2.51Mpa, 2.50MPa, 2.52MPa and 2.51MPa. The uniaxial tensile strength average of the specimen group is 2.51MPa.

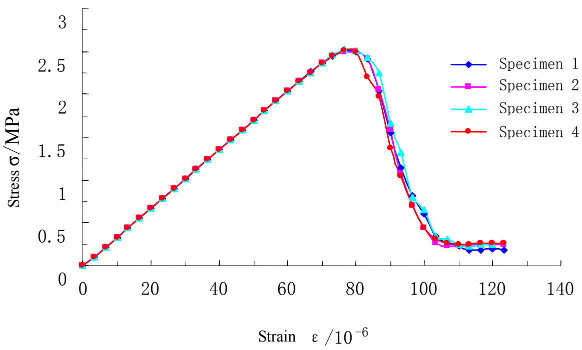


Figure 8. Stress-strain curve of RAC with 150mm×150mm×150mm

For the size 300mm×300mm×300mm of tension specimen, the numbers of coarse aggregate particles can be obtained according to Equation (20). By Monte Carlo method, random to create the centroid coordinates of all kinds of coarse aggregate particles, namely to generate random aggregate model as Figure 9:

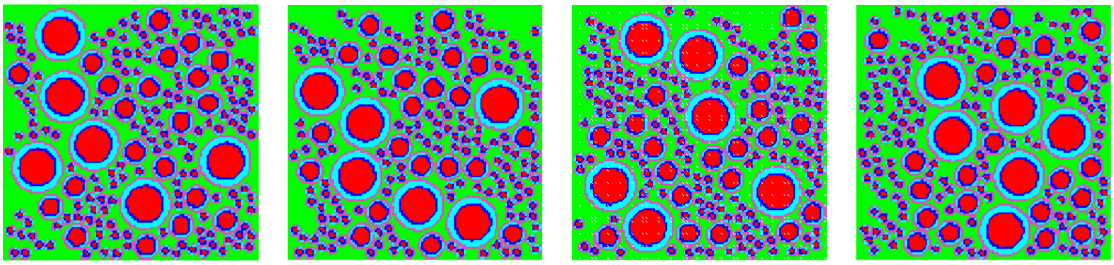


Figure 9. Random aggregate model with 300mm×300mm×300mm

The uniaxial tensile stress -strain curve of recycled concrete is getting as shown in Figure 10. The tensile strengths of the four specimens were 2.29Mpa, 2.30MPa, 2.31MPa and 2.28MPa. The uniaxial tensile strength average of the specimen group is 2.30MPa.

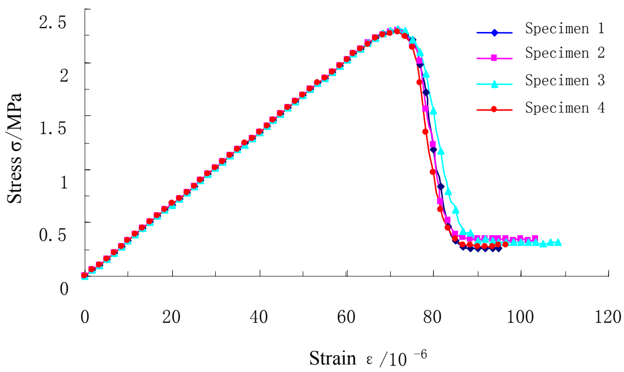


Figure 10. Stress-strain curve of RAC with 300mm×300mm×300mm

The size effects of mechanics properties of RAC under uniaxial tension loading are shown in table 2.

Table 2. Different sizes of recycled concrete compressive strength under uniaxial tension loading

Different sizes of recycled concrete	Tensile strength/MPa
100mm×100mm×100mm	2.67
150mm×150mm×150mm	2.51
300mm×300mm×300mm	2.30

7. Conclusions

(1) In this paper, a model of the base force element method (BFEM) is proposed for the damage analysis problem and is used to simulate the relations of meso-structure and macro-strength of recycled aggregate concrete (RAC). The characteristics of the BFEM are that the expression of the stiffness matrix $\boldsymbol{K}^{IJ}$ is a precise expression, and it is not necessary to introduce the Interpolation function and the Gauss' integral for calculating the stiffness coefficient at a point. The numerical results show that this method can be used for damage analysis of the RAC.

(2) In order to simulate the meso-structure of recycled concrete material, the recycled aggregate concrete is taken as five-phase composites consisting of natural coarse aggregate, new mortar, new interfacial transition zone (ITZ) , old mortar and old ITZ on meso-level in this paper. The random aggregate model is used for the numerical simulation of uniaxial tensile performance of recycled aggregate concrete. The results by the BFEM show that the uniaxial tensile strengths of specimens are approximately coincident with the experiment results, and the size effect of specimens is agree with the common rule.
(3) The numerical simulation provides a new way for research on mechanical properties of recycled aggregate concrete.

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