Mixed mode computation of the dynamic stress intensity factor for cracked 2D structures with inclusion using XFEM

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Abstract. This work deals with dynamic modelings of cracked structures containing inclusions using the eXtended Finite Element Method (X-FEM). The proposed theoretical developments focused on the case of the dynamic analysis, which constitutes the originality of the work. A computer code has been set up based on this theoretical framework. Several applications have been treated in order to demonstrate the effectiveness and robustness of the X-FEM based code for modeling cracked structures containing inclusions subjected to dynamic loadings.

Keywords: the eXtended Finite Element Method, Stress Intensity factor, inclusion, Dynamic Loads.

1. Introduction

In linear fracture mechanics, the Dynamic Stress Intensity Factor (DSIF) is used to characterize the cracking of fragile and quasi-fragile structures under dynamic loadings. In literature, we find many techniques to evaluate this parameter, among which we mention the finite element method (FEM) [1], the finite difference method (FDM) [2], the boundary element method (BEM) [3] and the symmetric-Galerkin boundary element method (SGBEM) [4]. We note that the FEM is the most popular for its flexibility and efficiency. However, it requires a special treatment of discontinuities and singularities of the unknown fields when cracks and inclusions that are present in the material are considered in the analysis as in the present study. To overcome such issues, a new FEM approach named eXtended Finite Element Method (XFEM) has been developed by Belytschko and Black [6] in 1999. It consists in taking into account the discontinuity at the crack edges and the singularity at the crack tip by enrichment of the neighboring nodes with new degrees of freedom, via the new shape functions, associated to elements containing those nodes. In 2004, a new enrichment function for inclusions has been proposed by Sukumar and Chopp [5]. Recently, J. M. PAIS (2010) used the eXtended Finite Element Method (XFEM) to study cracks and their propagation subjected to static and fatigue loadings [7]. In the authors knowledge, the earliest published papers treating the dynamic problems by using XFEM, but without inclusion, are due to Belytschko and Chen [8], Réthoré et al. [9], and Grégoire [10]. Other work has addressed this problem with a new approach such the one of A.V. Phan et al. [4] who used the symmetric-Galerkin boundary element method SGBEM.

In the present work, a modeling approach based on the XFEM method is proposed to describe the behavior of structures containing stationary cracks and inclusions, that are subjected to different types of dynamic loads (Heaviside step loading and triangular blast loading). Hence, the Dynamic Stress Intensity Factor (DSIF) will be evaluated by XFEM using the J integral. Moreover, the effect of the relative position of the inclusion with regard to the crack will be examined. For validation purpose, the results given by our approach will be compared to those of A.V. Phan et al. [4] obtained by using the SGBEM.

2. Presentation of the XFEM method

The XFEM introduces in the approximation of the displacement field three types of enrichments [6]:
- A discontinuous function H (Heaviside function) that enriches the split nodes (Fig. 1):

$$H(x) = \begin{cases} +1 & if \ \ \phi(x) \geq 0 \\ -1 & if \ \ \phi(x) \leq 0 \end{cases} . \tag{1}$$

Where ϕ is the level set function that determines the normal position of node *(x)* from the crack.
- Four singular functions for each tip node (Fig. 1):

$$F(x) = \sqrt{r}\,\{\sin(\theta/2),\ \sin(\theta/2)\sin(\theta),\ \cos(\theta/2),\cos(\theta/2)\sin(\theta)\}. \tag{2}$$

- One function associated with the interface nodes of the inclusion:

$$\upsilon(x) = \begin{cases} +1 & if \ \ \zeta(x) = 0 \\ 0 & if \ \ not \end{cases} \tag{3}$$

Where ζ is the level set function of the inclusion.

The approximate displacement fields are as follows:

$$u(x) \approx \sum N_i(x)u_i + \sum_{i \in I_H} N_i(x)H(x)a_i + \sum_{i \in I_{Br}} N_i(x)\left(\sum_{k=1}^{4} F_k(x)b_{i,k}\right) + \upsilon(\mathrm{x})\mathrm{a}_{\mathrm{e}}\,. \tag{4}$$

In addition to traditional unknowns u_i, we consider the unknowns a_i,b_k and a_e corresponding to the enrichment functions H, F_k and υ respectively.

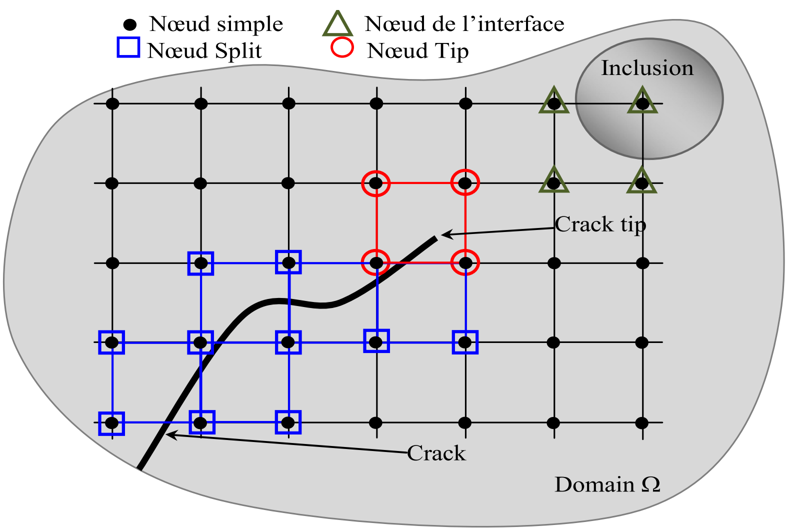


Fig. 1 Types of XFEM enrichments of the meshed domain.

3. Interaction integral method for DSIF computation

There are several methods to evaluate the DSIF. In this work, we use the method of the *J* integral by using the interaction integral (Fig.2). Because of its global character, it is the most stable technique.

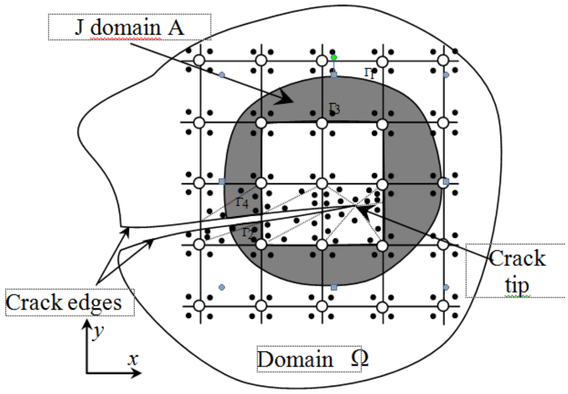


Fig. 2 Computing method of DSIF: interaction integral technique.

This method introduced by Sih et al [11], combines with the actual field an auxiliary field satisfying the boundary conditions of the problem. In this case, The J integral is given as follows:

$$J = J_{act} + J_{aux} + M. \tag{5}$$

Where J_{act}, J_{aux} are the J integrals in the actual and auxiliary fields, respectively, and M is the interaction integral that we are interested in, defined by :

$$M = \int_A \left[\sigma_{ij} \frac{\partial u_i^{aux}}{\partial x_1} + \sigma_{ij}^{aux} \frac{\partial u_i}{\partial x_1} - W^M \delta_{1i} \right] \frac{\partial q}{\partial x_i} d\Gamma = \frac{2}{E^{'}} \left(K_I K_I^{aux} + K_{II} K_{II}^{aux} \right). \tag{6}$$

With $W^M = \left(\sigma_{ij} \varepsilon_{ij}^{aux} + \sigma_{ij}^{aux} \varepsilon_{ij} \right) / 2$ is the strain energy of interaction and $E^{'} = E$ in plane stress and $E^{'} = E/(1-\nu^2)$ in plane strain. Therefore, the stress intensity factors in mode I and II take the form:

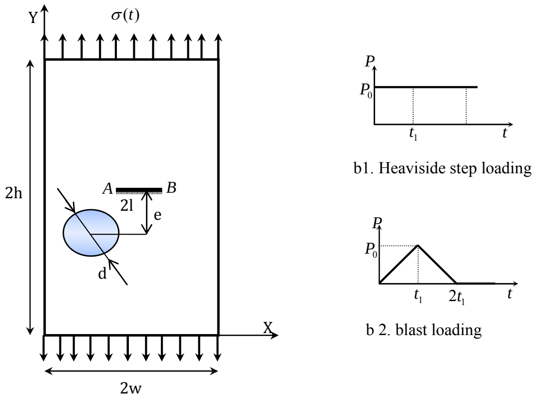
$$K = \frac{E^{'}}{2} M. \tag{7}$$

We take $K_I^{aux} = 1, K_{II}^{aux} = 0$ in mode I and $K_I^{aux} = 0, K_{II}^{aux} = 1$ in mode II. The computing procedure of M is based on the Gauss technique, the integration points are within the elements describing the area A of the J domain (see Fig 2).

4. Applications

We consider a plate of size 2w × 2h = 30mm × 40 mm containing an internal crack of initial length 2l = 4.8 mm and an inclusion of diameter d = 4 mm as shown in Fig. 3. The plate is subjected to a uniaxial Heaviside step tension loading σ(t) or a triangular blast loading with t1 = 2 μs and t2 = 8 μs. The inclusion is eccentrically positioned relatively to the crack center as shown Fig 3. The material properties for the plate and the inclusion are respectively: E = 260 GPa and 640 GPa, υ = 0.08 and 0.01, and ρ = 3.220 kg/m3 and 3.515 kg/m3. The DSIFs evaluated at crack tip A, and normalized with respect to the SIF of a similar situation in infinite plate under a uniaxial tension σ_0 without inclusion. The normalized DSIFs for this problem are defined as:

$$\overline{K_I} = \frac{K_I}{\sigma_0 \sqrt{\pi a}} \qquad \overline{K_{II}} = \frac{K_{II}}{\sigma_0 \sqrt{\pi a}} \tag{8}$$



a) b)

Fig.3 The validation problem: a) Cracked plate with inclusion, b) Different types of loadings.

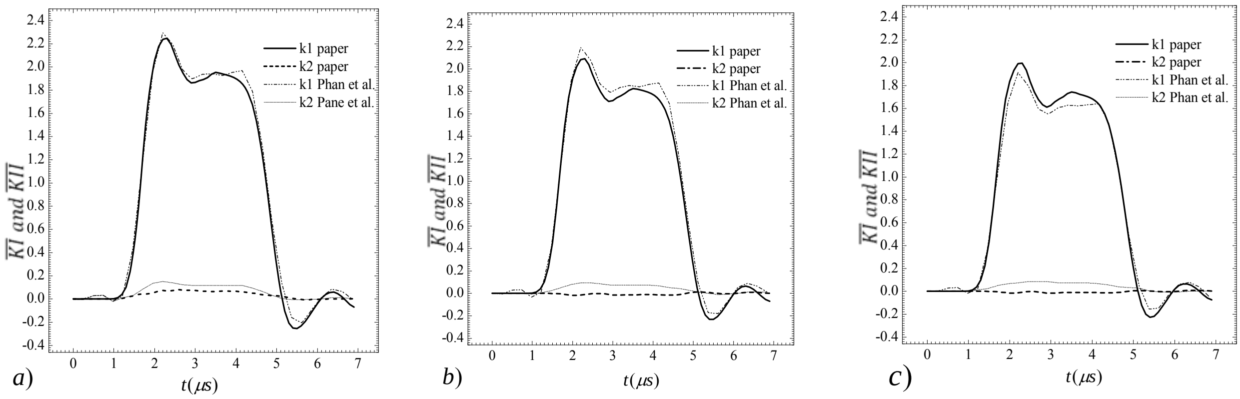


Fig.4 The plate under Heaviside step loading with different positions of the inclusion; (a) e= 3d/4 (b) e= d/2, (c) e= d/4.

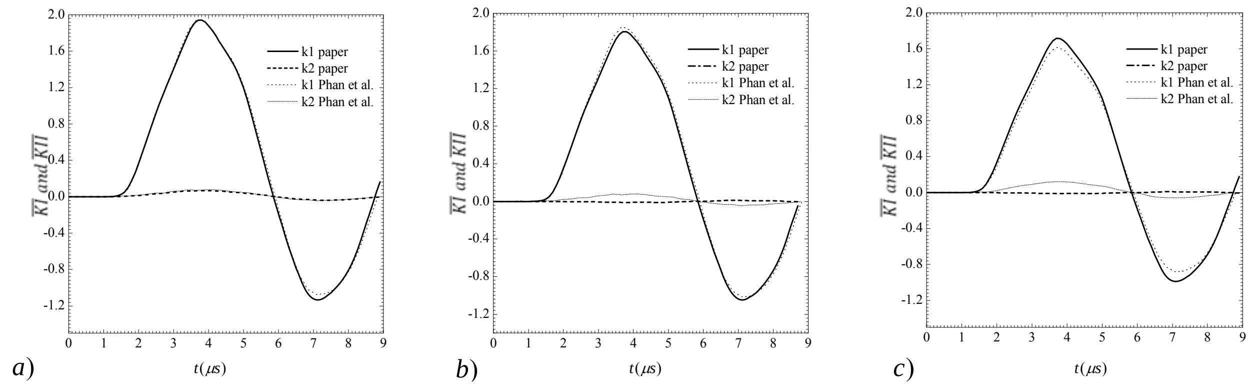


Fig.5 The plate under triangular blast loading with different positions of the inclusion; (a) e= *3d/4* (b) *e= d/2*, (c) *e= d/4*.

Figures 4 and 5 can be shown, there is an acceptable correlation between the obtained results and those of Phan et al. [4] using SGBEM, for different positions of the inclusion as well as for KI and KII.

We can note here that for DSIF KII, our results are closer to the exact solution (zero) compared to those obtained by Phan et al.[4].

Compared to the Heaviside step loading, the triangular blast loading increases more the negative value of KI and decreases more the positive peaks. This shows that the Heaviside step loading is more dangerous than the triangular blast loading. The quality of the obtained results demonstrates well the effectiveness of the proposed approach and the resulting computer code.

5. Conclusion

This study presents a computational procedure to evaluate the DSIF for stationary cracks in plate containing inclusions using the XFEM method. The agreement of the obtained results with those found in the literature for several treated configurations demonstrates the effectiveness and the robustness of the proposed procedure. As a perspective, this work will be extended to problems of multi inclusions, multi cracks, different form of the inclusion and dynamic crack propagation.

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