Modeling of cracked structures containing voids subjected to fatigue and dynamic loads using XFEM

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Abstract. In this paper, we present a modeling of planar structures under dynamic loading containing stationary cracks in order to determine the dynamic stress intensity factor (DSIF). This parameter will be evaluated by using the eXtended Finite Element Method (XFEM) coupled with the interaction integral technique. Some examples of validation of the computer code developed in this work were tested. The good correlation of the obtained results in fatigue with the literature proves the effectiveness of the method as well as the developed computer code. In the dynamic case, a parametric study on the presence, position and size of the void with respect to the crack and also on the crack type (crack edge and central crack) was conducted for some practical applications.
Keywords: Stress Intensity Factor, Extended Finite Element Method, Dynamic Loads, Fatigue, Void.

1. Introduction and stat of the art

In the cracking of fragile and quasi-fragile structures containing voids (holes) and subjected to quasi-static and dynamics loadings, the characterized parameter is the Stress Intensity Factor (SIF). Many techniques have been used in literature to evaluate this parameter. Among which we mention the finite element method FEM [1], the boundary element method BEM [2], the finite difference method FDM [3], and the symmetric-Galerkin boundary element method SGBEM [4]. We note that the FEM is the most popular for its flexibility and efficiency. However, it requires a special treatment of discontinuities and singularities of fields due to the presence of the crack. For this purpose, a new FEM approach has been developed by Belytschko and Black [5] named eXtended Finite Element Method (XFEM). It consists to take into account the discontinuity at the crack edges and the singularity at the crack tip by enrichment of neighboring nodes with new degrees of freedom via the new shape functions associated with elements containing these nodes. Among the first who addressed the problem of voids by using XFEM in static are Sukumar and Chopp [9], by introducing a new enrichments for voids. Recently, J M Pais [10] has treated voids problem using XFEM but limited on static and quasi-static loadings.

In this context, this work seeks to model the behavior of structures containing simultaneously voids and stationary cracks and subjected to different types of loads (fatigue loads and dynamic Heaviside step loading). The SIF will be evaluated using a global approach; based on the J integral. Also, in this work, we will test the effect of size and position of the void. The obtained results will be compared with other works in literature.

2. XEFM formulation

The XFEM introduces in the approximation of the displacement field three types of enrichments [5]:
-A discontinuous function H (Heaviside function) that enriches the split nodes (Fig. 1):

$$H(x) = \begin{cases} +1 & if \ \phi(x) \geq 0 \\ -1 & if \ \phi(x) \leq 0 \end{cases}. \quad (1)$$

Where ϕ is the level set function that determines the normal position of node *(x)* from the crack.
-Four (04) singular functions for each tip node (Fig. 1):

(2)

$$F(x) = \sqrt{r}\,\{\sin(\theta/2),\ \sin(\theta/2)\sin(\theta),\ \cos(\theta/2), \cos(\theta/2)\sin(\theta)\}.$$

-For void nodes, we add the following enrichment [9]:

$$V(x) = \begin{cases} 0 & if\ \ \chi(\mathrm{x}) < 0 \text{ (inside of the void)} \\ 1 & if\ \ \chi(x) > 0 \text{ (outside of the void)} \end{cases}. \tag{3}$$

Where χ is the level set function of voids
The approximate displacement fields are as follows:

$$u(x) \approx V(x)\left[\sum N_i(x)u_i + \sum_{i\in I_H} N_i(x)H(x)a_i + \sum_{i\in I_{Br}} N_i(x)\left(\sum_{k=1}^{4} F_k(x)b_{i,k}\right)\right] \tag{4}$$

In addition to traditional unknown u_i, we consider the unknowns a_i and b_k corresponding to the enrichment functions H et F_k, respectively.

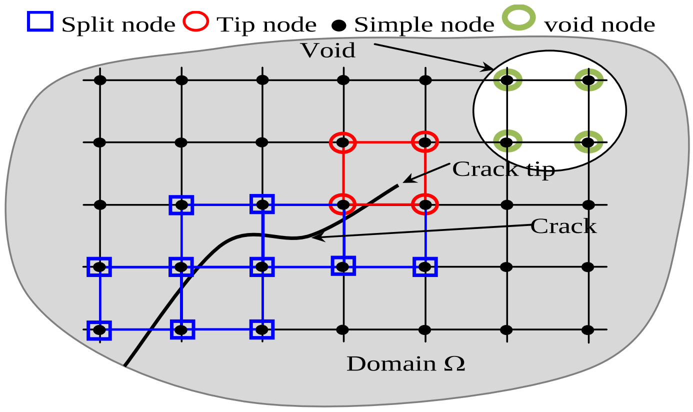

Fig. 1 Types of XFEM enrichments of the meshed domain.

3. Interaction integral method for SIF computation

There are several methods to evaluate the SIF. In this work we use the J integral method by using the interaction integral (Fig.2). Because its global character, this method is the most stable technique.

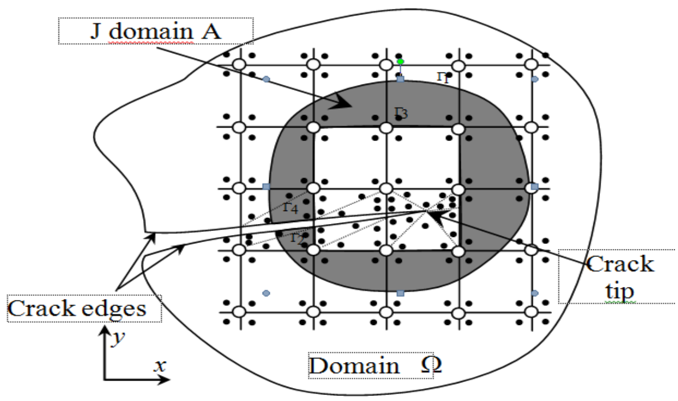

Fig.2 Method of SIF computing: interaction integral technique.

This method introduced by Sih et al [8], combines with the actual field an auxiliary field satisfying the boundary conditions of the problem. In this case, The J integral is given as follows:

$$J = J_{act} + J_{aux} + M. \tag{5}$$

Where J_{act}, J_{aux} are the J integrals in the actual and auxiliary fields, respectively, and M is the interaction integral that we are interested in, defined by :

$$M = \int_A \left[\sigma_{ij} \frac{\partial u_i^{aux}}{\partial x_1} + \sigma_{ij}^{aux} \frac{\partial u_i}{\partial x_1} - W^M \delta_{1i} \right] \frac{\partial q}{\partial x_i} d\Gamma = \frac{2}{E'} \left(K_I K_I^{aux} + K_{II} K_{II}^{aux} \right). \tag{6}$$

With $W^M = \left(\sigma_{ij} \varepsilon_{ij}^{aux} + \sigma_{ij}^{aux} \varepsilon_{ij} \right) / 2$ is the strain energy of interaction and $E' = E$ in plane stress and $E' = E / (1 - \nu^2)$ in plane strain. Therefore, the stress intensity factor in mode I and II take the form:

$$K = \frac{E'}{2} M. \tag{7}$$

We take $K_I^{aux} = 1, K_{II}^{aux} = 0$ in mode I and $K_I^{aux} = 0, K_{II}^{aux} = 1$ in mode II. The computing procedure of M is based on the Gauss points within the elements of J domain area A (see Fig 2).

4. Fatigue application

We validate the computer software carried out in this study and based on the above developments, in quasi-static (fatigue) loading, we consider a plate (Fig. 3) of size $2L \times 2l = 120mm \times 65mm$ with an edge crack of length $2a$, with $a = 10mm$, and 3 holes (one is of diameter 20 *mm* and the two others for the load action are both of 13 *mm*). The material properties are $E = 71.7 \times 10^9$ Pa, $\nu = 0.3$. The stress state is plane strain with a mesh of 60x120 elements. The plate is under uniaxial fatigue load with a variation of $\Delta p = 20KN$ with 12 increments of *da=3 mm*.

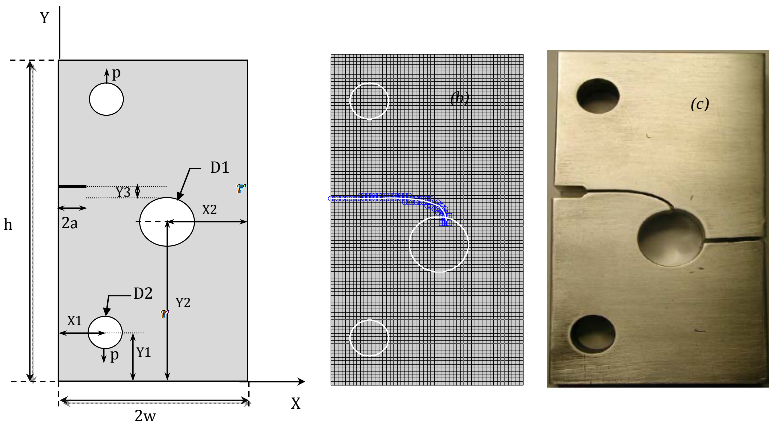

Fig.3 the considered validation: (a) Cracked plate containing voids,(b) results using XFEM, (c) experimental results [11]

In this case, the crack growth path is followed and compared with that obtained by Giner et al. [11], the results are regrouped in Fig 3. The obtained results as shown in Fig 3b are approximately close to the experimental ones Fig 3c proved so the accuracy of this approach.

5. Dynamic applications

For dynamic loads, we are limited to present our results only.

5.1. Plate with edge crack

We consider a reference problem of a plate (Fig. 5.a) of size $2L \times 2l = 400mm \times 300mm$ with an edge crack of length $2a$, with $a/l = 0.24$. The material properties are $E = 2.1 \times 10^{11}$ Pa, $\nu = 0.3$ and $\rho = 3220\text{Kg/m}^3$. With plane strain state and a mesh of 60x120 elements. The plate is under uniaxial dynamic tensile $\sigma_y(t, \sigma_0)$ of Heaviside step load (Fig. 4.b) with $\sigma_0 = 20 \times 10^6$ Pa ,

We're going to evaluate the no normalised SIF $K_{I\,ad}$ at the crack tip and the maximum of y component of normalised stress σ_{ad} on A point situated at the nearest node to the crack tip defined as:

$$K_{I\,ad} = K_I / (\sigma_0 \sqrt{\pi a}) \tag{8}$$

$$\sigma_{ad} = \sigma_{yy} / \sigma_0 \tag{9}$$

Curves on Fig 4.c were found with sliding the void horizontally with a step of (1/7)a. These Figures represent the variation of $K_{I\,ad}$ and σ_{ad} versus the relative position x/2w.

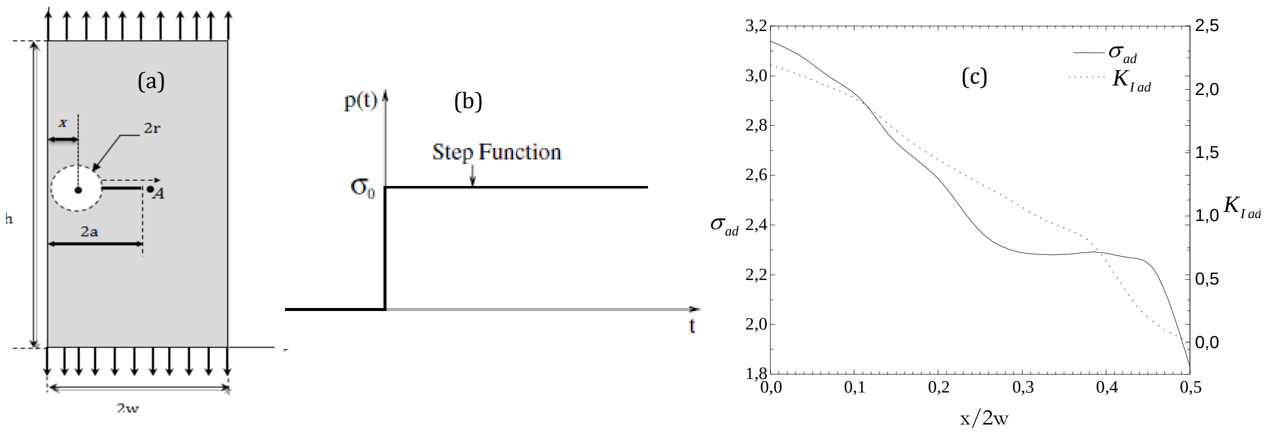

Fig.4 (a) considered geometries, (b). Dynamic load Heaviside (c) SIF and maximum stress

Fig 4c shows that both $K_{I\,ad}$ and σ_{ad} decreases with the distance of the hole to the crack tip; it is like the crack length decreases. The SIF continues to decline up then vanishes when the hole reached the crack tip. That is why holes at the crack tips are considered a very practical solution to stop their growth.

5.2. Plate with central crack

We reanalysis the precedent example but with a central crack crossed by a hole of a diameter varying from 2a/10 to 2a/1.1 as shown in Fig 4a.

In this example, we are going to evaluate the dimensionless SIF given in relation (8) for different diameters of hole to verify its role to increase the cracking plate resistance. Effectively, from the curves in Figure 5.b, the hole more and more bigger extinct more and more the SIF and therefore the risk of crack growth.

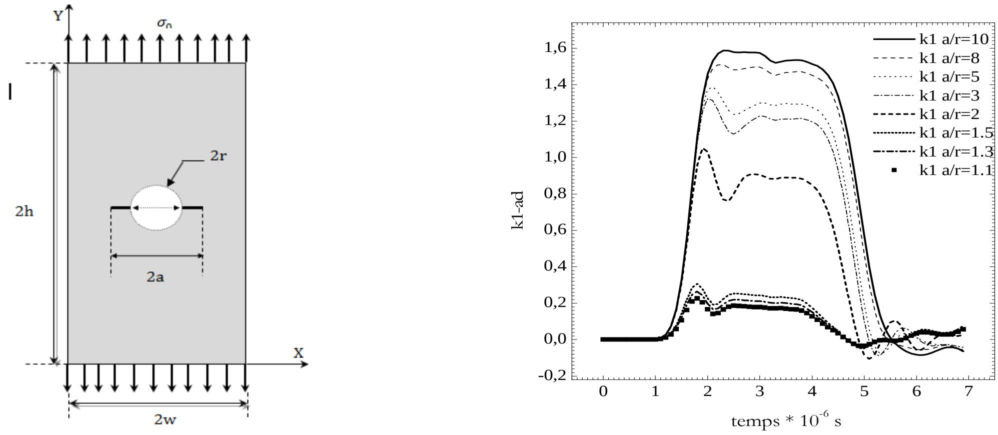

Fig.5 (a) considered geometries, (b). DSIF for different sizes of hole

6. Conclusion

This study presents a computational procedure to evaluate the SIF for cracked structures with void using XFEM. The correlation of the obtained results with the literature for the fatigue application demonstrates the effectiveness of this procedure. The obtained results of dynamic applications agree very well with the attended physical results which approve so the robustness of our approach. As perspectives of this study the present approach can be extended to problems of multi-voids and dynamic crack propagation.

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